ECE-205

Exam 3

Spring 2010

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 _________/20

Problem 2 _________/20

Problem 3 _________/20

Problem 4 _________/20

Problem 5 _________/20

Solutions

Total _____________

1) (20 points) For a system with transfer function

$$H(s) = \frac{3}{s^2 + 4s + 6}$$

determine the *unit step response* of the system:

Do not forget any necessary unit step functions.

$$Y(s) = \frac{3}{s[(s+2)^2+2]} = \frac{A}{s} + \frac{B\sqrt{2}}{(s+2)^2+2} + \frac{C(s+2)}{(s+2)^2+2} = \frac{3}{s(s^2+4s+6)}$$

$$A = \frac{3}{6} = \frac{1}{2}$$

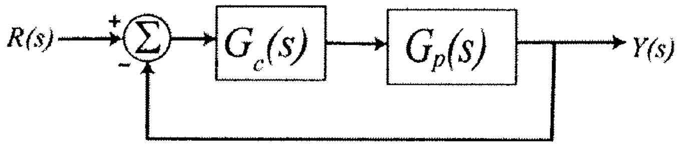
$\times s$, let $s \to \infty$ $\quad 0 = A + 0 + C \quad C = -\frac{1}{2}$

let $s = -2$ $\quad \frac{3}{-4} = \frac{-1}{4} + \frac{B\sqrt{2}}{2}$

$$\frac{-1}{2} = \frac{B}{\sqrt{2}} \qquad B = -\frac{\sqrt{2}}{2} = \frac{-1}{\sqrt{2}}$$

$$\boxed{y(t) = \left[\frac{1}{2} - \frac{1}{\sqrt{2}} e^{-2t} \sin(\sqrt{2}\,t) - \frac{1}{2} e^{-2t} \cos(\sqrt{2}\,t)\right] u(t)}$$

2) (20 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \frac{2}{s+4}$



a) Determine the settling time of the plant alone (assuming there is no feedback)

b) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$ and then

i) the settling time, in terms of k_p

ii) the steady state error for a unit step, in terms of k_p

c) For and integral controller, $G_c(s) = \frac{k_i}{s}$, determine the closed loop transfer function $G_0(s)$ and the steady state error for a unit step in terms of k_i

a) $T_s = \frac{4}{4} = \boxed{1\,\text{sec} = T_s}$

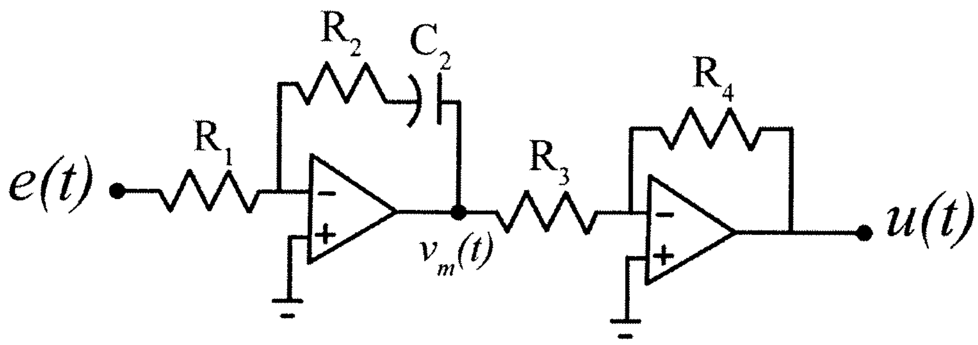
b) $G_0(s) = \frac{2k_p}{s+4+2k_p}$ $\quad T_s = \frac{4}{4+2k_p} = \boxed{\frac{2}{2+k_p} = T_s}$

$e_{ss} = 1 - \frac{2k_p}{4+2k_p} = \frac{4+2k_p-2k_p}{4+2k_p} = \frac{4}{4+2k_p} = \boxed{\frac{2}{2+k_p} = e_{ss}}$

c) $G_0(s) = \frac{2k_i}{s^2+4s+2k_i}$ $\quad e_{ss} = 1 - G_0(0) = 1 - 1 = \boxed{0 = e_{ss}}$

3) (20 points) Show that the following circuit can be used to implement the PI controller

$$G_c(s) = \frac{U(s)}{E(s)} = k_p + k_i \frac{1}{s}$$



Determine expression for both k_p and k_i in terms of the parameters R_1, R_2, R_3, R_4, C_2

$$\frac{E}{R_1} + \frac{V_m}{R_2 + \frac{1}{C_2 s}} = 0 \qquad\qquad \frac{V_m}{R_3} + \frac{U}{R_4} = 0$$

$$V_m = -\frac{E}{R_1}\left(R_2 + \frac{1}{C_2 s}\right)$$

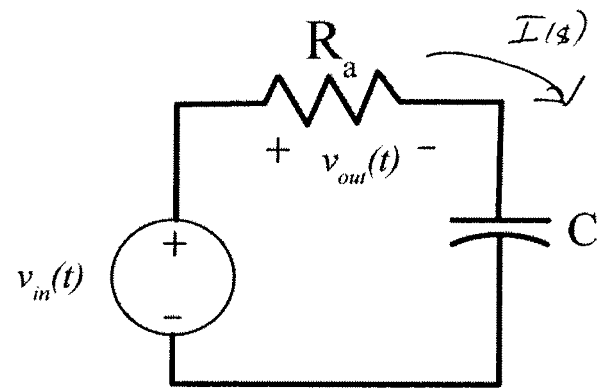
$$= -\frac{E}{R_1}\left(\frac{R_2 C_2 s + 1}{C_2 s}\right) \qquad\qquad U = -\frac{R_4}{R_3} V_m$$

$$U = \left(-\frac{R_4}{R_3}\right)\left(-\frac{E}{R_1}\right)\left(\frac{R_2 C_2 s + 1}{C_2 s}\right)$$

$$\frac{U}{E} = \frac{R_2 R_4 C_2}{R_1 R_3 C_2} + \frac{R_4}{R_1 R_3 C_2}\frac{1}{s}$$

$$\boxed{\frac{U}{E} = \underbrace{\left(\frac{R_2 R_4}{R_1 R_3}\right)}_{K_p} + \underbrace{\left(\frac{R_4}{R_1 R_3 C_2}\right)}_{K_i}\frac{1}{s}}$$

4) (20 points) For the following circuit determine the *transfer function* and the corresponding *impulse response*.



$$I(t) = \frac{V_{in}(s)}{R_a + \frac{1}{Cs}}$$

$$V_{out}(t) = R_a I(s) = \frac{R_a V_{in}(s)}{R_a + 1/Cs} = \frac{R_a Cs}{R_a Cs + 1} V_{in}(s)$$

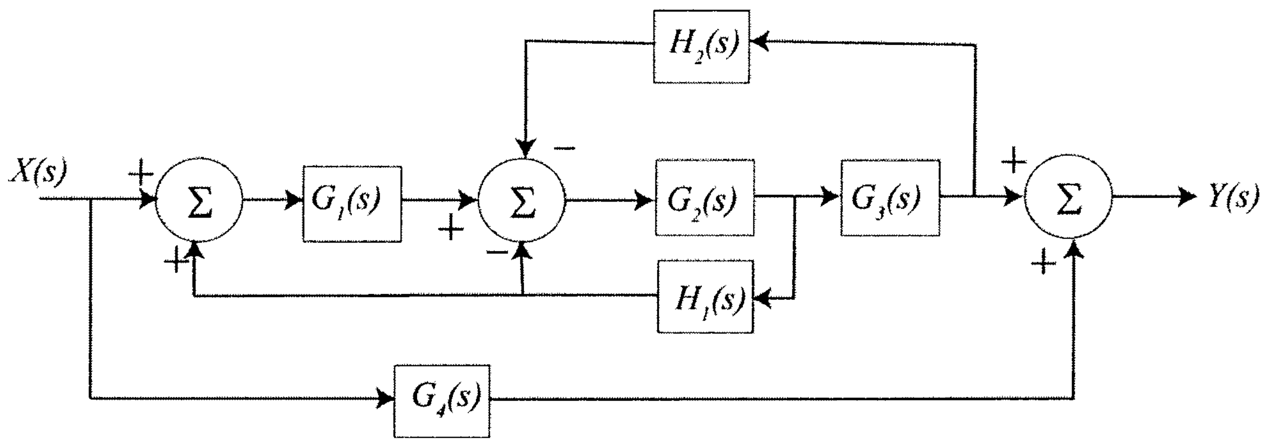
$$H(s) = \frac{V_{out}(s)}{V_{in}(t)} = \boxed{\frac{R_a C s}{R_a C s + 1} = H(s)}$$

$$\begin{array}{r|l} & \quad 1 \\ R_a C s + 1 & R_a C s \\ & R_a C s + 1 \\ \hline & \quad -1 \end{array}$$

$$\frac{R_a C s}{R_a C s + 1} = 1 - \frac{1}{R_a C s + 1} = 1 - \frac{1}{R_a C} \frac{1}{s + 1/R_a C} = H(s)$$

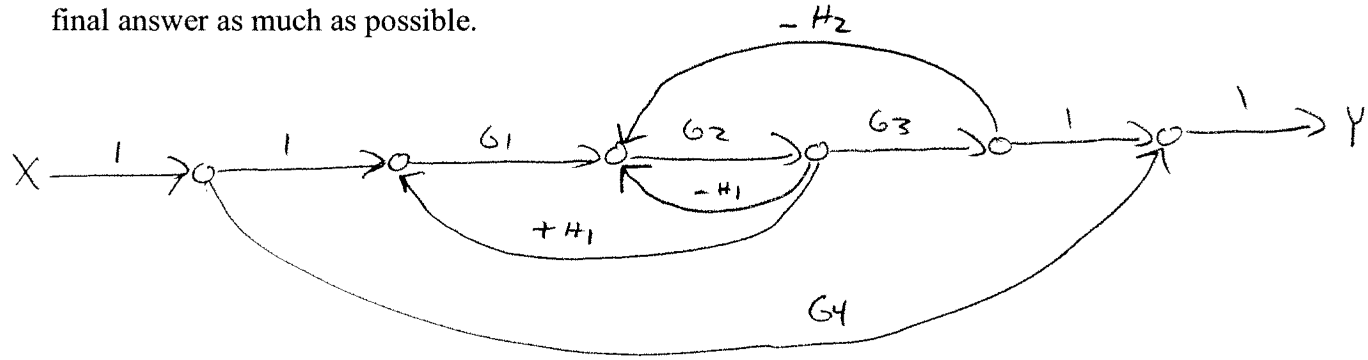
$$\boxed{h(t) = \delta(t) - \frac{1}{R_a C} e^{-t/R_a C} u(t)}$$

5) **(20 points)** For the following block diagram



a) Draw the corresponding signal flow graph, labeling each branch and direction

b) Determine the system transfer function using Mason's gain rule. You must simplify your final answer as much as possible.



$P_1 = G_1 G_2 G_3$ $\quad P_2 = G_4$ $\quad L_1 = -G_2 G_3 H_2$ $\quad L_2 = -H_1 G_2$ $\quad L_3 = G_1 G_2 H_1$

$\Delta = 1 - (L_1 + L_2 + L_3)$ $\quad \Delta_1 = 1$ $\quad \Delta_2 = \Delta$

$= 1 + G_2 G_3 H_2 + H_1 G_2 - G_1 G_2 H_1$

$$\frac{Y}{X} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \boxed{\frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 + H_1 G_2 - G_1 G_2 H_1} + G_4}$$