

ECE-205 : Dynamical Systems

Homework #2

Due : Thursday March 18 at the beginning of class

1) An RC circuit has parameters $R_{th} = R_a \parallel R_b$, $\tau = R_{th}C$, $K = R_{th}$ and is described by the first order equation

$$\tau \frac{dv_c(t)}{dt} + v_c(t) = Ki_{in}(t)$$

For this circuit $R_a = 1 k\Omega$, $R_b = 5 k\Omega$, $C = 0.1 \text{ mf}$ and the *initial voltage* is $v_c(0) = 0.5V$

The input current is

$$i_{in}(t) = \begin{cases} 0 & t \leq 0 \\ -2 \text{ mA} & 0 \leq t < 0.2 \text{ sec} \\ 4 \text{ mA} & 0.2 \leq t < 0.3 \text{ sec} \\ -3 \text{ mA} & t \geq 0.3 \text{ sec} \end{cases}$$

a) Determine an analytical expression for the voltage across the capacitor in each of these regions

b) Enter your analytical expression as an anonymous function in Matlab, and simulate the system using Simulink. Run the simulation for 1 second (from 0 to 1 second). Show that all three of your answers are identical. Turn in your Matlab (driver file) code and your neatly labeled plots. Be sure to set the initial value of the integrator to the correct value.

2) An RL circuit has parameters $R_{th} = R_a \parallel R_b$, $\tau = L / R_{th}$, $K = \frac{1}{R_a}$ and is described by the first order equation

$$\tau \frac{di_L(t)}{dt} + i_L(t) = Kv_{in}(t)$$

For this circuit, $R_a = 100 \Omega$, $R_b = 400\Omega$, $L = 30 \text{ mh}$ and the initial current through the inductor is zero. The input current is

$$v_{in}(t) = \begin{cases} 0 & t \leq 0 \\ 1V & 0 \leq t < 0.6 \text{ ms} \\ -2V & 0.6 \text{ ms} \leq t < 2 \text{ ms} \\ 6V & t \geq 2 \text{ ms} \end{cases}$$

a) Determine an analytical expression for the current through the inductor in each of these regions

b) Enter your analytical expression as an anonymous function in Matlab, and simulate the system using Simulink. Run the simulation for 5 ms. Show that all three of your answers are identical. Turn in your Matlab (driver file) code and your neatly labeled plots. Be sure to set the initial value of the integrator to the correct value.

3) For each of the following first order differential equations, use an integrating factor to write $y(t)$ as a function of its initial value $y(t_0)$ and an integral of the input (plus some other functions)

$$a\dot{y}(t) + by(t) = cx(t) \quad \frac{1}{2}\dot{y}(t) - ty(t) = x(t+2)$$

$$\dot{y}(t) + \frac{1}{t}y(t) = x(t) \quad \dot{y}(t) + \cos(t)y(t) = e^t x(t)$$

Answers:

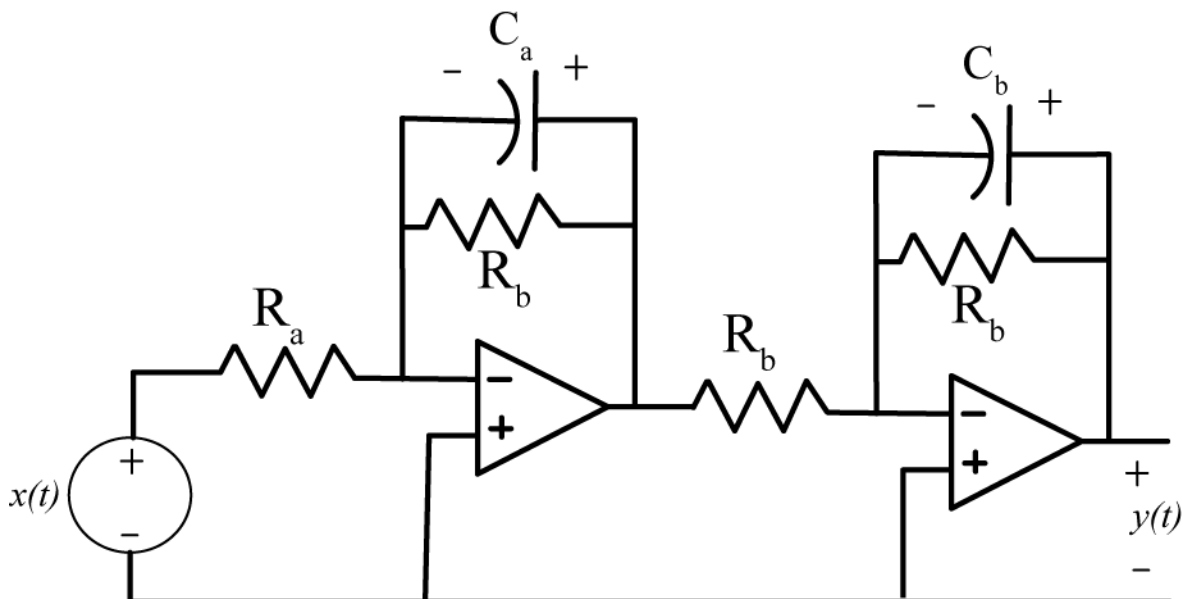
$$y(t) = e^{-\frac{b}{a}(t-t_0)} y(t_0) + \int_{t_0}^t \frac{c}{a} e^{-\frac{b}{a}(t-\lambda)} x(\lambda) d\lambda \quad y(t) = e^{t^2-t_0^2} y(t_0) + \int_{t_0}^t 2e^{(t^2-\lambda^2)} x(\lambda+2) d\lambda$$

$$y(t) = \left(\frac{t_0}{t}\right) y(t_0) + \int_{t_0}^t \left(\frac{\lambda}{t}\right) x(\lambda) d\lambda \quad y(t) = e^{-\sin(t)+\sin(t_0)} y(t_0) + \int_{t_0}^t e^{-\sin(t)} e^{\lambda+\sin(\lambda)} x(\lambda) d\lambda$$

4) Show that the governing differential equation for the following circuit is given by

$$R_b^2 C_a C_b \ddot{y}(t) + R_b (C_a + C_b) \dot{y}(t) + y(t) = \frac{R_b}{R_a} x(t)$$

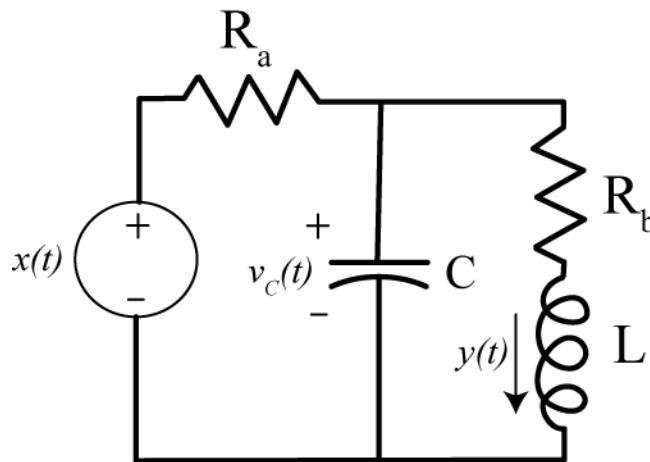
and that for this circuit $\omega_n = \frac{1}{R_b \sqrt{C_a C_b}}$, $\zeta = \frac{C_a + C_b}{2\sqrt{C_a C_b}}$, and $K = \frac{R_a}{R_b}$



5) Show that the governing differential equation for the following circuit is given by

$$\left(\frac{LCR_a}{R_a + R_b}\right)\ddot{y}(t) + \left(\frac{L + R_a R_b C}{R_a + R_b}\right)\dot{y}(t) + y(t) = \left(\frac{1}{R_a + R_b}\right)x(t)$$

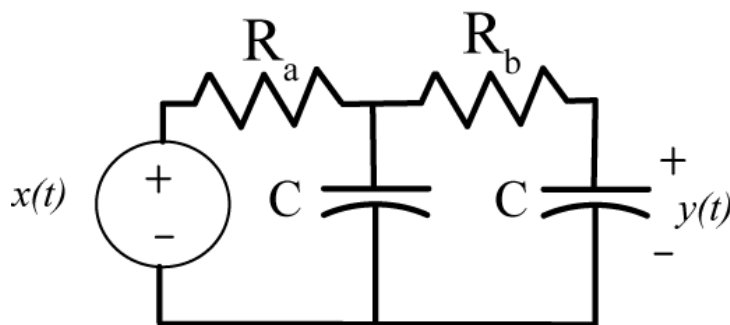
and that for this circuit $\omega_n = \sqrt{\frac{R_a + R_b}{LCR_a}}$, $\zeta = \frac{L + R_a R_b C}{2\sqrt{LCR_a}\sqrt{R_a + R_b}}$, and $K = \frac{1}{R_a + R_b}$.



6) Show that the governing differential equation for the following circuit is given by

$$(R_a R_b C^2)\ddot{y}(t) + (2R_a C + R_b C)\dot{y}(t) + y(t) = x(t)$$

and that for this circuit $\omega_n = \frac{1}{C\sqrt{R_a R_b}}$, $\zeta = \frac{2R_a + R_b}{2\sqrt{R_a R_b}}$, and $K = 1$.



7) PreLab/Matlab/Simulink Problem

One of the standard forms for a second order system is

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2y(t) = \omega_n^2Kx(t)$$

where ζ is the damping ratio, ω_n is the natural frequency, and K is the static gain. Use this form of the standard second order system in the remainder of this problem.

We want to simulate a system described by this differential equation using a Matlab driver. Just as you did for a first order system, you need to solve for the highest power derivative (as a function of the input and lower power derivatives). If you then take $\ddot{y}(t)$ and run it through an integrator you will get $\dot{y}(t)$ and if you run this through an integrator you will get $y(t)$. Hence you will need two integrators and two feedback loops and one input into your summing block (click on the summing block and modify it to get three inputs). You may need to click on the gain block and then choose **flip block** to get the correct direction.

- We want to look at the step response, so the input to your system should be a step. The amplitude of the step should be controlled by the Matlab program, the length of the simulation should be controlled by the Matlab program, and the step should start at time zero.
- Your Simulink file should only contain variables (static gain, natural frequency, damping ratio, amplitude of the step, length of the simulation)
- Plot the transient output of the system and the steady state output KA on the same graph.
- If you use the parameters $\zeta = 0.1$, $\omega_n = 20$, $K = 2$, $A = 1$, and $Tf = 3$ (final simulation time), you should get results like that shown below. Print out your figure, your Matlab code, and your Simulink model and turn them in.

