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ECE-205

Exam 2

Winter 2009

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/20

Problem 2 ________/15

Problem 3 ________/20

Problem 4 ________/25

Problem 5-9 _______/20

Total ____________

90-100 1
80-89 6
70-79 2
60-69 2
<60 3
median = 78

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1) (20 points) Fill in the following table with a Y (yes) or N (no) for each of the system models given. Assume $-\infty < t < \infty$ for all of the systems and all initial conditions are zero.

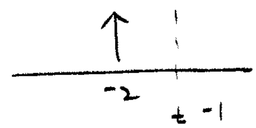
System	System Model	Linear?	Time-Invariant?	Causal?	Memoryless?
1	$y(t) = e^{t+1}\cos(t)x(t)$	Y	N	Y	Y
2	$y(t) = \lvert x(t-1) \rvert$	N	Y	Y	N
3	$y(t) = x(1-t)$	Y	N	N	N
4	$\dot{y}(t) + y(t) = e^{-t}x(t+1)$	Y	N	N	N
5	$y(t) = \int_{-\infty}^{t} e^{-(t-\lambda)}x(\lambda+1)d\lambda$	Y	Y	N	N

Name ____________________ Mailbox ____________

2) (15 points) Determine the impulse responses for the following systems

a) $y(t) = x(t-1) + \int_{-\infty}^{t-1} e^{-(t-\lambda-2)} x(\lambda+2)\,d\lambda$

$$h(t) = \delta(t-1) + \int_{-\infty}^{t-1} e^{-(t-\lambda-2)} \delta(\lambda+2)\,d\lambda$$



$t-1 > -2$

$t > -1$

$$\boxed{h(t) = \delta(t-1) + e^{-t} u(t+1)}$$

b) $\tau \dot{y}(t) + y(t) = Kx(t)$

$$\tau \dot{h}(t) + h(t) = K\,\delta(t)$$

$$\dot{h}(t) + \frac{1}{\tau} h(t) = \frac{K}{\tau}\,\delta(t)$$

$$\frac{d}{dt}\left(h(t)\,e^{t/\tau}\right) = \frac{K}{\tau} e^{t/\tau}\delta(t) = \frac{K}{\tau}\,\delta(t)$$

$$\int_{-\infty}^{t} \frac{d}{d\lambda}\left(h(\lambda)\,e^{\lambda/\tau}\right) d\lambda = h(t)\,e^{t/\tau} = \int_{-\infty}^{t} \frac{K}{\tau}\,\delta(\lambda)\,d\lambda = \frac{K}{\tau} u(t)$$

$$\boxed{h(t) = \frac{K}{\tau} e^{-t/\tau} u(t)}$$

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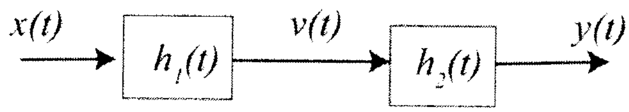
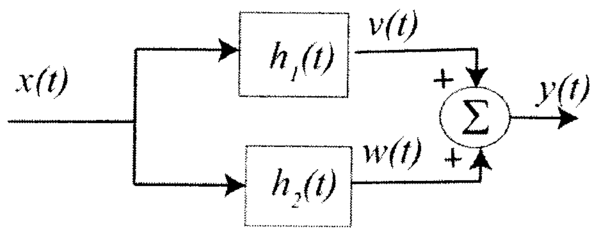
3) (20 points) For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input $x(t)$ and output $y(t)$) and

ii) determine if the system is causal.





a) $h_1(t) = u(t-1),\ h_2(t) = u(t+1)$

b) $h_1(t) = e^{-(t-1)}u(t-1),\ h_2(t) = \delta(t-2)$

(a) parallel $h(t) = h_1(t) + h_2(t) = \boxed{u(t-1) + u(t+1)}$ non causal

series $h(t) = \int_{-\infty}^{\infty} h_1(t-\lambda) h_2(\lambda) d\lambda = \int_{-\infty}^{\infty} u(t-\lambda-1) u(\lambda+1) d\lambda$

$= \int_{-1}^{t-1} d\lambda = \boxed{t\,u(t)}$ causal

(b) parallel $h(t) = h_1(t) + h_2(t) = \boxed{e^{-(t-1)} u(t-1) + \delta(t-2)}$ causal

series $h(t) = \int_{-\infty}^{\infty} h_1(t-\lambda) h_2(\lambda) = \int_{-\infty}^{\infty} e^{-(t-\lambda-1)} u(t-\lambda-1)\, \delta(\lambda-2) d\lambda$

$= \boxed{e^{-(t-3)} u(t-3)}$ causal

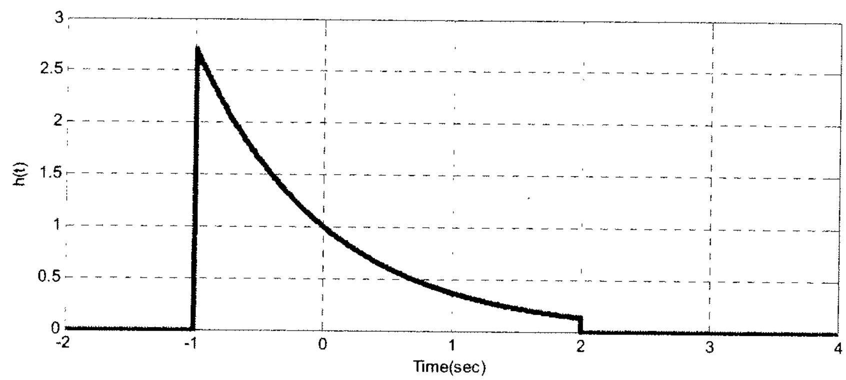
Name ______________________________ Mailbox ____________

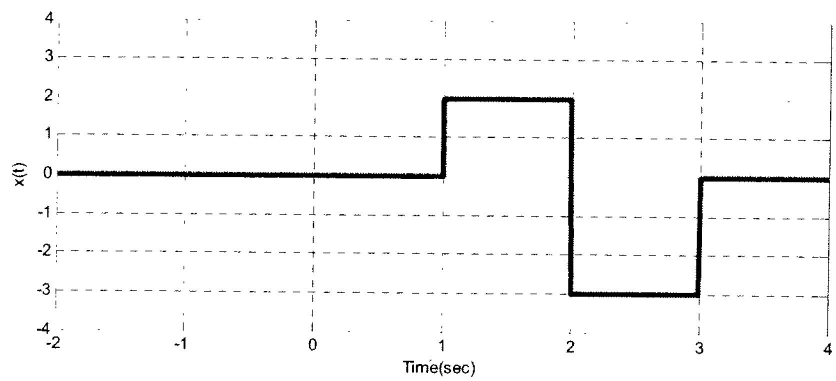
4) (25 points) Consider a noncausal linear time invariant system with impulse response given by

$$h(t) = e^{-t}[u(t+1) - u(t-2)]$$

The input to the system is given by

$$x(t) = 2u(t-1) - 5u(t-2) + 3u(t-3)$$





Using ***graphical convolution***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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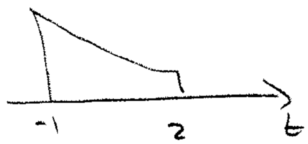
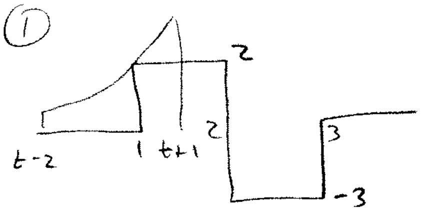
$h(t)$



$$h(-1) = h(t-\lambda) \qquad -1 = t-\lambda \qquad \lambda = t+1$$

$$h(2) = h(t-\lambda) \qquad 2 = t-\lambda \qquad \lambda = t-2$$

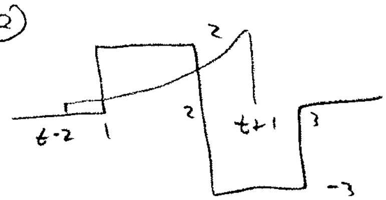
(1)



$0 \le t \le 1$

$$y(t) = \int_1^{t+1} e^{-(t-\lambda)}(2)\,d\lambda$$

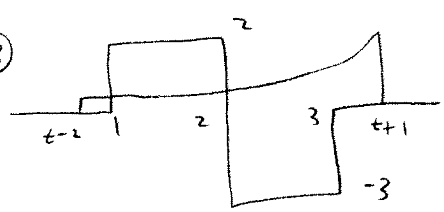
(2)



$1 \le t \le 2$

$$y(t) = \int_1^{2} e^{-(t-\lambda)}(2)\,d\lambda + \int_2^{t+1} e^{-(t-\lambda)}(-3)\,d\lambda$$

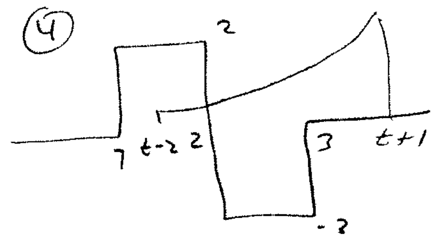
(3)



$2 \le t \le 3$

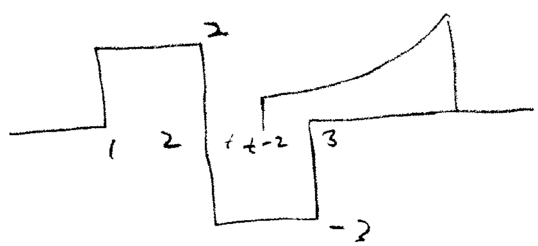
$$y(t) = \int_1^{2} e^{-(t-\lambda)}(2)\,d\lambda + \int_2^{3} e^{-(t-\lambda)}(-3)\,d\lambda$$

(4)



$3 \le t \le 4$

$$y(t) = \int_{t-2}^{2} e^{-(t-\lambda)}\,2\,d\lambda + \int_2^{3} e^{-(t-\lambda)}(-3)\,d\lambda$$



$4 \le t \le 5$

$$y(t) = \int_{t-2}^{3} e^{-(t-\lambda)}(-3)\,d\lambda$$

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Multiple Choice Problems (4 points each)

5) Assume we have a first order system in standard form, and the input is a step. The usual form used to compute the response of the system is

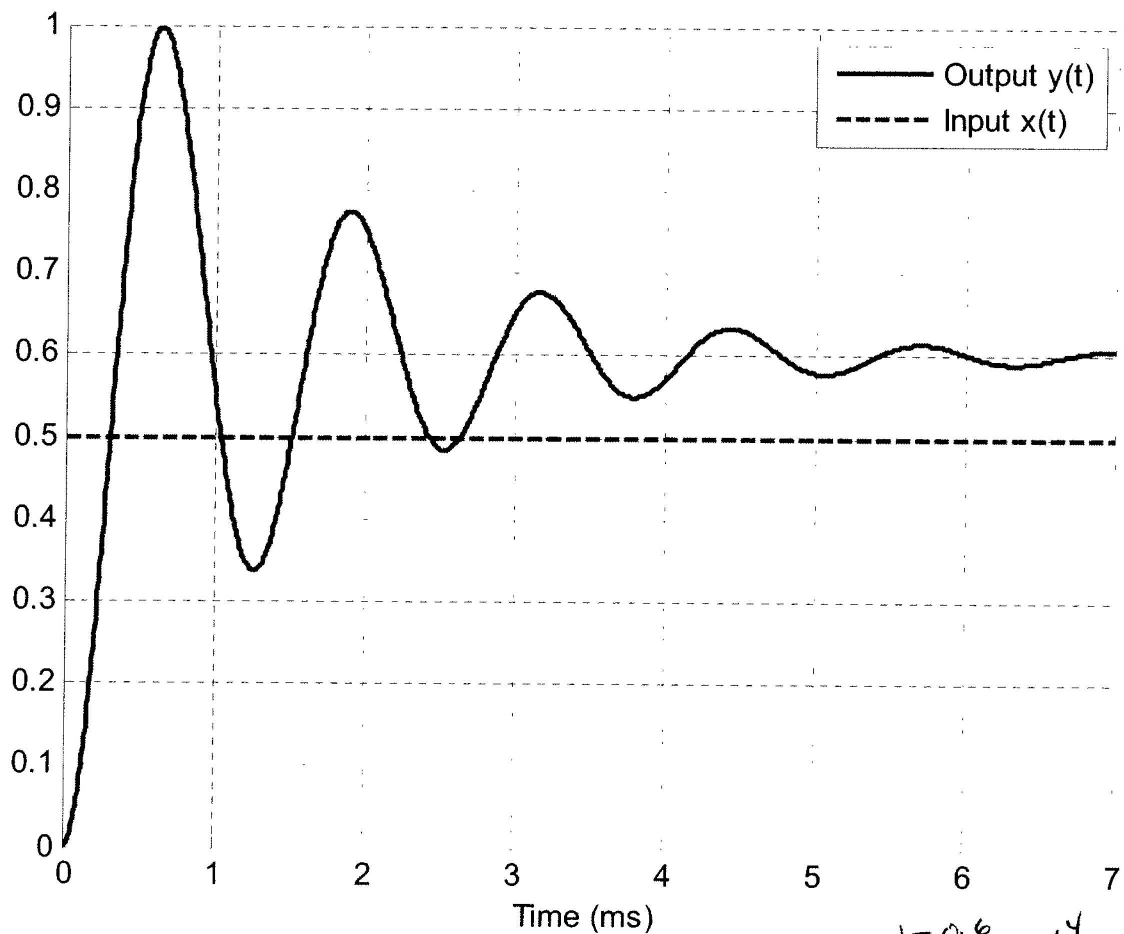
check $y(t)\big|_{t=0} = y(0)$

$y(t)\big|_{t=\infty} = y(\infty)$

a) $y(t) = [y(0) - y(\infty)]e^{-t/\tau} + y(0)$ b) $y(t) = [y(\infty) - y(0)]e^{-t/\tau} + y(0)$

c) $y(t) = [y(\infty) - y(0)]e^{-t/\tau} + y(\infty)$ $\boxed{\text{d) } y(t) = [y(0) - y(\infty)]e^{-t/\tau} + y(\infty)}$

Problems 6 and 7 refer the following graph showing the response of a second order system to a step input.



6) The percent overshoot for this system is best estimated as

$PO = \frac{1-0.6}{0.6} = \frac{.4}{.6} = 2/3$

$= 67\%$

a) 200 % b) 150 % c) 100% $\boxed{\text{d) 67 \%}}$ e) 50 % f) 33%

7) The static gain for this system is best estimated as

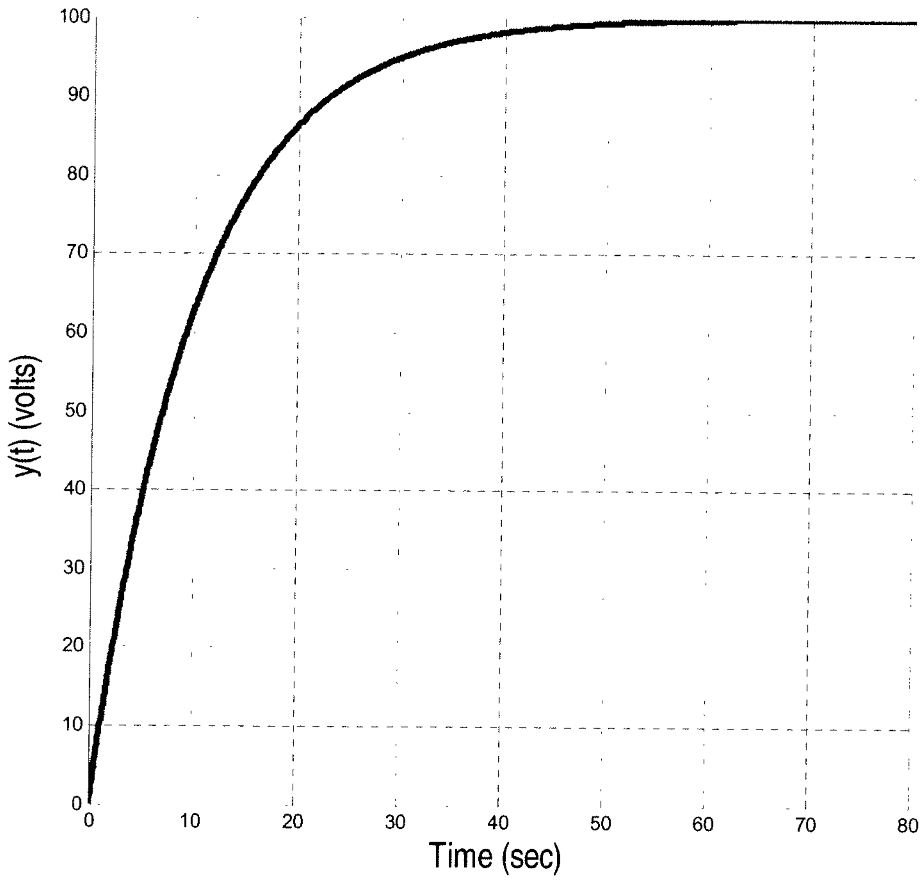
$K(0.5) = 0.6$

$K = \frac{6}{5} = 1.2$

a) 0.1 b) 0.5 c) 1.0 $\boxed{\text{d) 1.2}}$ e) 1.5 d) 2.0

Name ______________________ Mailbox ____________

8) The following figure shows a capacitor charging.



Based on this figure, the best estimate of the **time constant** for this system is $4\tau = 98 \approx 40 sec$

a) 5 sec $\boxed{\text{b) 10 sec}}$ c) 15 sec d) 20 sec e) 30 sec f) 40 sec $\tau = \frac{40}{4} = 10 sec$

9) For the second order equation $\ddot{y}(t) + 4\dot{y}(t) + 5y(t) = x(t)$ with an input $x(t) = u(t)$, we should look for a solution of the form

a) $y(t) = ce^{-2t}\sin(t+\theta)+1$ b) $y(t) = ce^{-t}\sin(2t+\theta)+1$ c) $y(t) = ce^{-t}\sin(2t+\theta)+5$

d) $y(t) = ce^{-2t}\sin(t+\theta)+5$ e) $y(t) = ce^{2t}\sin(t+\theta)+5$ $\boxed{\text{f) none of these}}$

$r^2 + 4r + 5 = 0$ $5y(\infty) = x(\infty)$ $5y(\infty) = 1$ $y(\infty) = 1/5$