Paper 2005-493

Frequency Domain System Identification of One, Two, and Three Degree of Freedom Systems in an Introductory Controls Class

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Abstract

We have developed a sequence of laboratories for our introductory controls classes to identify and control one, two, and three degree of freedom mass-spring-damper systems. Initial estimates of damping ratios and natural frequencies are made using the log-decrement method with only one cart free to move at a time. By exciting the system at various frequencies the magnitude portion of the Bode plot for the transfer functions between the input and the position of each cart is determined, and the resonant frequencies are estimated. Once the parameters for the transfer functions have been determined and the corresponding state variable model is identified the students control the positions of the carts with state variable control. This sequence of experiments has a number of practical benefits. By exciting these systems with various frequencies, the students understand transient response and sinusoidal steady state in terms of the physical behavior of mechanical systems. The students also develop a better appreciation for the use of Bode plots. Finally, nearly all of the parameters can be estimated in at least two ways and the students must compare these estimates to check for consistency.

Background

Eleven ECP-210a spring/mass/damper rectilinear systems were purchased through an NSF CCLI grant obtained by investigators from both the Electrical and Computer Engineering and Mechanical Engineering departments at Rose-Hulman Institute of Technology. These systems allow for easy implementation of different standard controller types and are easily reconfigurable. Figure 1 shows one of the "carts" of the system, connected with two springs. The position encoder is shown toward the back of the system. The carts are moved via a motor with a rack and pinion mechanism.

Faculty from both the Mechanical Engineering and Electrical and Computer Engineering departments have developed weekly 3 hour labs utilizing these systems in their introductory control systems classes. In the ECE introductory class the students regularly utilize these systems to first identify and then control a one degree of freedom system (only one cart moves) utilizing classical control techniques including root locus methods and various model matching methods. Once the students have been introduced to state variable methods about halfway though the course, they first try to control the position of the cart in the one degree of freedom system. In the final two labs in the class the students utilize both time domain and frequency domain methods to identify the systems for the two and three degree of freedom systems, and then use state variable methods to control the systems.

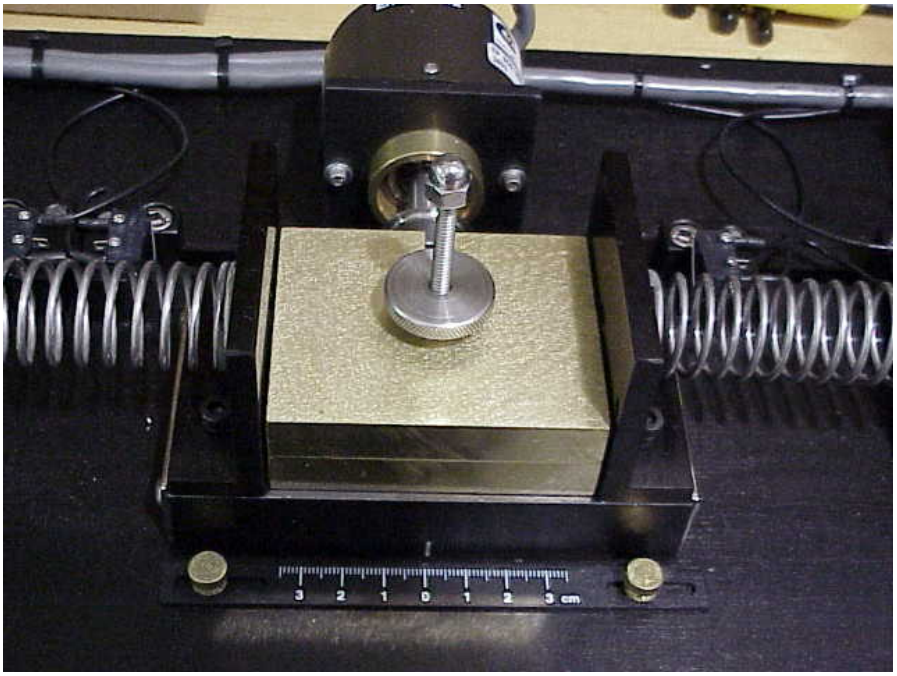

Figure 1: One cart (with variable masses) connected to two (variable) springs of the ECP-210a system. The position encoder is shown in the background.

During lab the students first determine the natural frequency and damping ratio for each cart individually, when it is the only cart free to move. Next they look at the response of a step input to determine the gain for each transfer function. Finally the students excite the systems at various frequencies to determine the magnitude portion of the Bode plot for the transfer functions between the input and the position of each cart, and the resonant frequencies are estimated. The students are given Matlab code to do the optimization to fit the transfer functions to the measured frequency response. Nearly all of the parameters can be estimated in at least two ways and the students must compare theses estimates to check for consistency.

System identification utilizing frequency domain response is a fairly common method of determining the transfer function of an unknown system.[1,2,3] Elementary circuit textbooks[4,5] introduce the concept of system identification utilizing the Bode plot as part pf their introduction to the frequency response characterization of circuits. Common experiments in typical signals and system courses include some type of frequency domain system identification of "black box" systems, usually low pass, high pass, or band pass filters[6]. Typical introductory control system texts[7,8] often include problems where students are given a Bode plot and asked to determine the system's transfer function. While students have clearly been presented with the idea of determining a system's transfer function from a Bode plot, they often do not understand how to construct a Bode plot experimentally, and are not used to putting the two ideas together.

In the next section the three different labs are discussed in more detail. In particular, the transfer functions and state models are presented. Then typical parameter values (initial and final estimates), and the fit between the estimated transfer functions and the measured frequency response is represented. The section concludes with examples of state variable feedback and sinusoidal steady state response for the three degree of freedom system.

Laboratory Descriptions

One Degree of Freedom System. The one degree of freedom system most commonly used in ECE-320 with classical control techniques can be modeled as shown in Figure 3. The transfer function for this model can easily be shown to be

$$\frac{X_1(s)}{F(s)} = \frac{K}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1}$$

where K is the static gain, ω_n is the natural frequency, and ζ is the damping ratio. In the time domain, both ζ and ω_n can be estimated using the log decrement method, and the static gain can be estimated by determining the ratio of the steady state value of the cart position $x_{1,ss}$ and the amplitude of the step response input. Next the system is excited at various frequencies, and a Bode plot of experimental data is constructed. This experimental frequency response data is compared with the frequency response of the transfer function to determine the parameters (K, ω_n, and ζ) again. A typical result is displayed in Figure 4, which shows the measured frequency response as discrete diamonds and the fit of the transfer function to the measured frequency response as a solid line. For this configuration the initial and final estimates of the transfer function parameters are shown in Table 1. These results are fairly typical, although the initial and final damping ratio estimates are not usually this close. It is emphasized to the students that they need to compare their initial estimates with the final estimates as a check on their work, and this comparison must be included in their report.

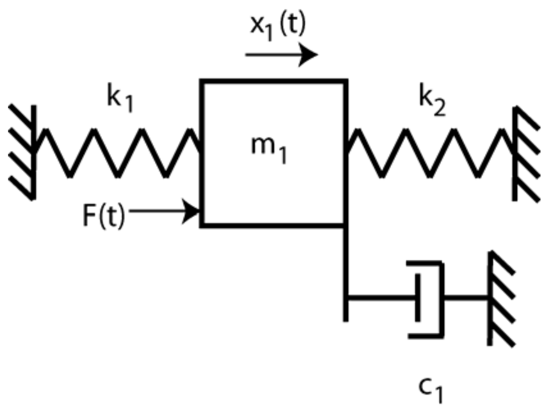

Figure 3. Model of a one degree of freedom system. Only one cart is free to move, there is at least one spring attached.

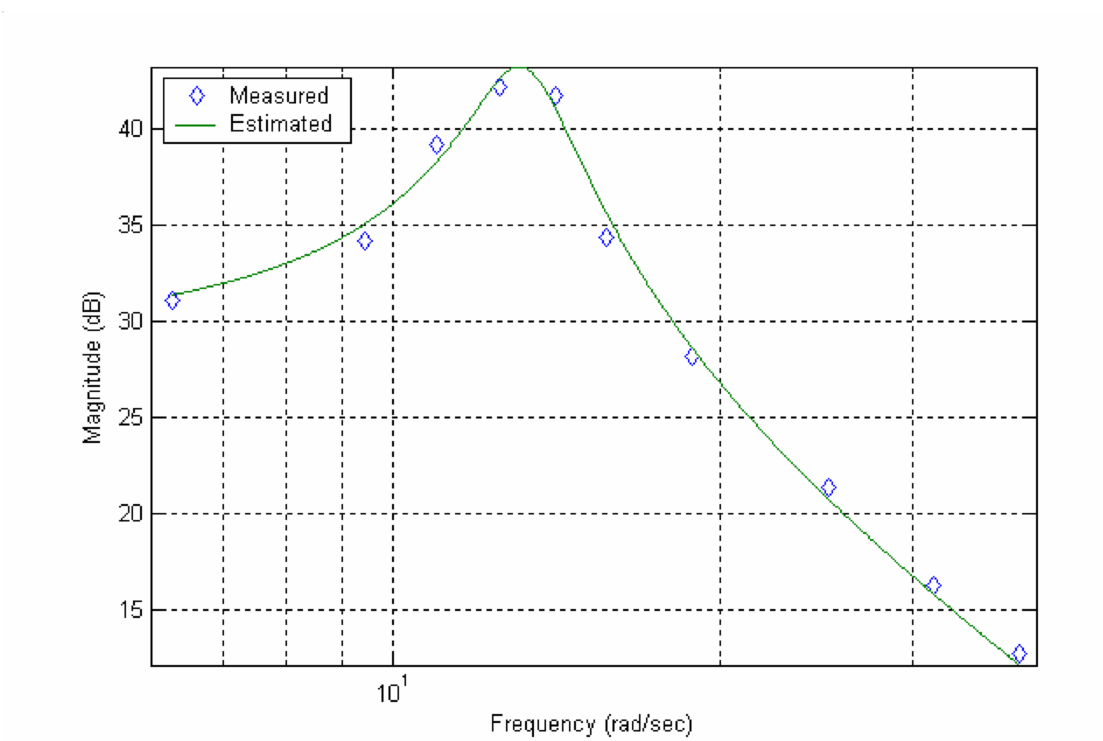

Figure 4: Fit of measured frequency response and estimated transfer function for the one degree of freedom system.

Parameter	Initial Estimate	Final Estimate
K	30.6	28.9
ω_n	13.2	13.2
ζ	0.10	0.10

Table 1. Initial and final estimates of the parameters for the one degree of freedom model.

In order to determine the state model, we define $q_1 = x_1$, and $q_2 = \dot{x}_1$, and obtain the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K\omega_n^2 \end{bmatrix} F$$

Clearly all of the parameters in the state variable model can be obtained from the transfer function for this system.

<u>*Two Degree of Freedom System.*</u> The two degree of freedom system we utilize can be modeled as shown in Figure 5. For our equations there must be at least two springs present. The transfer function between the position of the second cart and the input can be shown to be

$$\frac{X_2(s)}{F(s)} = \frac{K_2}{(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1)(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1)}$$

and the transfer function between the position of the first cart and the input can be shown to be

$$\frac{X_1(s)}{F(s)} = \frac{K_1\left(\frac{1}{\omega_2^2}s^2 + \frac{2\zeta_2}{\omega_2}s + 1\right)}{(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1)(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1)}$$

Here ω_a and ω_b are the resonant frequencies of the two cart system, which can be estimated while constructing the measured frequency response plots. K_1 and K_2 are the static gains, which are estimated by examining the step response of the system and computing the ratio of the steady state positions of the two carts and the amplitude of the step input. ω_2 and ζ_2 are the natural frequency and damping ratio of the second cart when it is the only cart moving. These are determined by the log decrement analysis. Finally, ζ_a and ζ_b are the damping ratios for the system. These are unknown, and we usually use the initial estimate that both of them are approximately 0.1. At this point we point out to the students the fact that both transfer functions have the same denominator (the characteristic polynomial) and this is very useful in attempting to fit the transfer function to the data. Since the transfer function for the second cart has no zeros, and the transfer function for the first cart has two zeros, it is easiest to match the frequency response for the second cart first. Once these parameters are fixed, the parameters of the numerator in the transfer function for the first cart can be determined. Typical transfer function estimates for the second cart and the first cart are shown in Figures 6 and 7, respectively.

For this configuration the initial and final estimates are shown in Table 2. Although the natural frequencies and damping ratios for the first cart do not explicitly appear in the transfer functions, they are recomputed as part of the transfer function computation. The results in this table are consistent with our usual results, in that our initial estimates of the natural frequencies and gains tend to match closely with the final estimates, but there are often large percentage changes in the damping ratios. Again, it is emphasized to the students that they need to compare their initial estimates with the final estimates as a check on their work, and include this comparison in their lab report.

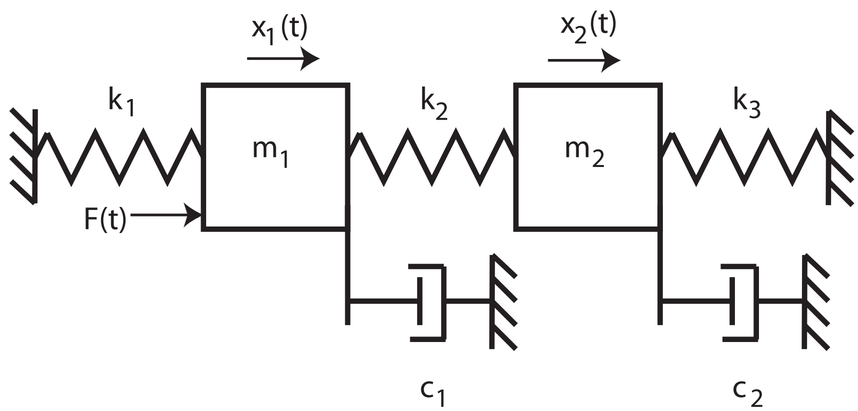

Figure 5. Two degree of freedom system. Two carts are free to move and there must be at least two springs.

Parameter	Initial Estimate	Final Estimate
K_1	25.6	24.3
K_2	17.2	16.4
ω_a	9.4	10.1
ω_b	37.7	37.3
ζ_a	0.10	0.11
ζ_b	0.10	0.03
ω_1	19.4	19.8
ω_2	32.1	32.9
ζ_1	0.03	0.05
ζ_2	0.02	0.04

Table 2. Initial and final estimates of the parameters for the two degree of freedom model.

In order to determine the state model, we define $q_1 = x_1$, $q_2 = \dot{x}_1$, $q_3 = x_2$, and $q_4 = \dot{x}_2$, and get the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\left(\frac{k_1+k_2}{m_1}\right) & -\left(\frac{c_1}{m_1}\right) & \left(\frac{k_2}{m_1}\right) & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{k_2}{m_2}\right) & 0 & -\left(\frac{k_2+k_3}{m_2}\right) & -\left(\frac{c_2}{m_2}\right) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{1}{m_1}\right) \\ 0 \\ 0 \end{bmatrix} F$$

As part of their homework assignment before this lab, the students are required to fill in the missing steps in the derivation of the transfer functions, the state variable model, and how to extract all quantities necessary from the transfer functions to determine the quantities needed in the state variable model.

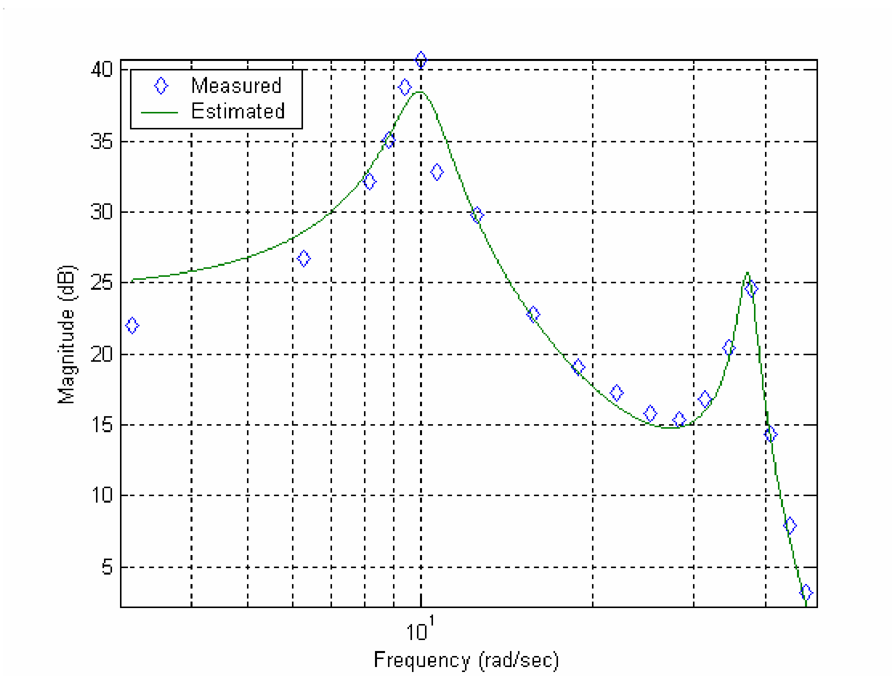

Figure 6. Fit of measured frequency response and estimated transfer function for the second cart, $X_2(s)/F(s)$, for the two degree of freedom system.

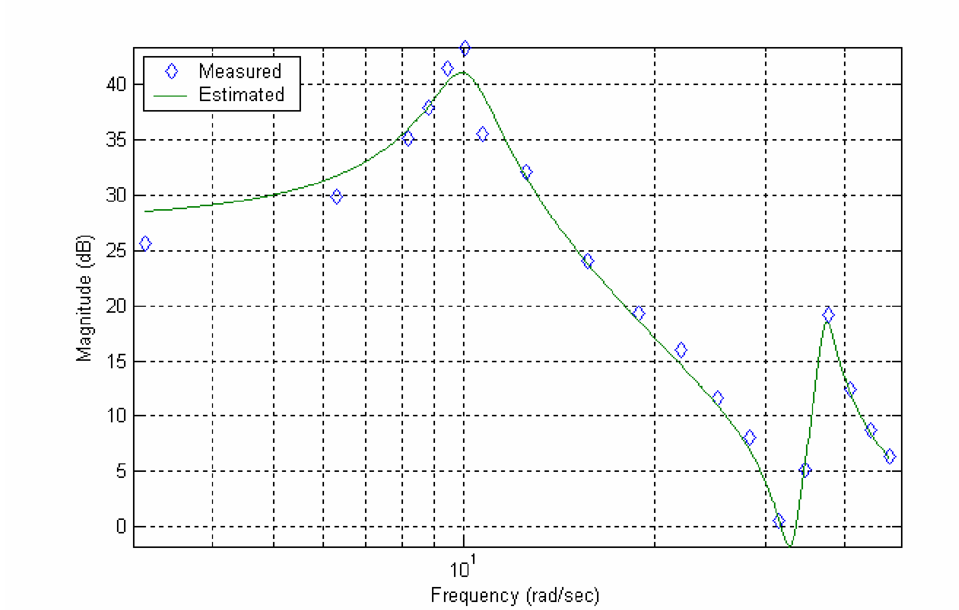

Figure 7. Fit of measured frequency response and estimated transfer function for the first cart, $X_1(s)/F(s)$, for the two degree of freedom system.

Three Degree of Freedom System. The three degree of freedom system we utilize can be modeled as shown in Figure 8. For our equations there must be at least three springs present. The transfer function between the position of the third cart and the input can be shown to be

$$\frac{X_3(s)}{F(s)} = \frac{K_3}{(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1)(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1)(\frac{1}{\omega_c^2}s^2 + \frac{2\zeta_c}{\omega_c}s + 1)}$$

Similarly the transfer function between the position of the second cart and the input is

$$\frac{X_2(s)}{F(s)} = \frac{K_2(\frac{1}{\omega_3^2}s^2 + \frac{2\zeta_3}{\omega_3}s + 1)}{(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1)(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1)(\frac{1}{\omega_c^2}s^2 + \frac{2\zeta_c}{\omega_c}s + 1)}$$

while the transfer function between the position of the first cart and the input is

$$\frac{X_1(s)}{F(s)} = \frac{K_1\left[(\frac{1}{\omega_x^2}s^2 + \frac{2\zeta_x}{\omega_x}s+1)(\frac{1}{\omega_y^2}s^2 + \frac{2\zeta_y}{\omega_y}s+1)\right]}{(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s+1)(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s+1)(\frac{1}{\omega_c^2}s^2 + \frac{2\zeta_c}{\omega_c}s+1)}$$

Here ω_a, ω_b, and ω_c are the resonant frequencies of the three cart system, which can be estimated while constructing the measured frequency response plots. K_1, K_2, and K_3 are the static gains, which are estimated by examining the step response of the system and computing the ratio of the steady state positions of the three carts and the amplitude of the step input. ω_3 and ζ_3 are the natural frequency and damping ratio of the third cart when it is the only cart moving. These are determined by the log decrement analysis. ζ_a, ζ_b, and ζ_c are the damping ratios for the system. These are unknown, and we usually use the initial estimate that they are all approximately 0.1.
ω_x and ω_y are the approximate frequencies of the nulls. Our initial estimate of these frequencies is determined by the initial frequency response measurements. Finally, ζ_x and ζ_y generally define the sharpness of the null, and are again approximated as 0.1. We again point out to the students the fact that all three transfer functions have the same denominator (the characteristic polynomial) and this is very useful in attempting to fit the transfer function to the data. Since the transfer function for the third cart has no zeros it is easiest to match the frequency response for the third cart first. Once these parameters are fixed, the parameters of the numerator in the transfer function for the second cart can be determined, since we have a good idea of all of the parameters. Finally the parameters for the transfer function for the first cart are determined. These are usually the parameters we have the least confidence in. Typical transfer function estimates for the third, second, and first cart are shown in Figures 9, 10, and 11, respectively. For this configuration the initial and final estimates are shown in Table 3. Although the natural frequencies and damping ratios for the first two carts do not explicitly appear in the transfer functions, they are recomputed as part of the transfer function computation. The results in this table are again consistent with our usual results, in that our initial estimates of the natural frequencies and gains tend to match closely with the final estimates, though there are often large percentage changes in the damping ratios.

Again, it is emphasized to the students that they need to compare their initial estimates with the final estimates as a check on their work, and include this comparison in their report.

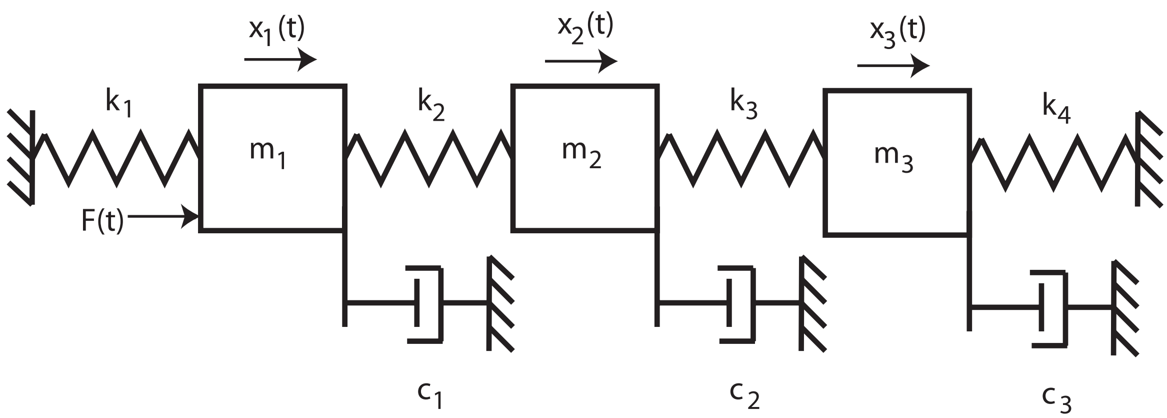

Figure 8: Three degree of freedom system. Three carts are free to move and there must be at least three springs.

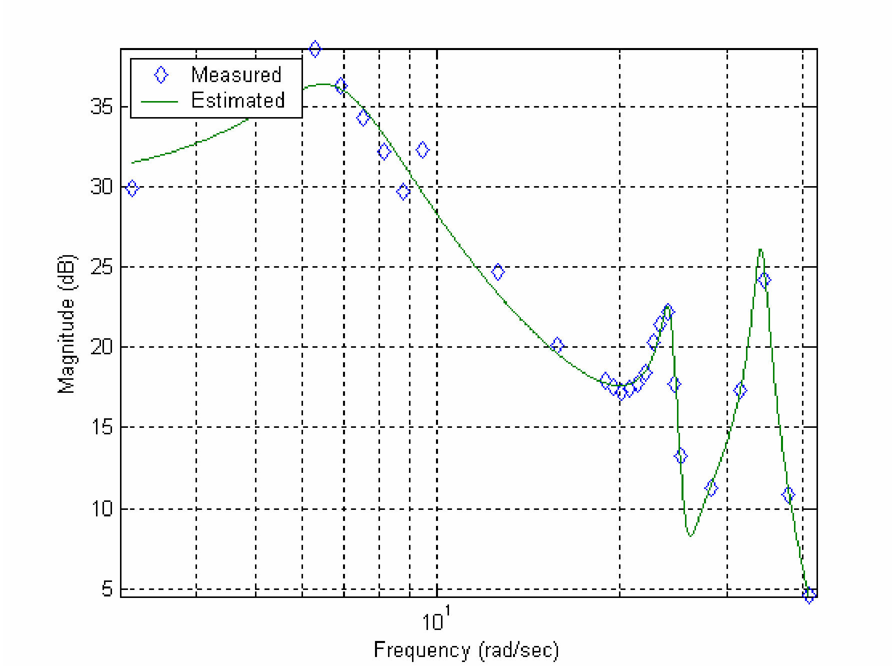

Figure 9. Fit of measured frequency response and estimated transfer function for the third cart, $X_3(s)/F(s)$, for the three degree of freedom system.

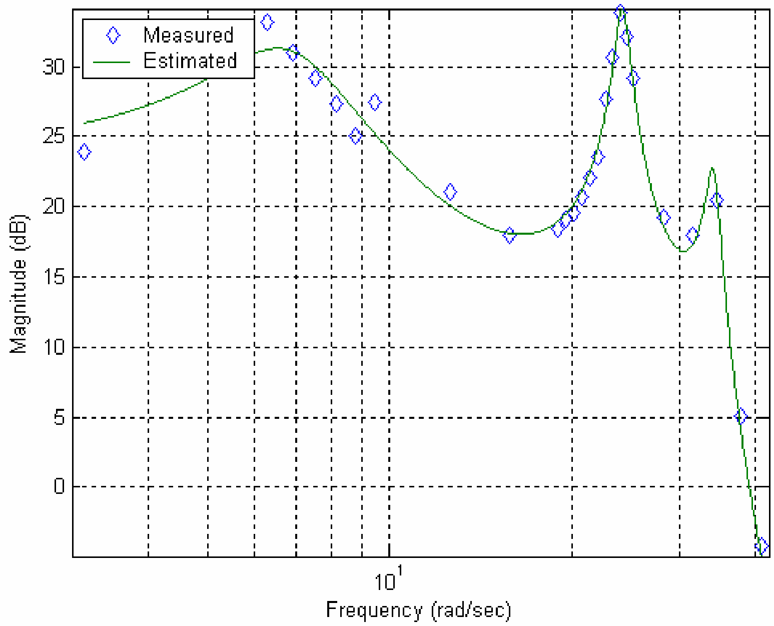

Figure 10. Fit of measured frequency response and estimated transfer function for the second cart, $X_2(s)/F(s)$, for the three degree of freedom system.

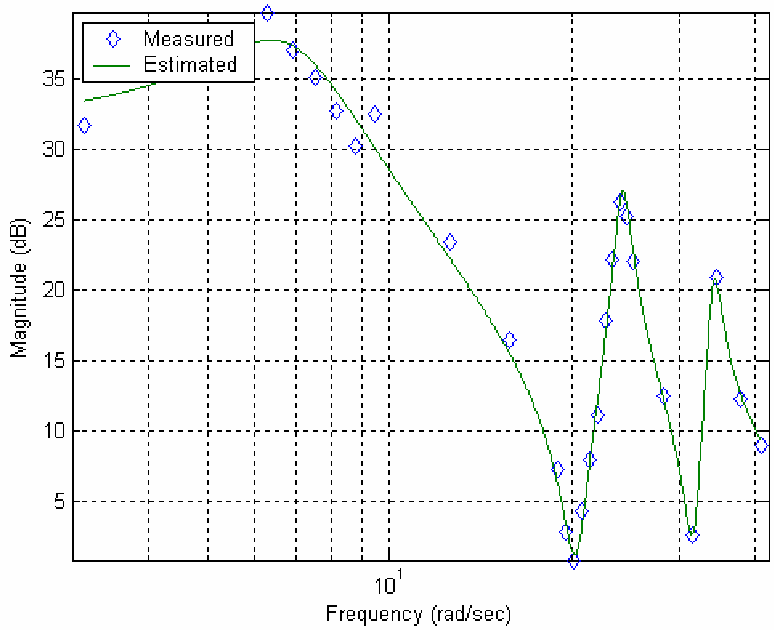

Figure 11. Fit of measured frequency response and estimated transfer function for the first cart, $X_1(s)/F(s)$, for the three degree of freedom system.

Parameter	Initial Estimate	Final Estimate
K_1	36.0	39.4
K_2	28.5	30.6
K_3	14.0	16.0
ω_a	6.3	6.9
ω_b	23.8	24.1
ω_c	36.6	34.1
ζ_a	0.10	0.25
ζ_b	0.10	0.03
ζ_c	0.10	0.03
ω_x	15.6	20.2
ω_y	31.4	31.6
ζ_x	0.1	0.04
ζ_y	0.2	0.03
ω_1	19.3	20.0
ω_2	26.2	27.2
ω_3	25.6	25.6
ζ_1	0.03	0.08
ζ_2	0.01	0.02
ζ_3	0.02	0.04

Table 3: Initial and final estimates of the parameters for the three degree of freedom model.

Defining $q_1 = x_1$, $q_2 = \dot{x}_1$, $q_3 = x_2$, $q_4 = \dot{x}_2$, $q_5 = x_3$, and $q_6 = \dot{x}_3$, we get the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\left(\frac{k_1+k_2}{m_1}\right) & -\left(\frac{c_1}{m_1}\right) & \left(\frac{k_2}{m_1}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \left(\frac{k_2}{m_2}\right) & 0 & -\left(\frac{k_2+k_3}{m_2}\right) & -\left(\frac{c_2}{m_2}\right) & \left(\frac{k_3}{m_2}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \left(\frac{k_3}{m_3}\right) & 0 & -\left(\frac{k_3+k_4}{m_3}\right) & -\left(\frac{c_3}{m_3}\right) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{1}{m_1}\right) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F$$

As part of their homework assignment before this lab, the students are required to fill in the missing steps in the derivation of the transfer functions, the state variable model, and how to extract all quantities necessary from the transfer functions to determine the quantities needed in the state variable model.

Sinusoidal Steady State. Although our students have been utilizing phasors in at least two previous circuits classes, in going through these labs it becomes quite clear that many of them do not really understand what sinusoidal steady state means. Figures 12 through 14 show the response of the three degree of freedom system analyzed previously when the system is excited by a 4 Hz sine wave. The motion of the first cart is displayed in Figure 12, of the second cart in Figure 13, and the final cart in Figure 14. These three responses are very typical of the types of sinusoidal responses the students see in measuring the frequency response of the system and constructing the Bode plot. Since the students need to find the ratio of the steady state output amplitude to the input amplitude at each frequency to construct the magnitude portion of the Bode plot, they often do not know which part of the response to use. Since these mechanical systems have relatively large time constants (compared to the circuits they have been analyzing) and provide a very visual physical demonstration of transient and steady state response, they make an ideal tool to illustrate the idea of sinusoidal steady state. In addition, the concept of the resonant frequency is also very clearly and visually demonstrated.

Step Response of the Three Degree of Freedom System. After modeling the systems in lab, the students then try and control the position of each cart, one at a time, using state variable feedback with a prefilter to adjust the gain based on the model. Specifically, they try and have each cart follow a 1 cm step input while limiting percent overshoot and settling time. In addition, they are required to compare the response of the real system with their model of the system. Typical results are shown in Figures 15 - 17, which show the step response of the three degree of freedom system we have modeled for a 1 cm step input. The goal in this case was to control the position of the second cart. As the figures show, the model and real system are fairly accurate for the first two carts, with more deviations in the third cart. In addition, the steady state values of the real system and the model often differ as they do in this example. Since these are the last labs in the course, the students are by now well aware that the discrepancies are most likely due to modeling errors, and are to be expected.

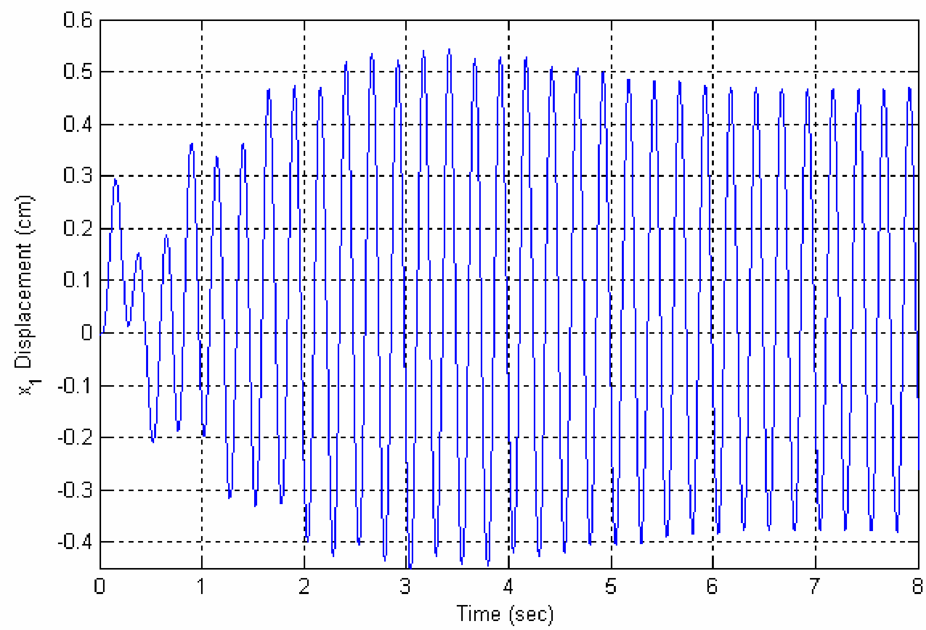

Figure 12. The motion of the first cart in the three degree of system when the system is excited by a 4 Hz sine wave.

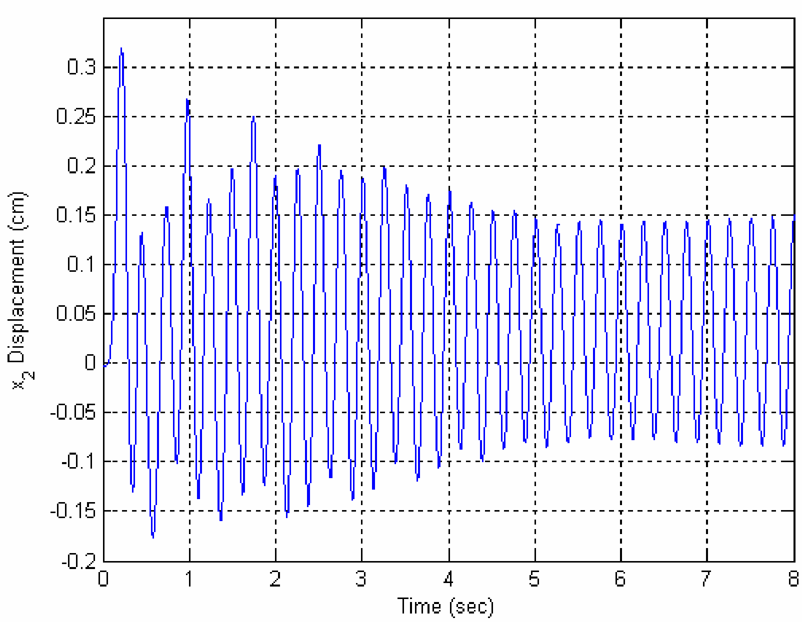

Figure 13. The motion of the second cart in the three degree of system when the system is excited by a 4 Hz sine wave.

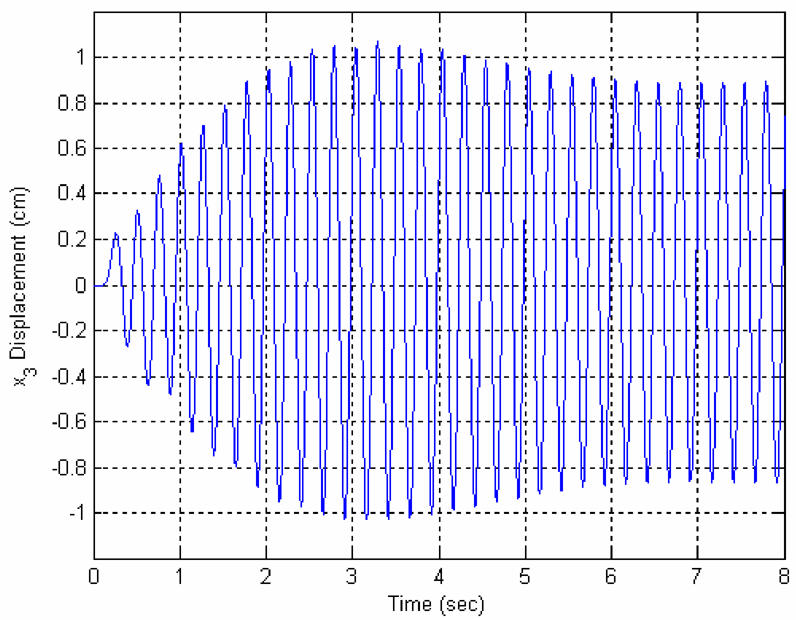

Figure 14. The motion of the third cart in the three degree of system when the system is excited by a 4 Hz sine wave.

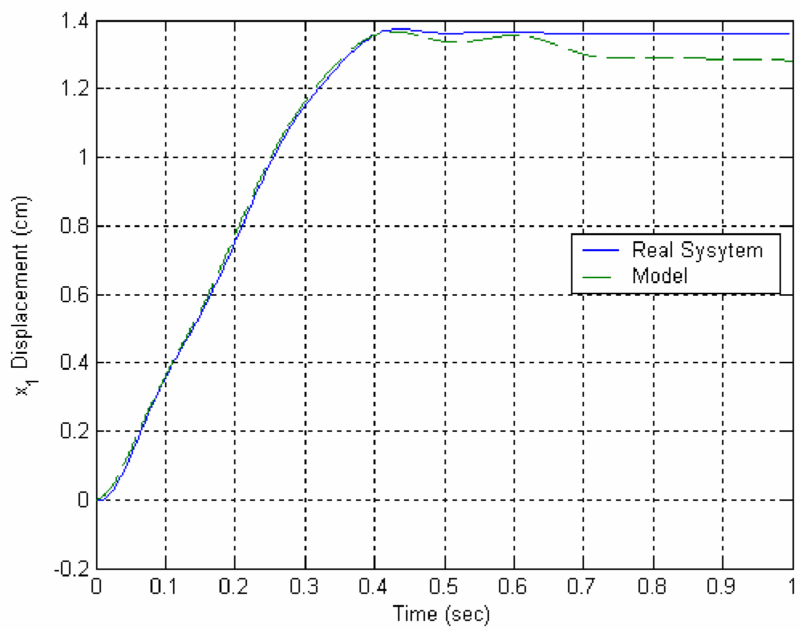

Figure 15: The position of the first cart for step response of the three degree of freedom system we have modeled. Both the response of the real system and the response of our model are shown. We were trying to have the second cart match a one cm step input.

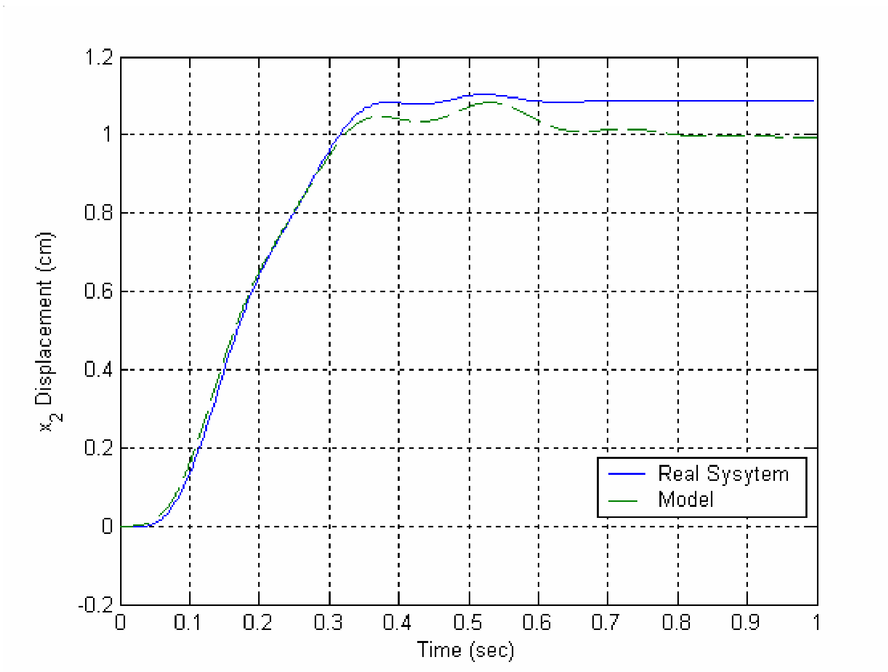

Figure 16: The position of the second cart for step response of the three degree of freedom system we have modeled. Both the response of the real system and the response of our model are shown. We were trying to have the second cart match a one cm step input.

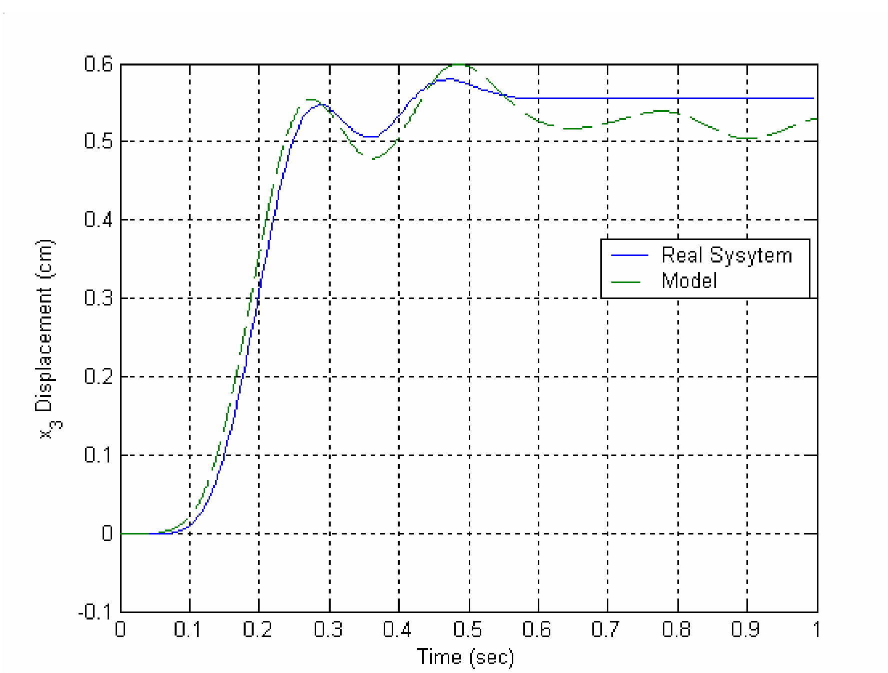

Figure 17: The position of the third cart for step response of the three degree of freedom system we have modeled. Both the response of the real system and the response of our model are shown. We were trying to have the second cart match a one cm step input.

Assessment

At the conclusion of the course, the thirteen students taking the class were asked to complete questionaires designed by Rose-Hulman's Office of Institutional Research, Planning, and Assessment. These questionaires had two types of questions: survey questions and questions about the level of knowledge and confidence in various course concepts. The results of the relevant survey questions are displayed in Table 4, which indicates that most of the students felt that the course helped them with the use and limitations of models in control systems and with the concept of the frequency response of a system. Table 5 presents the results for the relevant average knowledge and average confidence part of the questionaire. The knowledge score was based on the following scale:

Pre Course
1 = No clue, this concept is new to me
2 = Low, I had only heard about the concept
3 = Moderate, I knew the concept but had not applied it
4 = High, I knew the concept and had applied it

Post Course
1 = No Clue, I do not know to concept
2 = Low, I have only heard about the concept, but do not know it well enough to apply it.
3 = Moderate, I know the concept but do not know it well enough to apply it
4 = High, I know the concept and have applied it in this course.

The Confidence score was based on the following scale:

Pre Course
1 = No Clue, I have no idea if I can apply the concept
2 = Low, I had heard of the concept but had little confidence that I could apply it
3 = Moderate, I was somewhat confident that I understood the concept and was fairly sure I could apply it to a new problem
4 = High, I was confident that I understood and apply it to a new problem

Post Course
1 = No Clue, I am not confident that I can apply the concept
2 = Low, I have heard of the concept but am not sure if I can apply it
3 = Moderate, I am somewhat confident that I understand the concept and I can apply it to a new problem
4 = High, I am confident that I understand and can apply the concept to problems.

As the average knowledge/confidence levels indicate, the student felt their knowledge level had increased during the course, and their confidence had also increased. This paper has presented three of the labs in this course, though there were a total of eight labs in the course, so it is not possible to attribute all of the increases in confidence and knowledge to just the labs presented in this paper. However, these were the labs in the course most closely associated with system identification and modeling.

Question	*Strongly Agree*	*Agree*	*Disagree*	*Strongly Disagree*	*I Don't Know*
As a result of this class, I understand the uses of models in designing control systems that will help me in future classes.	75 %	25 %	0 %	0 %	0 %
As a result of this class, I understand the limitations of models in designing control systems that will help me in future classes.	58 %	33 %	8 %	0%	0%
This class helped me to better understand the frequency response of a system.	42 %	58 %	0%	0 %	0%

Table 4: Results of student questionaires at end of the course.

Concept	*Pre Course Knowledge*	*Pre Course Confidence*	*Post Course Knowledge*	*Post Course Confidence*
Various approaches to modeling dynamical systems.	2.83	2.75	3.67	3.67
Distinctions between a model and a real dynamical system.	3.00	2.92	3.67	3.58
Comparison between the predicted response of a mathematical model and the response of a physical system.	2.75	2.92	3.75	3.75

Table 5: Pre and post course knowlwdge and confidence (average score).

Concluding Remarks

We have presented our sequence of laboratories for identification and control one, two, and three degree of freedom mass-spring-damper systems. This sequence of experiments has a number of practical benefits. By exciting these systems with various frequencies, the students understand transient response and sinusoidal steady state in terms of the physical behavior of mechanical systems. The students also develop a better appreciation for the use of Bode plots. Nearly all of the parameters can be estimated in at least two ways and the students must compare these estimates to check for consistency. Finally, the students use state variable feedback to control the systems and compare the difference between the response of the model and the response of the real system.

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