A Control Systems Lab Sequence Designed to Foster Understanding[1]

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Abstract

Rose-Hulman Institute of Technology has a unique sophomore curriculum that culminates in a multi-disciplinary system dynamics course. Because of this curriculum, seniors entering their only required control systems course in the mechanical engineering curriculum have a background that differs from typical engineering upperclassmen. In particular, they have already seen the topics of modeling using transfer functions, state space and simulation diagrams, frequency response, and the rules for sketching Bode plots, and typical response of first and second order linear systems. With this in mind, the author has developed a series of laboratories to foster understanding of control systems topics. In particular, labs 2 and 4 seek to enhance understanding of the Bode and Root Locus plots respectively by requiring the students to generate these plots from experimental data. The sequence begins with time and frequency domain system identification, segues through basic control actions, experimental determination of the Root Locus plot, and ends with four design methodologies applied to single and two degree of freedom plants.

In ME 406, we emphasize *model based* design. In particular, we find parametric models of the rectilinear plant in one degree of freedom (1 DOF) and two degree of freedom (2 DOF) modes. The quality of control delivered from the ensuing model is directly limited by the quality of the model. A model of the system is necessary for the initial design of a controller, but the predicted response of the system may not match the true system response due to the simplified models being used.

Introduction

As recipients of an NSF CCLI grant, the mechanical engineering and electrical engineering faculty at Rose-Hulman (RHIT) are currently upgrading the system dynamics and control laboratory. The investigators have chosen the Educational Control Products (ECP) Rectilinear Control System[1], shown in Figure 1, as the primary hardware plant used in the system

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dynamics and control courses. This plant has three adjustable mass carts on low friction slider bearings. The masses can be connected to each other and to the bench by various springs, and an adjustable air damper. The actuator is a direct current servo-motor which is rigidly attached to the first mass through a rack and pinion. The position of each mass is detected by a high-resolution optical encoder with a precision of 2196 counts per centimeter of travel. Open loop and closed loop control of the plant is facilitated by A/D and D/A interface with a desktop personal computer. ECP supplies a Windows based software interface with a wide assortment of control architectures, including implementation of continuous time transfer function controllers, direct digital designs, and state feedback.

This laboratory upgrade provided the author with an opportunity to rethink the introductory control theory course taught to mechanical engineering seniors. The course schedule was amended to provide a weekly three-hour lab session. Course topics were rearranged to leverage the students' previous experience with system modeling in the time and frequency domain, then to provide the essential knowledge for success in root locus control design. System modeling was limited to translational mechanical systems with emphasis on the various forms of system models including differential equations, transfer functions, simulation diagrams, and state space descriptions. The design topics also included direct pole placement design using state feedback, and dynamic feedback path design by the Diophantine equation. These final topics demonstrated the algorithmic approach to control design, and were intended to illustrate the advantages of including more sensors in the control system.
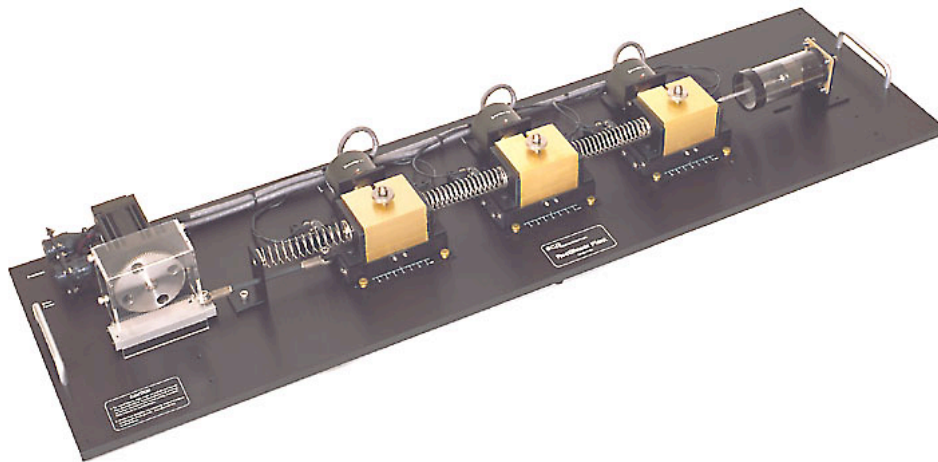
Fig. 1. The ECP Rectilinear Control System

The rest of this paper is organized into two main sections describing the system modeling labs and the control design labs respectively. It then concludes with lessons learned, and thoughts for improvement in the lab sequence.

System Modeling Labs

Lab 1. In this laboratory, the students compare the response of several 1 DOF systems. They first investigate the effects of varying system mass, stiffness, and damping. By using the known differences between effective system inertia, they are then able to solve an over-determined system of equations to get a good estimate of system effective mass and stiffness. System

damping then follows from a lumped parameter model of the system. Although the linear models match the experimental model well, there are some differences due to unmodeled non-linearities, and violation of the lumped parameter assumption in the real system. This lab emphasizes second order response characteristics, and basic mechanical system modeling.

The students are required to measure experimental step response peak time (t_p) and percent overshoot (M_p). They can then compute damped natural frequency (ω_d) directly from peak time using Eq. 1, and damping ratio (ζ) from Eq. 2

$$t_p = \frac{\pi}{\omega_d} \tag{1}$$

$$\zeta = \sqrt{\frac{\left(\ln(M_p)\right)^2}{\left(\ln(M_p)\right)^2 + \pi^2}} \tag{2}$$

The cart mass can be varied in increments of 500g. It is then possible to determine a parametric second order transfer function model of the system by combining the results from four or more cases. That is, the known difference between system effective mass values is combined in an over-determined system that can be solved for the mass values and the common stiffness.

$$\begin{bmatrix} \omega_{d1}^2 & 0 & \cdots & \left(\zeta_1^2 - 1\right) \\ -1 & 1 & \cdots & 0 \\ 0 & \omega_{d2}^2 & \cdots & \left(\zeta_2^2 - 1\right) \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{Bmatrix} m_1 \\ m_2 \\ \vdots \\ k \end{Bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ \vdots \end{bmatrix} \tag{3}$$

For complete details of this lab, see Burchett.[2]

Lab 2. In this laboratory, the students use the system experimental frequency response to determine a transfer function model of the system in 2 DOF mode with rigid body dynamics included. Building on the parameters determined in Lab 1, and using lumped parameter modeling, they are able to construct a parametric state-space model. This lab emphasizes the meaning of Bode plots, rigid body dynamics, transmission zeros, and advanced mechanical system modeling.

For this lab, the system is set up with the first two masses connected to each other by a stiff spring, as shown in Figure 2. Neither mass is anchored to the base, that is, the overall system is free to roll on the bearings. Cart 1 is loaded with two 500g masses, and cart 2 has four 500g masses. This set-up is an example of position control through a flexible structure. The state space and transfer function models were derived in class, illustrating that this is a *gray box* identification problem. That is, the form of the system model is known, but nearly all of the parameters are unknown. The effective mass of cart 1, including inertia of the motor, rack and pinion, is known from the results of Lab 1.
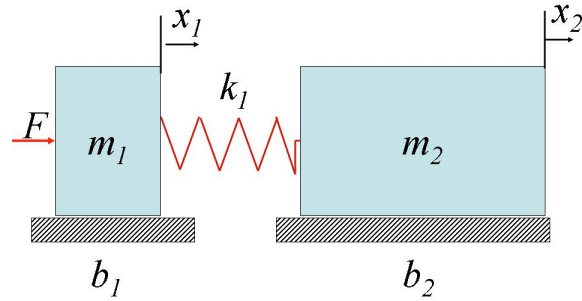
Fig. 2. Plant Schematic for Lab 2

This contrived configuration results in a Bode plot that is relatively simple yet emphasizes almost every feature imaginable. Figure 3 is a typical result, demonstrating the features of: the rigid body mode, seen as the -20 decibel per decade slope at low frequency; the resonant peak on both curves due to the lightly damped vibrational mode; the lightly damped trough due to the underdamped zero pair in the transfer function from force to x_1, and the difference in high frequency slopes since the transfer function from force to x_2 has four poles and no zeros, and the other transfer function has four poles and two zeros. This plot also illustrates that a system has a common system determinant regardless of which input / output pair is chosen. Finally, the deep trough of the zero pair provides an opportunity to explain zeros using the physical example of a passive vibration damper.
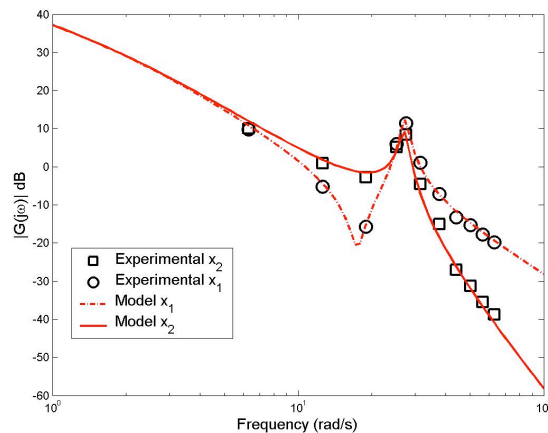
Fig. 3. Typical System Identification Results using Frequency Response.

The best state space model is found by comparing theoretical Bode magnitude values in dB with the experimental data. The Nelder-Mead direct search algorithm, provided in the Matlab *fminsearch* function, is used to optimize the unknown system parameters. This search algorithm finds the nearest local minimum, and thus convergence is sensitive to the initial guess. Students found that the quality of their model fit was hindered if their experimental data did not capture the steady state behavior. For complete details of lab 2, see Burchett and Layton[3].

Lab 3. In this laboratory, the students investigate the properties of basic control actions. Using the system mass determined in Lab 1, and system gain from the user's manual, they form a double-integrator plant model. The hardware is configured in 1 DOF mode with no spring or damper. The theoretical transfer function model is $G(s) = 1/ms^2$. The students are then able to directly select the P I and D gains to match closed-loop requirements. Students are encouraged to physically feel the control forces due to P, D, and I control. They should then be able to better quantify the effects of each type of feedback. Interested students are encouraged to try Ziegler-Nichols tuning of P and PID gains for extra credit.

Lab 4. In this laboratory, the students construct a Root Locus plot from experimental system response data. The double integrator plant of Lab 3 is revisited. The Students first collect experimental closed-loop response data using proportional control only. At least ten gains are used, preferably exploring the region of marginal stability. The root locus plot is then constructed by inferring closed-loop pole locations from 2nd order response characteristics. I.E., the peak time is used to determine system damped natural frequency, and the percent overshoot is used to determine damping ratio. These quantities can then be converted to the real and imaginary coordinates of the system closed-loop poles.

The students should discover that the hardware does not behave exactly as the model would predict. Due to internal friction, and delays in the control system, the root locus appears to be at least third order. Stability varies from strictly stable to marginally stable to unstable, whereas the double-integrator plant expected behavior is marginal stability for all gains. By requiring the student to perform a physical experiment, and construct a root locus plot from their experimental data, understanding of the significance of the root locus plot is greatly enhanced. As one lab group commented in their lab 4 write-up "The mystery that was the root locus plot has now been unraveled."
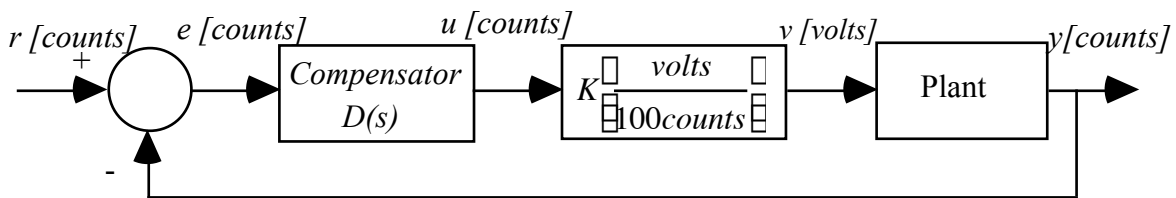
Control Design Labs

The last five labs focused on five distinct control design techniques. Lab 5 provides the students an opportunity to revisit the PID control method in the context of root locus, and uses virtual system response only. Labs 6 and 7 used the 1 DOF plant with forward path cascade control. The control architecture is shown in Figure 4. Labs 8 and 9 used the 2 DOF plant. Lab 8 uses the state feedback control architecture shown in Figure 5. Lab 9 uses a dynamic feedback path and prefilter architecture shown in Figure 6. By plotting control effort of a simple proportional controller, the author discovered that when operating in closed-loop mode, the ECP software includes a hidden gain of 1/100 between the controller output and the voltage sent to the servo motor. This gain is shown explicitly in Figures 4 and 6.

The 1 DOF plant is fairly easy to control. Labs 6 and 7 show simple position control, i.e. how a feedback controller can speed up the system, and decrease steady state errors. Students are encouraged to compare open-loop and closed-loop performance. Labs 7 and 8 require position control through a flexible structure, the spring. This plant set-up is much more difficult to control. Open-loop step response is unbounded, and simple proportional control gives unacceptable performance.

Lab 5. This is a software lab, used to review some of the features of the Matlab Control Toolbox. The CAD package SISOTOOL is introduced. The students perform computer aided design of P, PI, PD, and PID controllers. This exercise helps to reinforce the relationship between closed-loop pole locations and system step response characteristics. The software dynamically links the Root Locus and Bode plots, and the closed-loop step response. The students are then able to quickly iterate through many possible designs, and immediately see the effects of control loop design changes.

Lab 6. In this lab the students design a lead-lag cascade controller for the 1 DOF system. They then implement the controller on the hardware plant, and iterate their design to meet a steady-state error requirement. The professor suggested relatively fast closed-loop pole locations in order to take advantage of as much actuator bandwidth as possible. Students found when using fast pole locations that the hardware step response was very similar to that predicted by their software model using the Matlab SISOTOOL and LTIVIEW features.

r [counts] + *e [counts]* → Compensator D(s) → *u [counts]* → $K\left[\dfrac{volts}{100counts}\right]$ → *v [volts]* → Plant → *y[counts]*
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Fig. 4. Control System Architecture for Labs 6 and 7.

Lab 7. In this lab the students design a notch-lag controller for the 1 DOF system identified in Lab 1. They use SISOTOOL extensively for this design. After implementing their design in the lab, they are encouraged to investigate ways to increase closed-loop gain and hence decrease steady state error.

In the fall quarter of 2004, Labs 6 and 7 were combined into a single experience. The control system architecture, shown in Figure 4, is the same for both labs. The cascade controller used in lab 6 is second order. In lab 7, the controller turns out to be third order--consisting of a second order notch zero / pole set and a first order lag controller. Students found that their plant model had to be very accurate in order to get good performance from the notch-lag controller. The lead-lag design technique was thus shown to be more *robust* to modeling errors. The use of SISOTOOL in control design allowed the students to rapidly iterate their designs. They were freed from tedious calculations, and thus able to focus on the physical effect of choosing faster or slower compensator pole and zero locations.

Table 1. Suggested Closed-Loop pole locations for Lab 8.

Set 1	Set 2	Set 3	Set 4
$s = -7.533 \pm 27.28j$	$s = -8.7164 \pm 26.7j$	$s = -9.3632 \pm 26.0j$	$s = -9.6046 \pm 25.516j$
$-18.53 \pm 10.35j$	$-23.59 \pm 9.53j$	-35.1	-60.124
		-26.96	-22.841

Lab 8. In this lab the students use Ackermann's formula to directly place the closed-loop poles at desired location using state feedback. The 2 DOF plant identified in Lab 2 is used. The professor provides four sets of good closed-loop poles based on the LQR root locus. The four pole sets

(shown in Table 1) were chosen by progressively decreasing the control effort penalty in the LQR solution. Pole set 4 pushes the system performance near the actuator saturation limit on most systems. Since the feedback gains are unique for any specified set of closed-loop poles, the feedback gains found using Ackermann's formula will be identical to the LQR gains. Performance is shown to improve drastically by using state feedback. The students are required to plot the actual control effort, that is the voltage sent to the servo motor. The absolute limit of control system performance should become obvious considering that the motor input voltage is limited to ±5 volts. Since the plant model varies significantly between set-ups, the actual LQR pole locations will also vary, however, the controller found using LQR is very robust, and thus the student designed state feedback controller is also very robust.
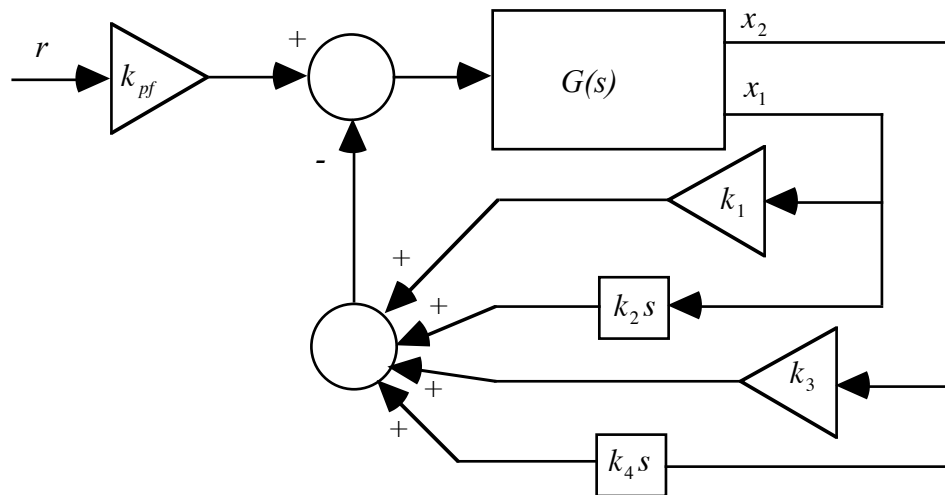

Fig. 5. Control System Architecture for Lab 8.

Lab 9. In this lab the students attempt to match the performance of the state feedback controller by choosing the desired closed loop poles and designing dynamic pre-filter and return path compensation by solving the Diophantine equations. The ECP software provides for this control architecture with a single input and output. The resulting control system is expected to exhibit somewhat degraded performance when compared to Lab 8, since only one feedback measurement is used as opposed to four. This design methodology demonstrates the use of a known design algorithm, and illustrates how such algorithms generally result in a compensator having the same order as the plant. Using this approach also provides an opportunity to discuss the concept of observers without exploring the theory of observer design *per se*. The dynamic return path effectively contains observer dynamics that are transparent to the user.

The professor provided a Matlab function designed to compute full and reduced order dynamic feedback path compensators given a plant model and set of desired closed-loop poles. The students were encouraged to first try the fastest and slowest pole locations provided in Lab 8. After determining the dynamic feedback path, the students were required to design a prefilter that canceled out the unwanted 'estimator' poles.
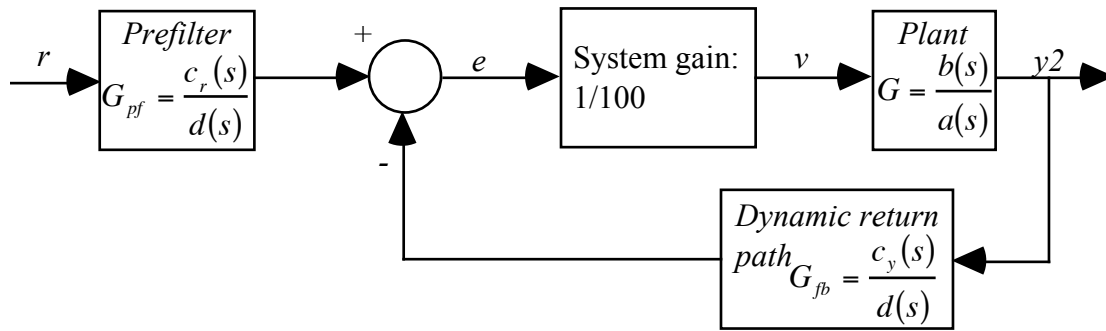
Fig. 6. Control System Architecture for Lab 9.

An interesting, unanticipated result of this method was that the full order design using the Diophantine equation resulted in a conditionally stable system. This is clearly seen in the system root locus plot of Figure 7. There are two poles at the origin, one from the plant and one from the compensator. The initial tendency of these poles it to migrate into the right half plane. Thus, the system would be unstable if the students set the gain too *low* as well as too *high*. This feature required the students to pay close attention to unit conversions, and provided a good experience of how unforgiving some systems can be.
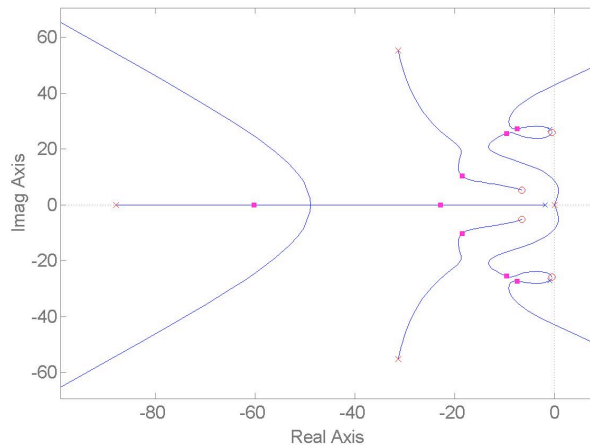

Fig. 7. Root Locus for Full Order Diophantine Direct Design

Lessons Learned and Thoughts for Improvement

The system modeling labs turned out to be a great way to illustrate the importance of theoretical lumped parameter modeling. Lab 1 provided an opportunity to emphasize how experimentation should provide an over-determined system, i.e. more equations than unknowns, resulting in a *least squares* solution. Many of our students who have extensively used a computer algebra system to solve systems of equations found this to be a major paradigm shift. Labs 1 and 2 demonstrated that knowing the form of the appropriate model is a major part of determining the system parameters. In lab 2, an optimization technique was used to determine the unknown parameters. Once again, a direct solution was not available, and the final result was based on a *least squares* fit. Students found this anaylsis cumbersome. However, those groups that checked the quality of their data every step of the way arrived at a good solution with a much smaller time investment. For instance, in the data collecting phase, insuring that the system reaches

steady state is very important, and worth the effort of checking immediately after each individual frequency response. After all frequencies are collected, the experimental Bode magnitude plot should be generated to see that it conforms to the expected shape. Outlier data are then easily detected as any frequency that does not conform to the pattern. In every case where these steps were performed, the optimization did converge to an appropriate model.

Using PID controllers in Lab 3 provided a tremendous opportunity to illustrate the basic control efforts, and how each contributes to transient and steady state response characteristics. By physically pushing on the controlled mass with the control loop active, students could actually feel that proportional control acts as a virtual spring, and derivative control acts as a virtual damper. By displacing and holding the mass with integral control, students can feel the force build up over time, thus emphasizing the steady state error canceling property of integral control.

By requiring the students to generate a root locus from experimental data, the utility and significance of the root locus plot was clearly demonstrated. They are afforded the opportunity to connect changing the forward path gain on the hardware with a resulting change in the closed-loop pole locations.

The control system CAD package SISOTOOL was introduced in Lab 5, and used throughout the subsequent labs. This author has concluded that SISOTOOL is tremendous for allowing students to avoid tedious calculations when iterating a control system design. Students in ME406 are required to use hand calculations to show that simple cascade controllers with real poles and zeros meet the angle and magnitude criteria. They are also required to sketch root locus and Bode plots by hand. However, when it comes to understanding the relationship between closed-loop pole locations and transient response, or open-loop Bode gain and steady state error performance, SISOTOOL is tremendously powerful at demonstrating these relationships, and at freeing the student to focus on the effect of control loop modifications, rather than the underlying calculations required to generate such plots.

Students discovered that the best way to deal with the hidden 1/100 gain is to lump it into the plant model used for design. When this gain is included in the plant, the gains determined in SISOTOOL could be directly applied to the hardware, thus avoiding an intermediate units correction calculation.

By combining Labs 6 and 7 as described above, the lab emphasized a comparison between two compensation methods as well as a comparison between open and closed-loop performance. Notch control was found to be very sensitive to modeling errors, while lead-lag control was considerably more robust.

Labs 8 and 9 demonstrated control through a flexible manipulator. The system was configured with a rigid body mode so that open loop control was impossible. The class was introduced to the idea of algorithmic control system design by the methods of Ackermann's formula and the Diophantine Equation. Lab 8 used full state feedback, and as such a very fast and robust controller was found. Students were required to plot the control effort, and thus realize that there are limitations to the achievable performance. Lab 9 used a dynamic feedback path to

reconstruct the system state from a single measurement, and place the closed loop poles at locations chosen using LQR.

Student comments included: 'The lab work helped reinforce the class work', 'Labs strongly reinforced the course material', and 'Labs really reflected what was going on in class', '…the class really seemed to grasp the concepts when we went through examples and working through the topic in the labs', 'I liked the emphasis on the lab and the way it stayed connected to the work done in class', 'The lab work really let you see the stuff you learned in a slightly different environment', and 'Fun labs, help understanding'. A few students complained that the basic ECP user interface made digital implementation of the control system transfer functions 'too transparent'. That is, in a few cases, students were interested in learning how to implement dynamic controllers in a low-level language, so their skills would be portable to any system. Perhaps by migrating to the SIMULINK real-time interface, such low-level implementation could be demonstrated in the lab next year.

In conclusion, the lab sequence seems to have accomplished its primary goal in its inaugural presentation. The course and lab sequence will see continued improvement next year by incorporating new torsional plants, and perhaps making use of the SIMULINK real-time interface.

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Author Biography
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