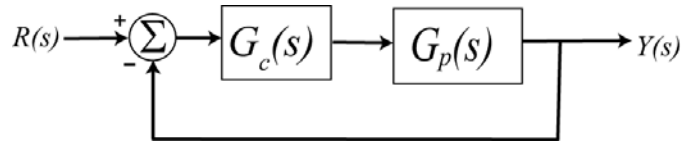# ECE-205 : Dynamical Systems

Homework #9

Due : <u>Tuesday</u> May 11 at the beginning of class

Exam 3, Thursday May 13

1) (Model Matching) Consider the following closed loop system, with plant $G_p(s)$ and controller $G_c(s)$.


One way to choose the controller is to try and make your closed loop system match a transfer function that you choose (hence the name model matching). Let's assume that our **desired** closed loop transfer function $G_o(s)$, our plant can be written in terms of numerators and denominators as $G_o(s) = \dfrac{N_o(s)}{D_o(s)}$ $G_p(s) = \dfrac{N_p(s)}{D_p(s)}$

Show that our controller is then $G_c(s) = \dfrac{N_o(s)D_p(s)}{N_p(s)\left[D_o(s) - N_o(s)\right]}$

Note that there are some restrictions here, in that for implementation purposes the controller must be stable, and it must be proper.

2) For the following system, with plant $G_p(s) = \dfrac{1}{s+1}$, and controller $G_c(s)$
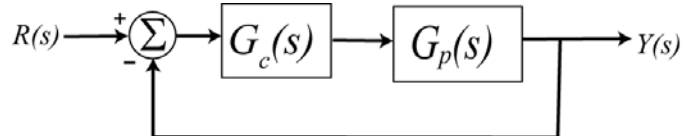
a) Using the results from problem 1, determine the controller so that the closed loop system matches a second order ITAE (**I**ntegral of **T**ime and **A**bsolute **E**rror) optimal system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^2}{s^2 + 1.4\omega_0 s + \omega_0^2}$$

Ans. $G_c(s) = \dfrac{\omega_0^2(s+1)}{s(s+1.4\omega_0)}$, *note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.*

b) Show that the damping ratio for this system is 0.7, the closed loop poles of this system are at $-0.7\omega_0 \pm j0.714\omega_0$. For faster response should ω_0 be large or small?

**c)** What is the steady state error for this system if the input is a unit step?

d) Determine the controller so that the closed loop system matches a third order **deadbeat** system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^3}{s^3 + 1.90\omega_0 s^2 + 2.20\omega_0^2 s + \omega_0^3}$$

Ans. $G_c(s) = \dfrac{\omega_0^3(s+1)}{s(s^2 + 1.9\omega_0 s + 2.20\omega_0^2)}$, *note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.*

e) What is the steady state error of this system for a unit step input?

3) When implementing a control system, we need to be careful the control effort does not get too large. Sometimes this is overcome by changing the feedback structure. Consider the following two feedback control structures,
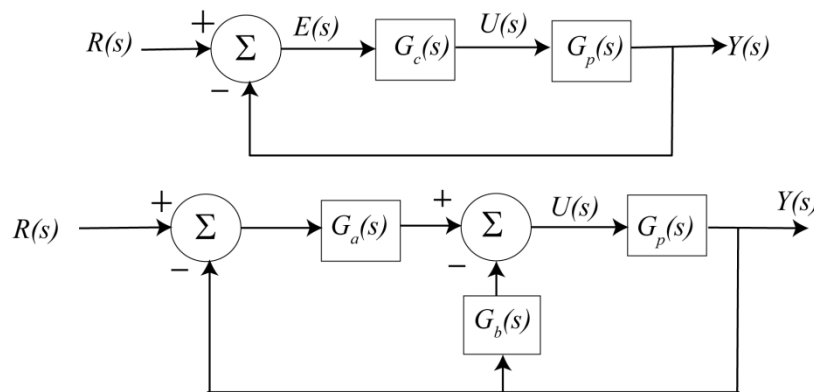
For the first structure, the initial control effort is given by

$$u(0^+) = \lim_{s \to \infty} sU(s) = \lim_{s \to \infty} \frac{sR(s)G_c(s)}{1 + G_c(s)G_p(s)}$$

And for the second structure the initial control effort is given by

$$u(0^+) = \lim_{s \to \infty} sU(s) = \lim_{s \to \infty} \frac{sR(s)G_a(s)}{1 + [G_a(s) + G_b(s)]G_p(s)}$$

Let's assume we can break up the controller so that we can write the transfer functions as

$$G_p(s) = \frac{N_p(s)}{D_p(s)}, G_c(s) = \frac{N_c(s)}{D_c(s)}, G_a(s) = \frac{N_a(s)}{D_a(s)}, G_b(s) = \frac{N_b(s)}{D_b(s)}$$

a) Show that if we can partition the controller so that

$$G_a(s) + G_b(s) = G_c(s)$$

Then we can write the closed loop transfer functions as

$$G_o(s) = \frac{N_c(s)N_p(s)}{D_c(s)D_p(s)+N_c(s)N_p(s)}, \quad G_o(s) = \frac{D_c(s)N_a(s)N_p(s)}{D_a(s)\left[D_c(s)D_p(s)+N_c(s)N_p(s)\right]}$$

Hence, if $D_a(s)$ is a constant, the two structures produce the same closed loop poles.

b) Assume we have the plant $G_p(s) = \dfrac{3}{s+2}$, and assume we are trying to use a PD controller. So
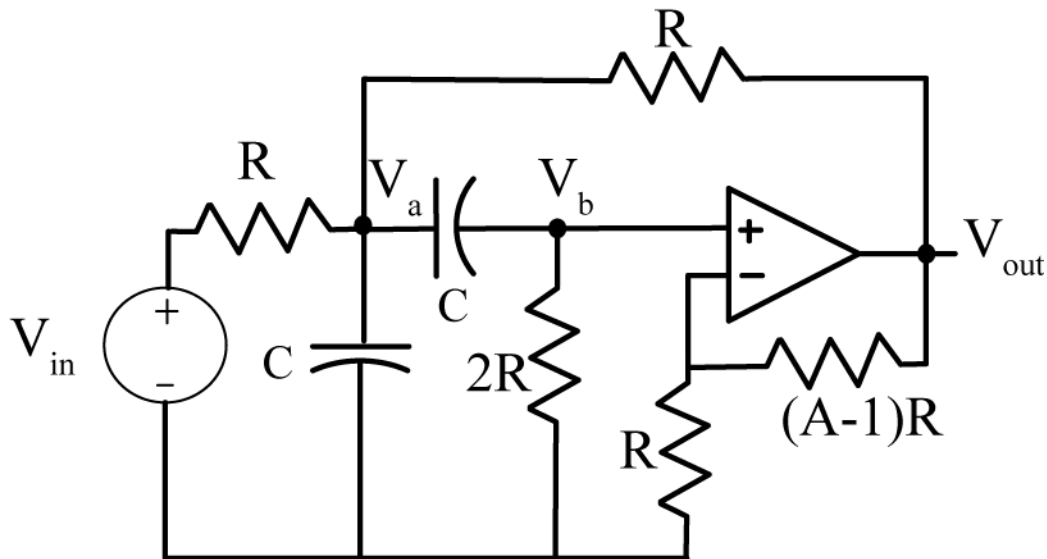
$$G_c(s) = k_p + k_d s, \quad G_a(s) = k_p, \quad G_b(s) = k_d s$$

Show that if the input is a step of amplitude A, then for the two control structures we have

$$u(0^+) = \infty, \ e_{ss} = \frac{A2}{2+3k_p}, \quad u(0^+) = \frac{Ak_p}{1+3k_d}, e_{ss} = \frac{A2}{2+3k_p}$$

This second control structure is often used with PID controllers, as I-PD and PI-D controllers.

4) (Matlab Problem) Consider the following Sallen-Key filter, shown in the figure below:

We are going to solve for the output of this circuit for a range of frequencies using Matlab.

a) Download the m-file **homework9.m** from the class website. You will need to modify this code in this problem.

b) Write node equations for this system in terms of the nodes **Va**, **Vb**, and **Vout** and the input voltage **Vin.** Note that you will not be able to write a node equation at the output of the op amp, but will have to relate **Vb** and

Vout using the ideal op-amp assumption. The impedance for a capacitor is $Z(j\omega) = \dfrac{1}{j\omega C}$. In Matlab, we can

use **j** as the square root of -1 (though you may have to multiply by **j** explicitly). The capacitor is 1 microfarad, the resistors are 1000 ohms, and the parameter A is 2.5. These parameters have already been entered in the program. The frequency will be changing however.

c) In the Matlab program, write the node equations in terms of the matrix M, and the input vector b. Assume we write the matrix equation as $Mx = b$, or

$$[M] \begin{bmatrix} V_a \\ V_b \\ V_{out} \end{bmatrix} = \begin{bmatrix} V_{in} \\ 0 \\ 0 \end{bmatrix}$$

d) Use Matlab to solve the matrix equation for the nodal voltages (x). In this problem we only care about the output voltage, the third element of x. Assume the input voltage is 1 volt with a phase of zero degrees.

e) Use the Matlab commands **abs** and **angle** to get the magnitude and phase of the output signal. The command angle gives the phase in radians, and you will need to convert this to degrees.

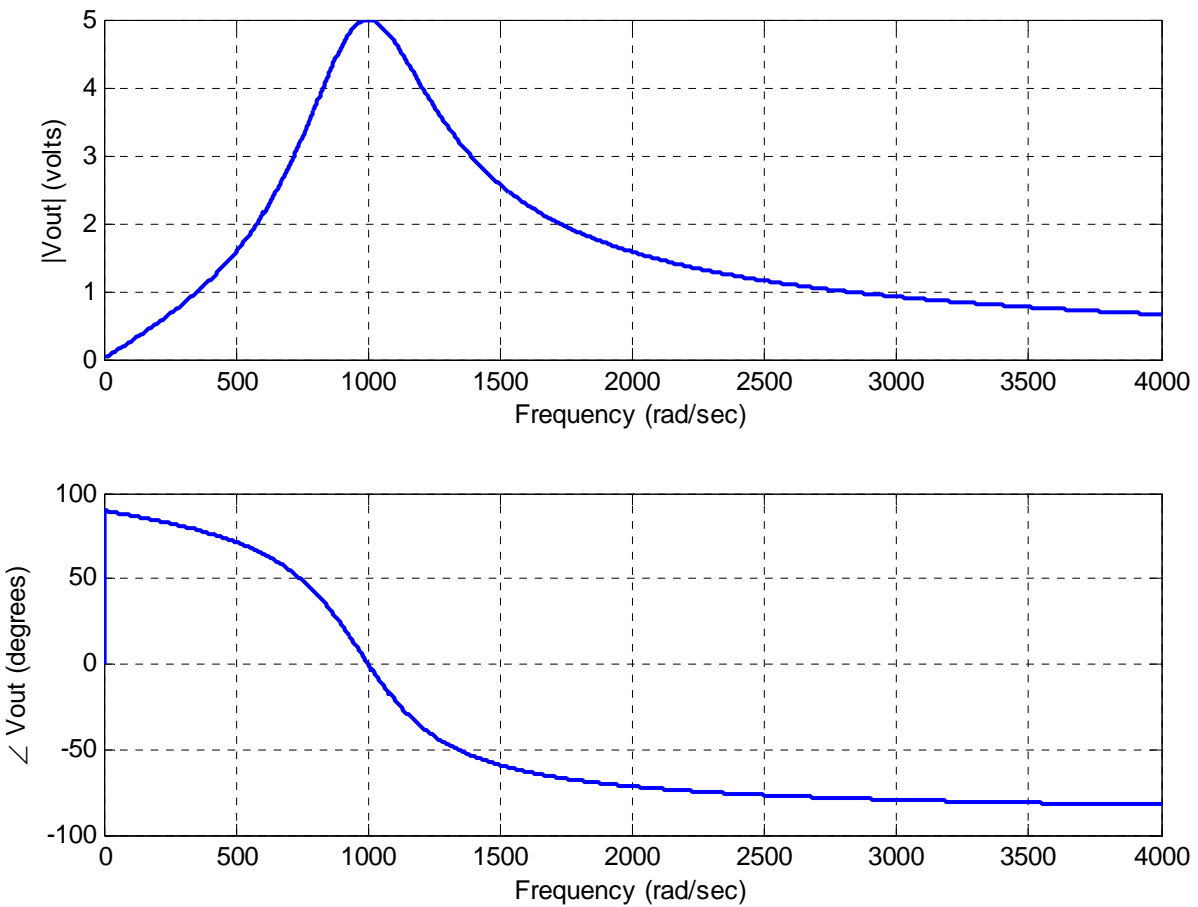Your final plot should look like that in Figure 1. Turn in your code and your plot.

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Figure 1. Frequency response of the Sallen-Key filter from ploblem 4.