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AN UNDERGRADUATE SYSTEM IDENTIFICATION LABORATORY

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ABSTRACT
One of the essential objectives of any undergraduate controls curriculum is understanding the frequency response, its physical meaning, and stability consequences. In addition, the student should gain an appreciation for simple system identification techniques, and the relationship between frequency response and the system complex plane pole-zero map. These skills allow the student to analyze "black box" systems. With these objectives in mind, the authors have developed a system identification laboratory using commercially available physical plants and digital computer controllers. This paper outlines the present course background, lesson objectives, data collection and reduction techniques for a system identification laboratory taught in the introductory system dynamics course in the Mechanical and Electrical Engineering programs at Rose-Hulman Institute of Technology.

INTRODUCTION
In 2002, the mechanical engineering and electrical engineering faculty at Rose-Hulman (RHIT) applied for a Course, Curriculum and Laboratory Improvement (CCLI) grant from the National Science Foundation to improve the system dynamics and controls lab at RHIT. RHIT has a unique sophomore engineering curriculum that culminates in a five-credit-hour course in system dynamics. This course, Analysis and Design of Engineering Systems (ADES) is a comprehensive overview of modeling, analysis and simulation of mechanical, electrical, thermal and fluid systems and is taught to Electrical and Mechanical Engineering majors in an interdisciplinary setting. One of the most important objectives of this course is that the student learn to model systems using lumped parameters, and understand the assumptions and limitations of such modeling.

Frequency response is one of the fundamental concepts in linear system theory and classical control. We have found that our mechanical engineering students have a particularly difficult time relating frequency response to the physical world. This laboratory gives the student an opportunity to gather frequency response data, while watching the response of a physical system, summarize this data on a Bode plot, and determine the underlying Laplace domain model. This is the eighth laboratory in the course.

LESSON OBJECTIVES
The objectives of this lab are as follows. Students will become familiar with the Educational Control Products (ECP) Rectilinear Control System [1], and Matlab [2] interface. They will collect input-output data for several sinusoid inputs to the system. In doing so, they will learn the physical meaning of frequency response. They will then compile their data on a Bode magnitude plot. This exercise will augment understanding of the Bode plot and spectral representations of frequency response data. Students will determine a fourth order transfer function model from the Bode magnitude plot. Since the phase data is not used, the student must infer stability and pole locations from the system physical characteristics. This emphasizes the relationship between frequency response plots and the system pole-zero map.

DATA COLLECTION
We developed this lab on the ECP Rectilinear Control System [1] which is shown in Figure 1. The system consists of the hardware plant which is essentially a three degree of freedom spring-mass-damper system, and a digital computer running ECP's own Interface Software "Executive Program". The ECP Interface Software provides an ASCII data output capability which allows for easy import of the data to Matlab for off-line analysis. The plant can be configured for one, two

or three degrees of freedom. For this lab, we prefer the two degree of freedom configuration since we would expect a fourth-order transfer function model, and have the ability to demonstrate two resonant mode shapes. The system has the flexibility of varying spring constants, masses of the carts, and the location of a viscous damper. For this lab, the instructor configures the system masses to approximately 1.0 kg per cart. Each set up may have a different spring configuration which will allow students' answers to vary from team to team. It is preferable to have a stiffness configuration that will provide two distinct resonant frequencies and mode shapes.

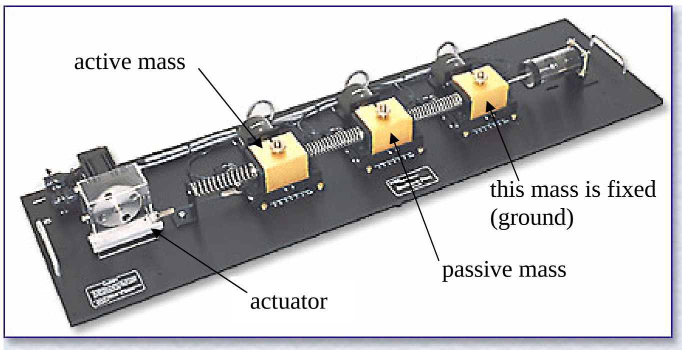

Figure 1: The ECP Rectilinear Control System

The student may obtain system frequency response data by following these steps:

a. Under the Command/Trajectory menu, set the open-loop input amplitude to 0.5 V with a frequency of 1 Hz. Set the number of repetitions equal to 10 times the frequency (in Hz).

b. Under Utility, select Zero Position

c. Select Command/Execute and press Run.

d. Watch the response. After the Upload successful dialog completes, click OK.

e. (Optional) To look at an individual data set, select Plotting/Setup Plot. Do this for some of the higher frequencies to insure that the system has reached steady state.

f. Export your data using the Data/Export Raw Data menu.

g. Repeat for the remaining frequencies of interest. As a minimum, use the frequencies [1 2 3 4 5 6 7 8 9 10] Hz.

DATA REDUCTION

Looking at a typical response such as Figure 2, the student should immediately recognize this as a linear system response to a sinusoidal input. That is, frequency is preserved in the output, and output can be related to input by a gain factor and phase lag. The input is a smooth sinusoid from the internal signal generator.

$$u(t) = A\sin\omega t \tag{1}$$

$$\omega = 2\pi f \tag{2}$$

The output shows some transient behavior for the first six seconds of motion. In steady state the response can be described by Eq. 3. In the case shown, the phase angle is approximately 180 degrees.

$$y(t) = |G(j\omega)| A\sin[\omega t + \arg G(j\omega)], \tag{3}$$

where $|G(j\omega)|$ is the transfer function magnitude. We subtract the mean from input and output signals prior to plotting them on Figure 2. The student should do likewise prior to comparing maximum or minimum amplitudes in order to determine $|G(j\omega)|$. We expect the student to exercise some judgment in determining whether to use the maximum or minimum of the output oscillation in calculating the transfer function magnitude.

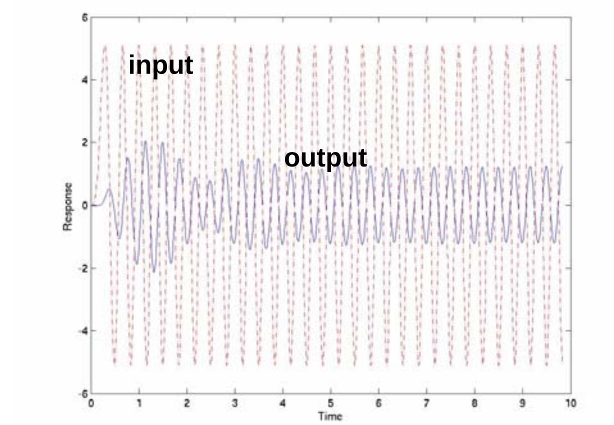

Figure 2: Typical System Sinusoidal Response

Figure 3a shows the ASCII format in which the ECP executive program saves the data when using Export Raw Data in step f above. Note that the open square bracket used in Matlab array assignments is placed at the beginning of the data, and data is arranged in columns using space delimiters, and semi-colon delimiters for rows. There is also a close square bracket at the end of the file which is not shown in the Figure.

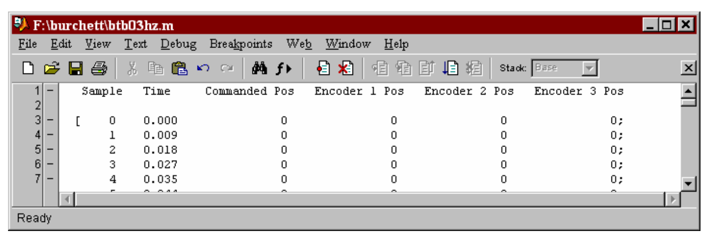

Figure 3a: ECP Data Export Format

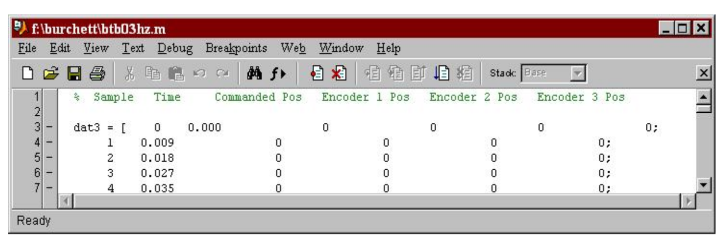

Figure 3b: Data file after Editing

Figure 3b shows the file after minimal editing by the student, changing the header to a comment, and the open square bracket to the array assignment “dat3”.

The Bode magnitude for each frequency can be determined from the ASCII data files using the following steps.

Extract the Encoder 2 Position data and subtract the mean.

Convert Encoder 2 Position to centimeters. The encoder sensitivity is 1604.1 counts/cm.

Now determine the ratio of output to input amplitude. Plot the data to determine where the oscillations have reached steady state. Output amplitude can then be determined by taking the maximum value from several steady state oscillations. Since the input amplitude was specified to be 0.5 volts, we divide by 0.5 to get the ratio of output magnitude over input magnitude.

Note, the array indices in this statement need to be adjusted so that you consider only oscillations after the system has reached steady state. It is probably easiest to do these calculations inside each data file.

The student should then write a top-level script that 1) executes each data file. 2) Converts the Magy vector to dB, and plots the experimental data on a semilog plot. 3) Uses fminsearch to find the fourth order transfer function that best fits the data. Use a transfer function form with no finite zeros.

$$G(s) = \frac{K\omega_1^2\omega_2^2}{\left(s^2 + 2\zeta_1\omega_1 + \omega_1^2\right)\left(s^2 + 2\zeta_2\omega_2 + \omega_2^2\right)} \quad (4)$$

You should field an initial guess based on where the resonant peaks appear to be in the experimental data. The system is very lightly damped, so guess each damping ratio to be 0.1. Reduce the transfer function to the following form, and use the x_i values for your initial guess.

$$G(s) = \frac{x_1}{x_2 s^4 + x_3 s^3 + x_4 s^2 + x_5 s + x_6} \quad (5)$$

Then put the coefficients of your guess into a column vector to serve as the initial guess for fminsearch. (A Matlab script for performing this optimization problem is given in the Appendix.) Fminsearch uses the Nelder-Mead Simplex approach which finds a local minima near the initial guess and does not require gradient information. The student should be reminded that his initial guess of the transfer function parameters can greatly influence the success of the search.

The student will need to write a function “lab3” that computes the sum squared error between experimental magnitudes and theoretical magnitudes (in dB) along the Bode plot. The student is encouraged to use the features of the control toolbox, including, in particular, the bode command. See the appendix for a sample script.

Plot the magnitude of your best fit transfer function, and determine the resonant frequencies. (You will need to re-run the bode function with the “best” transfer function coefficients, and use a fine frequency vector like ww=logspace(0,2,100)).

Time permitting, the student should try exciting the system at the resonant frequencies. You will probably need to reduce the input amplitude to 0.25 volts to avoid exceeding the travel limits of the device. Incorporate this data into your analysis, and re-do the numerical fit analysis. In the example shown below, this step produced two new near-resonant frequencies at 18 rad/s (2.86 Hz) and 28.2 rad/s (4.48 Hz).

Figure 4 shows a typical Bode Magnitude plot with experimental data and the best theoretical fit transfer function response.

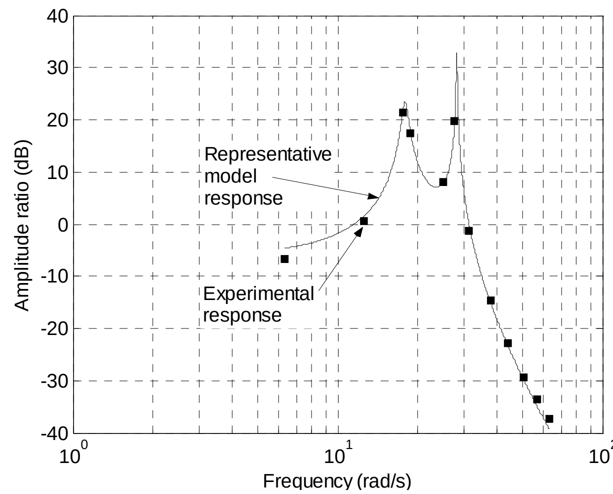

Figure 4: Example Experimental Frequency Response with Theoretical Fit.

The best theoretical transfer function for the example shown in Figure 4 is given by $G(s)$ =

$$\frac{0.489}{3.88\left(10^{-6}\right)s^4 + 5.28\left(10^{-6}\right)s^3 + 4.34\left(10^{-3}\right)s^2 + 3.56\left(10^{-3}\right)s + 1} \quad (6)$$

The characteristics of this transfer function are given in Table 1. Uncertainties are at a 95% confidence interval based on ten numerical trials using the optimization routine with different starting points for the search. While the percentage uncertainty in the second damping ratio is large, in absolute terms the damping ratio confidence interval is $0.003 \le \zeta_2 \le 0.005$.

Table 1: System Identification Results.

Characteristics	Mean value	Uncertainty
SS Gain	$K = 0.489$	±0.07%
Natural frequencies	$\omega_1 = 18.0$ rad/s	±0.003%
	$\omega_2 = 28.2$ rad/s	±0.002%
Damping ratios	$\zeta_1 = 0.028$	±0.3%
	$\zeta_2 = 0.004$	±26%

From a transfer function like that shown in Eqn. 6, the student may obtain poles that are clearly unstable, that is, in the right-hand s-plane. However, since we only considered the frequency response magnitudes, the identified poles could lie in either the left or right half plane. This is illustrated in Figure 5 which shows the relationship between the system pole-zero map and frequency response. An animated version of this figure is available from the authors. The × symbols represent system poles and the circles represent system zeros. The dot on the imaginary axis represents the current input frequency.

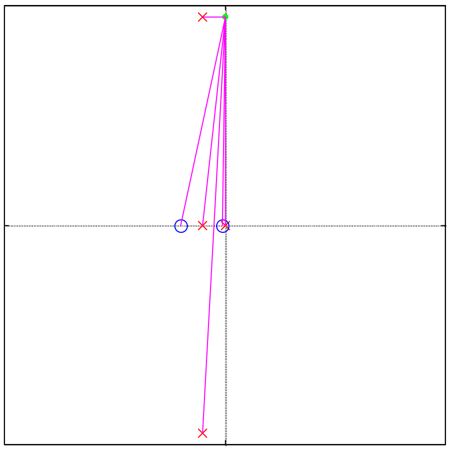

Figure 5: Graphical Evaluation of Frequency Response

The system Bode magnitude at this frequency is the ratio of product of lengths of line segments from zeros to excitation frequency over product of lengths of line segments from poles to excitation frequency or:

$$|G(j\omega)| = \frac{\prod_i |z_i|}{\prod_j |p_j|} \quad (7)$$

Note that the two lightly damped poles could lie either in the left or right half plane, and this would not change the Bode magnitude of the system at all excitation frequencies. Thus, we would have the liberty to arbitrarily change the real part of the second pair of poles to be negative.

The system will clearly oscillate in two distinct mode shapes. At the lower resonant frequency, the masses will move in phase. At the higher frequency the masses will move exactly 180 degrees out of phase.

TEACHING CONSIDERATIONS

This lab provides the students an opportunity to see first hand what is meant by frequency response. Although the input signal is an electrical voltage, and not readily observable during data collection, using a two degree of freedom spring mass configuration introduces obvious phase differences between the controlled mass and the passive mass. This configuration also provides a transient response of significant length that the students can clearly observe, especially in frequencies near the resonant peaks.

Although the phase shift could be measured, doing so would add greatly to the data reduction effort. Also, ignoring the phase data provides an opportunity for the professor to emphasize the relationship between complex plane pole-zero map and system frequency response. Non-minimum phase implies that the significant difference between left and right half plane poles and zeros is that right half plane ones contribute significantly more phase lag or lead.

CONCLUSIONS

In this paper, we have presented course background, lesson objectives, data collection and reduction techniques for a system identification laboratory. This laboratory is particularly well suited to introducing students to the physical meaning of frequency response. The data reduction is readily done in Matlab, since the ECP export format is easy to import into Matlab. The student can determine experimental frequency response magnitudes fairly easily by, but should plot each input/output data set to insure their calculations are based on steady-state behavior. The hardware we have chosen has imperfect sensors and this provides opportunity to address uncertainty in experimental data.

REFERENCES

[1] Manual for Model 210/210a Rectilinear Control System, Educational Control Products, Bell Canyon, CA, 1999. http://www.ecpsystems.com

[2] MATLAB is Copyright The Mathworks, Inc. 24 Prime Park Way, Natick, MA 01760-1500. http://www.mathworks.com

APPENDIX

Listed below is an example Matlab script for performing some of the functions described in the paper.

```
% Extract the steady-state position data and
% subtract the mean
enc2cm = dat4(:,5)-mean(dat4(:,5));
% Run the optimization
options = optimset(@fminsearch)
options = optimset(options,'Display','iter');
coeffs = fminsearch(@lab3,x0,options)
% Compute the sum squared error in dB between
% experimental and theoretical magnitudes.
function J = lab3(x)
num = x(1); den = x(2:6); sys = tf(num,den);
w = 2*pi*[1 2 2.8 3 4 4.4 5 6 7 8 9 10]';
mag = [0.4592; 1.0650; 11.8708; 7.3933;
       2.5025; 9.6076 0.8568; 0.1852; 0.0733;
       0.0341; 0.0212; 0.0136];
magdB = 20*log10(mag);
maggie = bode(sys,w);  maggie = maggie(:);
maggiedB = 20*log10(maggie);
J = norm(magdB - maggiedB);
```