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ECE-205

Exam 1

Spring 2010

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1	**________/19**
Problem 2	**________/20**
Problem 3	**________/20**
Problem 4	**________/20**
Problem 5-11	**________/21**

Total ____________

90-100	10
80-89	17
70-79	10
60-69	9
<60	6

median = 75
average = 76

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1) (19 points) For a first order system described by the differential equation

$$\tau \dot{y}(t) + y(t) = Kx(t)$$

we can use integrating factors to determine the solution is

$$y(t) = y(t_0)e^{-(t-t_0)/\tau} + \int_{t_0}^{t} e^{-(t-\lambda)/\tau} \frac{K}{\tau} x(\lambda) d\lambda$$

(This equation is being given to you, do not derive it!)

Show that if the initial time is zero, $t_0 = 0$, and the input is a step of amplitude A, $x(t) = A$ for $t \geq 0$, then the above solution reduces to

$$y(t) = \left[y(0) - y(\infty)\right]e^{-t/\tau} + y(\infty)$$

Hint: (1) remove everything from the integral that is not a function of λ

(2) what is $y(\infty)$ equal to?

$$y(t) = y(0)e^{-t/\tau} + \frac{KA}{\tau}e^{-t/\tau}\int_0^t e^{\lambda/\tau} d\lambda \qquad KA = y(\infty)$$

$$\int_0^t e^{\lambda/\tau} d\lambda = \tau e^{\lambda/\tau}\Big|_0^t = \tau\left[e^{t/\tau} - 1\right]$$

$$y(t) = y(0)e^{-t/\tau} + y(\infty)e^{-t/\tau}\left[e^{t/\tau} - 1\right]$$

$$\boxed{y(t) = \left[y(0) - y(\infty)\right]e^{-t/\tau} + y(\infty)}$$

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2) (20 points) Assume we have a first order system with the governing differential equation

$$3\dot{y}(t) + 2y(t) = 6x(t)$$

The system is initially at rest, so $y(0) = 0$. The input to this system is

$$x(t) = \begin{cases} 0 & t \le 0 \\ -3 & 0 < t \le 2 \\ 4 & 2 < t \le 5 \\ 0 & 5 < t \end{cases}$$

Determine the output of the system in each of the above time intervals. *Simplify your final answer as much as possible and box it.*

$\frac{3}{2}\dot{y}(t) + y(t) = 3x(t) \qquad \tau = 3/2 = 1.5 \quad K = 3$

1 $\quad 0 \le t \le 2 \quad y(\infty) = KA = (3)(-3) = -9 \quad y(0) = 0$

$$\boxed{y(t) = -9(1 - e^{-t/1.5})}$$

2 $\quad 2 \le t \le 5 \quad y(\infty) = KA = (3)(4) = 12 \quad$ "$y(0)$" $= y(2) = -9(1 - e^{-2/1.5}) = -6.628$

$$y(t) = [-6.628 - 12]e^{-(t-2)/1.5} + 12$$

$$\boxed{y(t) = -18.628e^{-(t-2)/1.5} + 12}$$

3 $\quad 5 \le t \quad y(\infty) = KA = (3)(0) = 0 \quad$ "$y(0)$" $= y(5) = -18.628e^{-3/1.5} + 12 = 9.479$

$$\boxed{y(t) = 9.479e^{-(t-5)/1.5}}$$

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3) (20 points) Assume we have a second order system with the governing differential equation

$$\ddot{y}(t) + 6\dot{y}(t) + 13y(t) = 39x(t)$$

The input to this system is $x(t) = u(t)$ (the input is one for time greater than zero), and the initial conditions are $y(0) = \dot{y}(0) = 0$

a) Determine the correct form of the solution (the roots are complex, but each part is an integer)

b) Solve for the unknown coefficients

c) Write out the final solution and put a box around it.

$y_p(t) = 3 = KA$ $\quad r^2 + 6r + 13 = 0$ $\quad r = \frac{-6 \pm \sqrt{36-52}}{2} = \frac{-6 \pm \sqrt{-16}}{2}$

$= -3 \pm 2j$

$$\boxed{y(t) = c e^{-3t} \sin(2t + \theta) + 3}$$

$y(0) = \boxed{0 = c\sin(\theta) + 3}$

$\dot{y}(t) = -3ce^{-3t}\sin(2t+\theta) + 2ce^{-3t}\cos(2t+\theta)$

$\dot{y}(0) = 0 = -3\sin(\theta) + 2\cos(\theta)$

$3\sin(\theta) = 2\cos(\theta)$

$\tan(\theta) = 2/3 \quad \theta = 33.69^\circ$

$c = \frac{-3}{\sin(\theta)} = \frac{-3}{\sin(33.69)} = -5.408$

$$\boxed{y(t) = 3 - 5.408e^{-3t}\sin(2t + 33.69^\circ)}$$

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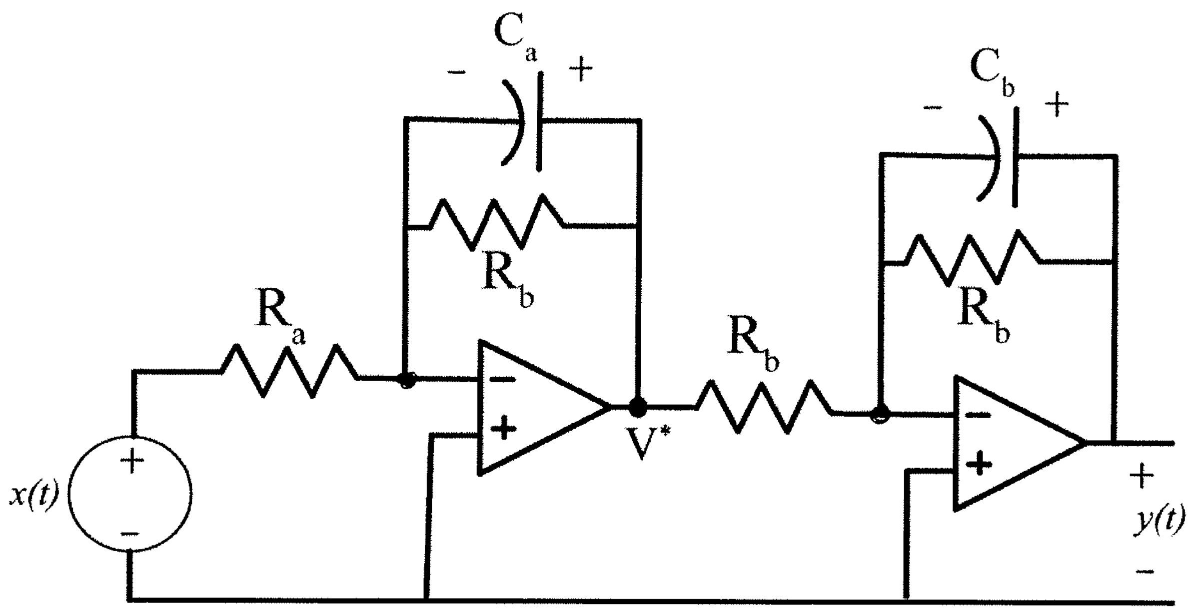
4) (20 points) For the second order circuit below, derive the governing second order differential equation for the output $y(t)$ and input $x(t)$. You do not need to put it into a standard form, but it must be simplified as much as possible.



Hint: Write the equations for each op amp in terms of V^*, and then eliminate this node voltage.

$$\frac{x}{R_a} + \frac{V^*}{R_b} + C_a \dot{V}^* = 0 \qquad \frac{V^*}{R_b} + \frac{y}{R_b} + C_b \dot{y} = 0$$

$$\frac{R_b}{R_a} x = -V^* - R_b C_a \dot{V}^* \qquad V^* = -y - R_b C_b \dot{y}$$

$$\frac{R_b}{R_a} x = y + R_b C_b \dot{y} + R_b C_a (\dot{y} + R_b C_b \ddot{y})$$

$$\boxed{\frac{R_b}{R_a} x = y + R_b (C_a + C_b) \dot{y} + R_b^2 C_a C_b \ddot{y}}$$

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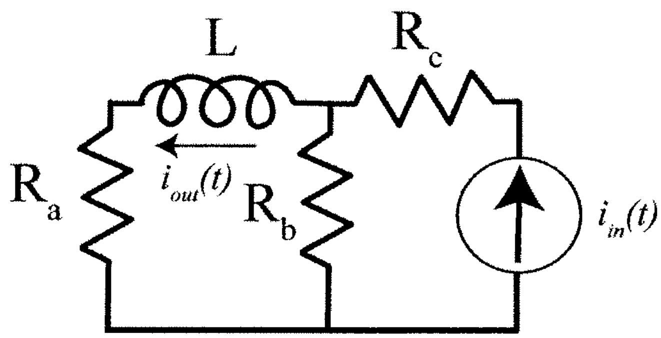
Problems 5-11, 3 points each, no partial credit (21 points)

Problems 5 and 6 refer to the following circuit



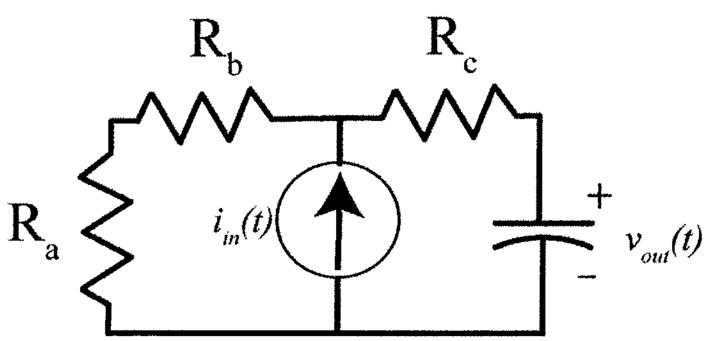
5) The Thevenin resistance seen from the ports of the inductor is

a) $R_{th} = R_a + R_b \,||\, R_c$ b) $R_{th} = R_c + R_a \,||\, R_b$ c) $R_{th} = R_a + R_b$ d) $R_{th} = R_a + R_c$ e) none of these

6) The static gain for the system is

a) $K = 1$ b) $K = \dfrac{R_b}{R_a + R_b}$ c) $K = \dfrac{R_a}{R_a + R_b}$ d) $K = \dfrac{R_b}{R_c + R_b}$ e) none of these

Problems 7 and 8 refer to the following circuit



7) The Thevenin resistance seen from the ports of the capacitor is

a) $R_{th} = R_a + R_b$ b) $R_{th} = R_c$ c) $R_{th} = R_c \,||\, (R_a + R_b)$ d) $R_{th} = R_a + R_b + R_c$ e) none of these

8) The static gain for the system is

a) $K = 1$ b) $K = R_c$ c) $K = R_a + R_b$ d) $K = R_c \,||\, (R_a + R_b)$ e) none of these

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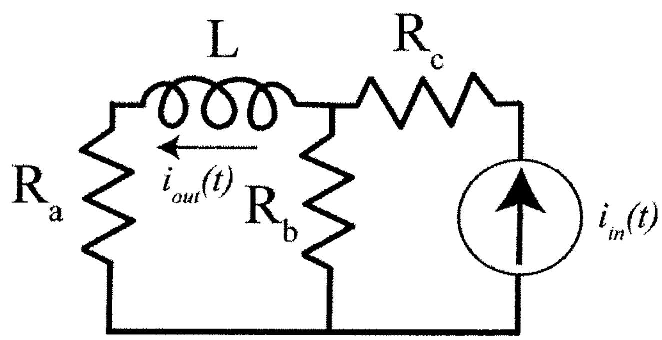

5) The Thevenin resistance seen from the ports of the inductor is

a) $R_{th} = R_a + R_b \,||\, R_c$ b) $R_{th} = R_c + R_a \,||\, R_b$ c) $R_{th} = R_a + R_b$ d) $R_{th} = R_a + R_c$ e) none of these

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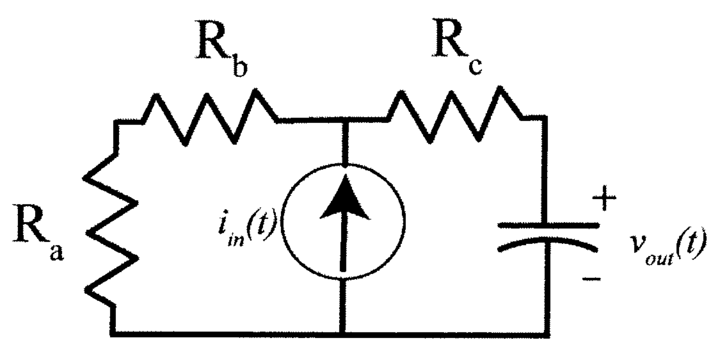

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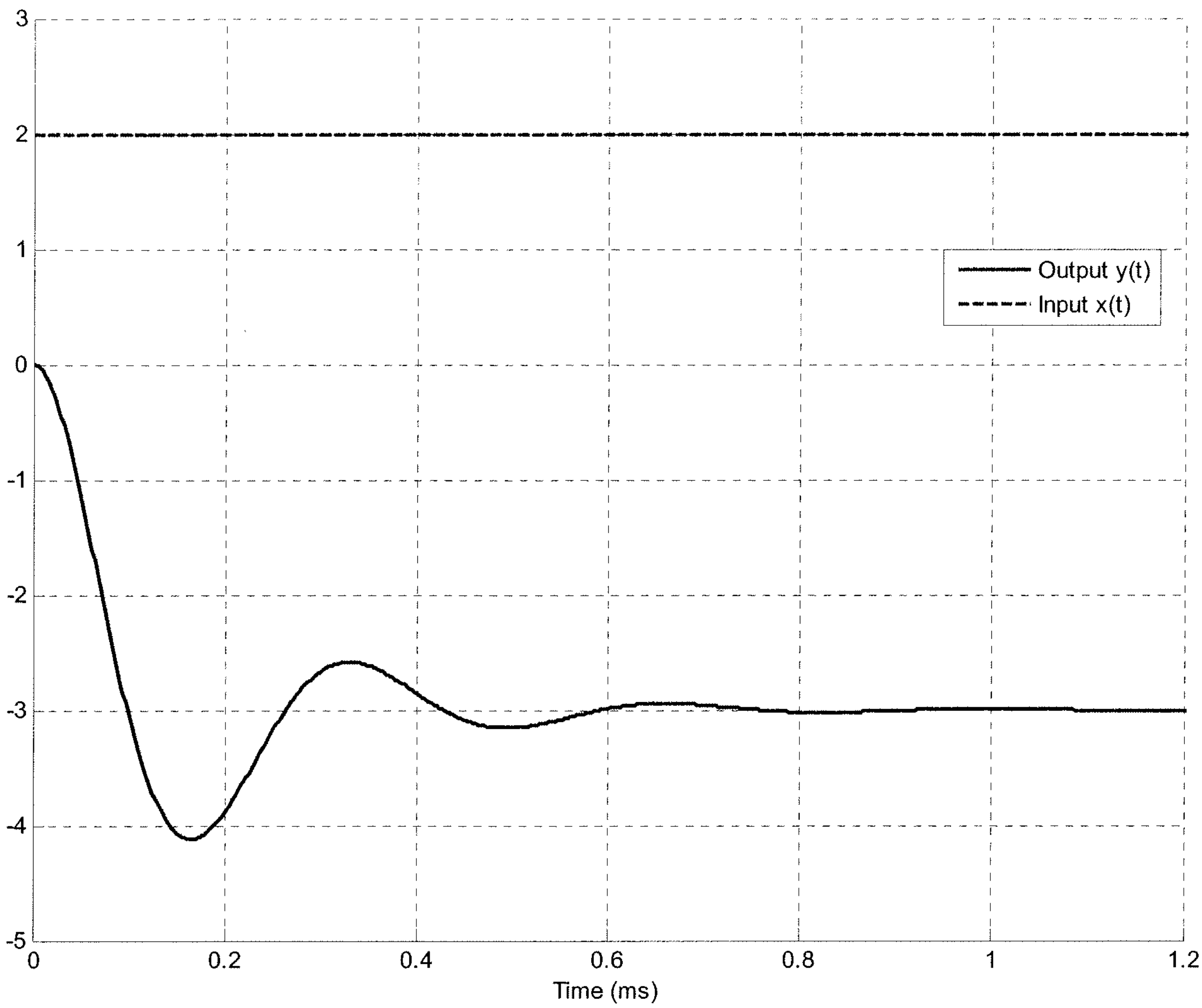
Problems 9-11 refer the following graph showing the response of a second order system to a step input.



9) The percent overshoot for this system is best estimated as

$\frac{-4-(-3)}{-3} = \frac{-1}{-3} = 1/3$

a) 400% b) -400 % c) 300% d) -300 % e) -33% f) 33%

10) The (2%) settling time for this system is best estimated as

a) 0.3 ms b) 0.6 ms c) 1.0 ms d) 1.2 ms

11) The static gain for this system is best estimated as

$K2 = -3$

$K = -3/2$

a) 1.5 b) 3 c) -1.5 d) -3