ECE-205 : Dynamical Systems

Homework #8

Due : <u>Thursday</u> May 6 at the beginning of class

1) For the following transfer functions

$$H(s)=\frac{2}{s^2+2s+2} \quad H(s)=\frac{3}{s^2+4s+6} \quad H(s)=\frac{5}{s^2+6s+10}$$
$$H(s)=\frac{4}{s^2-4s+7} \quad H(s)=\frac{1}{s^2+4}$$

By computing the inverse Laplace transform show that the step responses are given by

$$y(t)=\left[1-e^{-t}\cos(t)-e^{-t}\sin(t)\right]u(t) \quad y(t)=\left[\frac{1}{2}-\frac{1}{\sqrt{2}}e^{-2t}\sin(\sqrt{2}t)-\frac{1}{2}e^{-2t}\cos(\sqrt{2}t)\right]u(t)$$
$$y(t)=\left[\frac{1}{2}-\frac{3}{2}e^{-3t}\sin(t)-\frac{1}{2}e^{-3t}\cos(t)\right]u(t) \quad y(t)=\left[\frac{4}{7}+\frac{8\sqrt{3}}{21}e^{2t}\sin(\sqrt{3}t)-\frac{4}{7}e^{2t}\cos(\sqrt{3}t)\right]u(t)$$
$$y(t)=\left[\frac{1}{4}-\frac{1}{4}\cos(2t)\right]u(t)$$

2) For the following transfer functions, determine

- the characteristic polynomial
- the characteristic modes
- if the system is (asymptotically) stable, unstable, or marginally stable

a) $H(s)=\frac{s-1}{s(s+2)(s+10)}$ b) $H(s)=\frac{s(s-1)}{(s+1)^2(s^2+s+1)}$ c) $H(s)=\frac{1}{s^2(s+1)}$

d) $H(s)=\frac{s^2-1}{(s-1)(s+2)(s^2+1)}$ e) $H(s)=\frac{1}{(s^2+2)(s+1)}$

Partial Answer: 1 stable, 2 unstable, 2 marginally stable

3) For a system with the following pole locations, estimate the settling time and determine the dominant poles

a) -1,-2,-4,-5 b) -4, -6, -7, -8

c) -1+j, -1-j, -2, -3 d) -3-2j, -3+2j, -4+j, -4-j

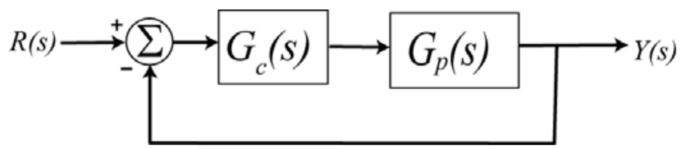
Scrambled Answers: 4/3, 4, 4, 1

4) Determine the static gain for the systems represented by the following transfer functions, and then the steady state output for an input step of amplitude 3:

$$H(s)=\frac{s+2}{s^2+s+1},\quad H(s)=\frac{1}{s^2+4s+4},\quad H(s)=\frac{s-4}{s^2+s+1}$$

Answers: 2, 0.25, -4, 6, 0.75, -12 (this should be very easy)

5) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s)=\frac{3}{s+2}$ and the controller is a proportional controller, so $G_c(s)=k_p$.



a) Determine the settling time of the plant alone (assuming there is no feedback)

b) Determine the closed loop transfer function, $G_0(s)$

c) Determine the value of k_p so the settling time of the system is 0.5 seconds.

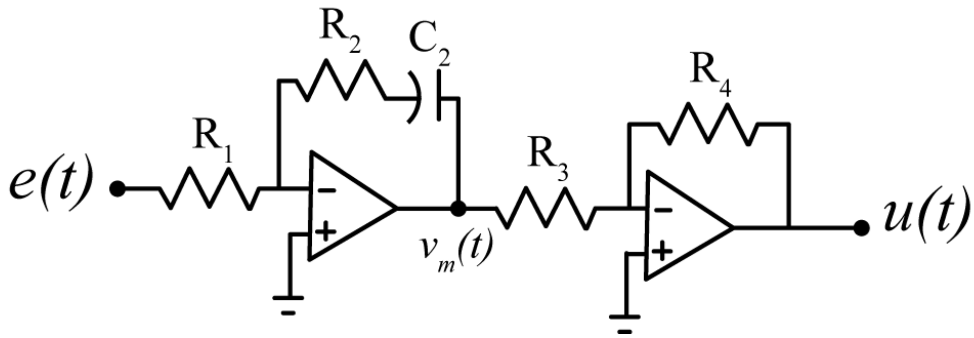
d) If the input to the system is a unit step, determine the output of the system.

e) The steady state error is the difference between the input and the output as $t\to\infty$. Determine the steady state error for this system.

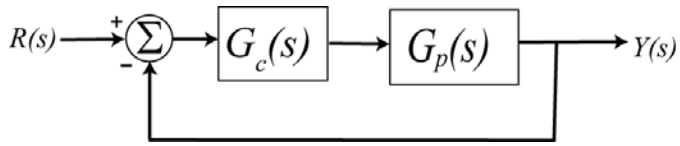
Partial Answer: $y(t)=\frac{3}{4}\left[1-e^{-8t}\right]u(t),\quad e_{ss}=0.25$

6) Show that the following circuit can be used to implement the PI controller

$$G_c(s)=\frac{U(s)}{E(s)}=k_p+k_i\frac{1}{s}=\frac{R_4R_2}{R_3R_1}+\frac{R_4}{R_3R_1C_2}\frac{1}{s}$$



7) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{3}{s+2}$ and the controller is an integral controller, so $G_c(s) = \dfrac{k_i}{s}$.



a) Determine the closed loop transfer function, $G_0(s)$

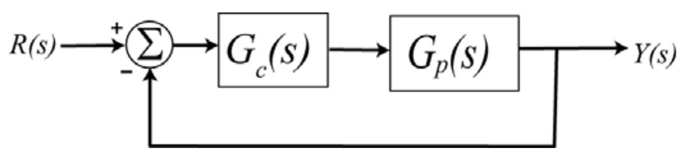
b) Determine the poles of value of $G_0(s)$ and show they are only real if $0 < k_i < \dfrac{1}{3}$. Note that the best possible setting time is 4 seconds. Use $k_i = \dfrac{1}{3}$ for the remainder of this problem.

c) If the input to the system is a unit step, determine the output of the system.

d) The steady state error is the difference between the input and the output as $t \to \infty$. Determine the steady state error for this system.

Partial Answer: $y(t) = \left[1 - e^{-t} - te^{-t}\right]u(t), \quad e_{ss} = 0$

8) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{2}{s+4}$. The input is a unit step.



a) Determine the settling time and steady state error of the plant alone (assuming there is no feedback)

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the steady state error for a unit step is 0.1, and the corresponding settling time for the system.

d) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the settling time is 0.5 seconds, and the corresponding steady state error.

e) Assuming an integral controller, $G_c(s) = k_i / s$, determine closed loop transfer function, $G_0(s)$

f) Assuming an integral controller, $G_c(s) = k_i / s$, determine the value of k_i so the steady state error for a unit step is less than 0.1 and the system is stable.

PartialAnswers: $T_s = 1, e_{ss} = 0.5, k_p = 18, k_p = 2, T_s = 0.1, e_{ss} = 0.5, k_i > 0$