ECE-205 : Dynamical Systems

Homework #7

Due : <u>Thursday</u> April 29 at the beginning of class

1) In this problem we will derive some useful properties of Laplace transforms starting from the basic relationship

$$X(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt$$

a) Let's assume $x(t)$ is a causal signal (it is zero for $t < 0$). We can then write $x(t) = x(t)u(t)$ to emphasize the fact that $x(t)$ is zero before time zero. If there is a delay in the signal and it starts at time t_0, then we can write the signal as $x(t-t_0) = x(t-t_0)u(t-t_0)$ to emphasize the fact that the signal is zero before time t_0.

Using the definition of the Laplace transform and a simple change of variable in the integral, show that $\mathcal{L}\{x(t-t_0)u(t-t_0)\} = X(s)e^{-st_0}$

b) Using the results from part **a**, determine the inverse Laplace transform of $X(s) = \dfrac{e^{-3s}}{(s+2)(s+4)}$

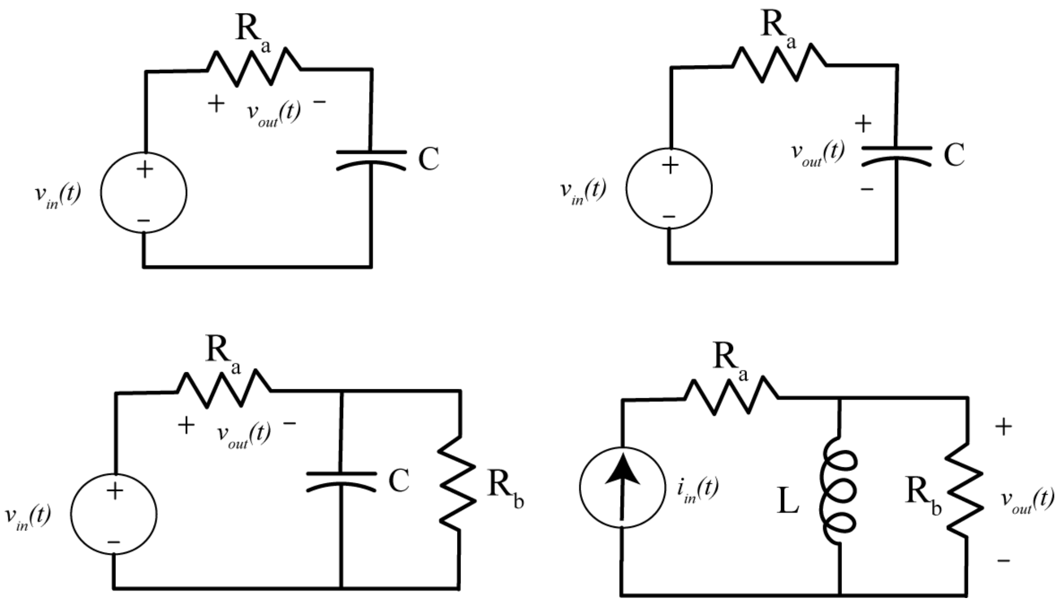
Answer: $x(t) = \frac{1}{2}\left[e^{-2(t-3)} - e^{-4(t-3)}\right]u(t-3)$

c) Starting from the definition of the Laplace transform, show that $\mathcal{L}\{tx(t)\} = -\dfrac{dX(s)}{ds}$.

d) Using the result from part c, and the transform pair $x(t) = e^{-at}u(t) \leftrightarrow X(s) = \dfrac{1}{s+a}$, and some simple calculus, show that

$$\mathcal{L}\{te^{-at}u(t)\} = \frac{1}{(s+a)^2}, \mathcal{L}\{t^2e^{-at}u(t)\} = \frac{2}{(s+a)^3}, \mathcal{L}\{t^3e^{-at}u(t)\} = \frac{6}{(s+a)^4}$$

2) For the following circuits, determine the transfer function and the corresponding impulse responses.



Scrambled Answers:

$$h(t) = R_b\delta(t) - \frac{R_b^2}{L}e^{-tR_b/L}u(t),\ h(t) = \delta(t) - \frac{1}{R_aC}e^{-t/R_aC}u(t),\ h(t) = \frac{1}{R_aC}e^{-t/RaC}u(t), h(t) = [\delta(t) - \frac{1}{R_aC}e^{-t\frac{R_a+R_b}{CR_aR_b}}]u(t)$$

3) For the following impulse responses and inputs, compute the system output using transfer functions.

a) $h(t) = e^{-t}u(t),\ x(t) = u(t)$ **b)** $h(t) = e^{-2t}u(t),\ x(t) = \delta(t)$ **c)** $h(t) = e^{-2(t-1)}u(t-1),\ x(t) = e^{-2t}u(t)$

d) $h(t) = e^{-t}u(t),\ x(t) = (t-1)u(t-1)$ **e)** $h(t) = e^{-2t}u(t),\ x(t) = u(t) - u(t-1)$

f) $h(t) = e^{-2(t-1)}u(t-1),\ x(t) = te^{-3t}u(t)$

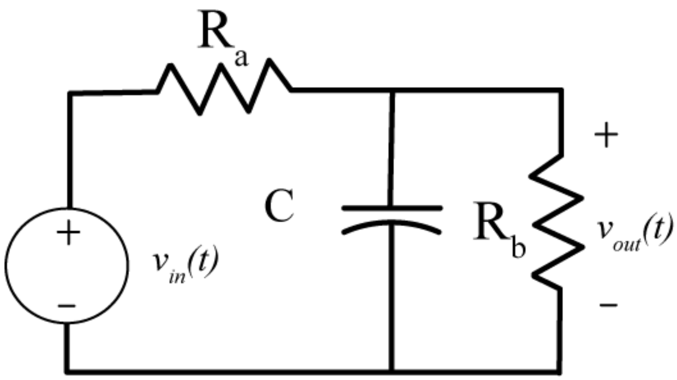
Scrambled Answers :

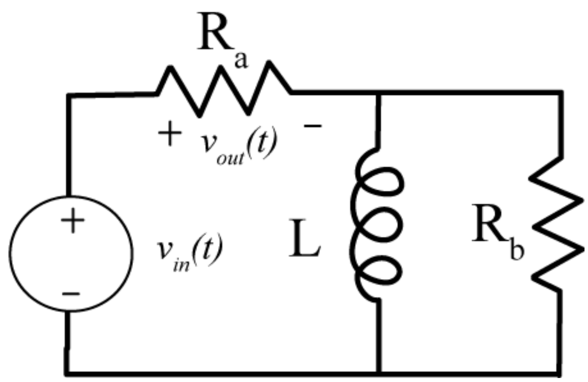
$$y(t) = \frac{1}{2}\left[1 - e^{-2t}\right]u(t) - \frac{1}{2}\left[1 - e^{-2(t-1)}\right]u(t-1),\ y(t) = \left(1 - e^{-t}\right)u(t),\ y(t) = (t-1)e^{-2(t-1)}u(t-1)$$

$$y(t) = e^{-2t}u(t),\ y(t) = [-1 + (t-1) + e^{-(t-1)}]u(t-1),$$

$$y(t) = [e^{-2(t-1)} - e^{-3(t-1)} - (t-1)e^{-3(t-1)}]u(t-1)$$

4) For the following circuits, determine and expression for the output $V_{out}(s)$ in terms of the ZSR and ZIR. Do not assume the initial conditions are zero. Also determine the system transfer function.



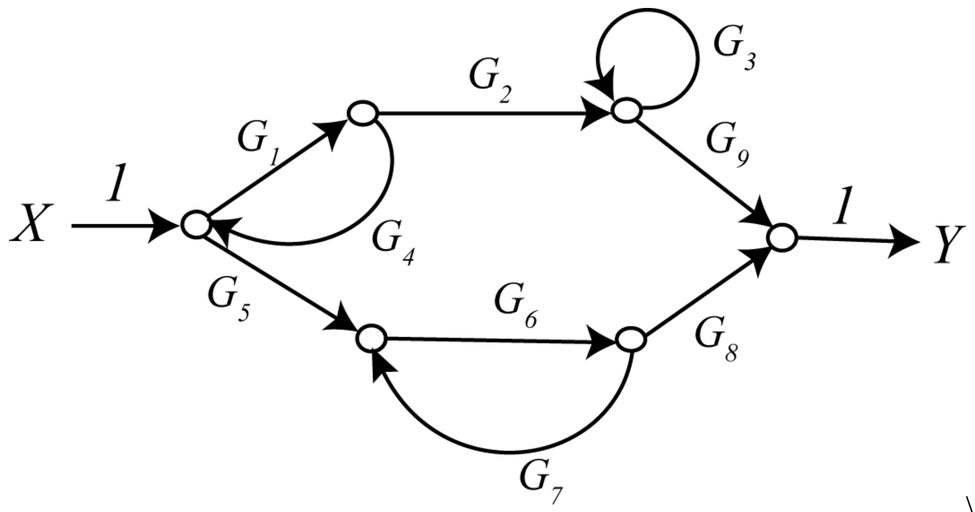


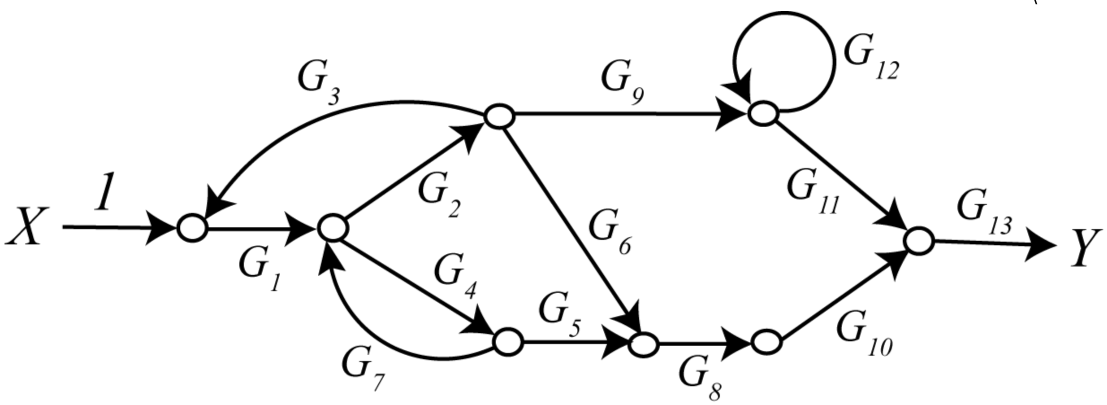
Answers:

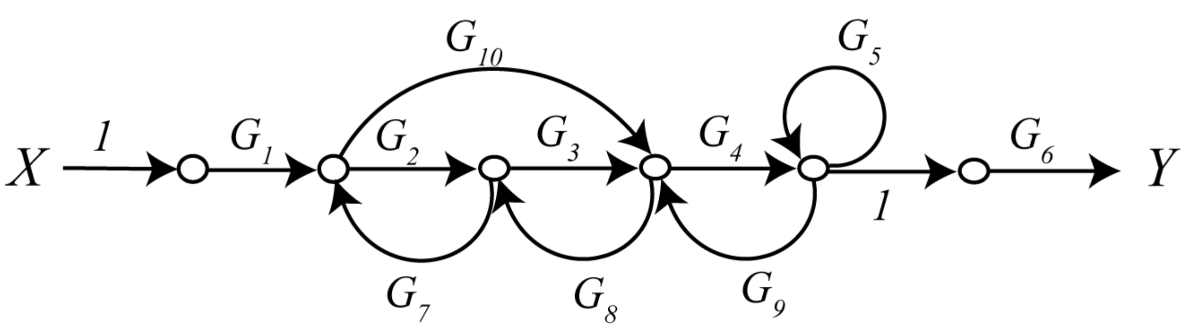
$$V_{out}(s) = \left[\frac{R_b}{R_a R_b C s + R_a + R_b}\right] V_{in}(s) + \left[\frac{R_a R_b C}{R_a R_b C s + R_a + R_b}\right] v(0^-)$$

$$V_{out}(s) = \left[\frac{R_a(R_b + Ls)}{(R_a + R_b)Ls + R_a R_b}\right] V_{in}(s) + \left[\frac{R_a R_b L}{(R_a + R_b)Ls + R_a R_b}\right] i(0^-)$$

5) For the following signal flow diagrams determine the system transfer function. *You may use Maple.*







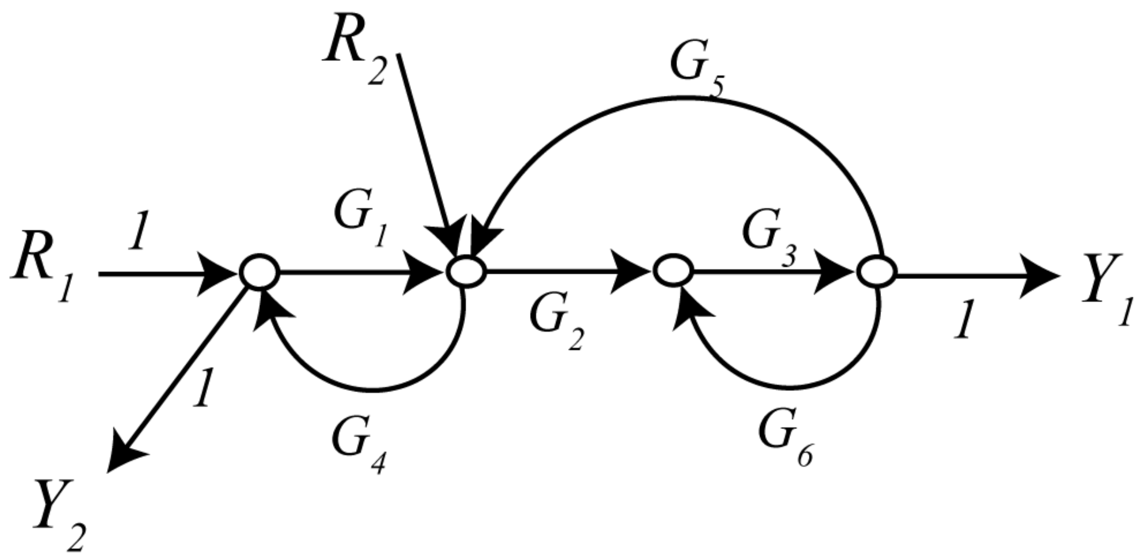
Answers:

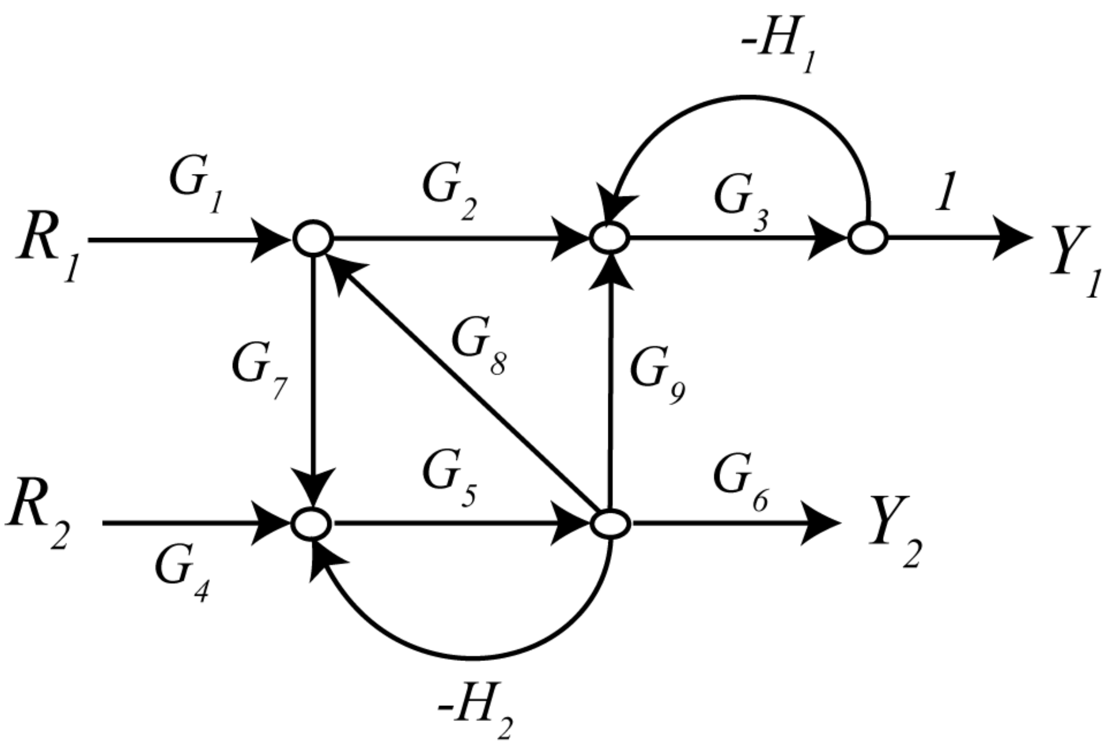
$$\frac{Y}{X}=\frac{G_1G_2G_9(1-G_6G_7)+G_5G_6G_8(1-G_3)}{1-G_1G_4-G_6G_7-G_3+G_1G_4G_6G_7+G_1G_3G_4+G_3G_6G_7-G_1G_3G_4G_6G_7}$$

$$\frac{Y}{X}=\frac{G_1G_2G_3G_4G_6+G_1G_4G_6G_{10}}{1-G_2G_7-G_3G_8-G_4G_9-G_7G_8G_{10}-G_5+G_2G_4G_7G_9+G_2G_5G_7+G_3G_5G_8+G_5G_7G_8G_{10}}$$

$$\frac{Y}{X}=\frac{G_1G_2G_9G_{11}G_{13}+G_1G_2G_6G_8G_{10}G_{13}(1-G_{12})+G_1G_4G_5G_8G_{10}G_{13}(1-G_{12})}{1-G_1G_2G_3-G_{12}-G_4G_7+G_1G_2G_3G_{12}+G_4G_7G_{12}}$$

6) We can also use Mason's rule for systems with multiple inputs and multiple outputs. To do this, we use superposition and assume only one input is non-zero at a time. The only things that changes are the paths (which depends on the input and the output) and the cofactors (which depends on the path). The determinant does not change, since it is intrinsic to the system. For the following systems, determine the transfer functions from all inputs (R) to all outputs (Y). *You may use Maple.*





Answers:

$$\frac{Y1}{R1}=\frac{G_1G_2G_3}{1-G_1G_4-G_2G_3G_5-G_3G_6+G_1G_3G_4G_6},$$

$$\frac{Y2}{R1}=\frac{1-G_2G_3G_5-G_3G_6}{1-G_1G_4-G_2G_3G_5-G_3G_6+G_1G_3G_4G_6}$$

$$\frac{Y1}{R2}=\frac{G_2G_3}{1-G_1G_4-G_2G_3G_5-G_3G_6+G_1G_3G_4G_6},$$

$$\frac{Y2}{R2}=\frac{G_4(1-G_3G_6)}{1-G_1G_4-G_2G_3G_5-G_3G_6+G_1G_3G_4G_6}$$

$$\frac{Y1}{R1}=\frac{G_1G_2G_3(1+G_5H_2)+G_1G_3G_5G_7G_9}{1+G_5H_2-G_5G_7G_8+H_1G_3+G_3G_5H_1H_2-H_1G_3G_5G_7G_8},$$

$$\frac{Y2}{R1}=\frac{G_1G_5G_6G_7(1+G_3H_1)}{1+G_5H_2-G_5G_7G_8+H_1G_3+G_3G_5H_1H_2-H_1G_3G_5G_7G_8}$$

$$\frac{Y1}{R2}=\frac{G_3G_4G_5G_9+G_2G_3G_4G_5G_8}{1+G_5H_2-G_5G_7G_8+H_1G_3+G_3G_5H_1H_2-H_1G_3G_5G_7G_8},$$

$$\frac{Y2}{R2}=\frac{G_4G_5G_6(1+H_1G_3)}{1+G_5H_2-G_5G_7G_8+H_1G_3+G_3G_5H_1H_2-H_1G_3G_5G_7G_8}$$