ME 406 ADVENTURE 3

PD and PID control of a double integrator plant

Deadline: COB Today

INTRODUCTION

We will investigate the benefits and limitations of controlling a double integrator plant with PD and PID control.

OBJECTIVE

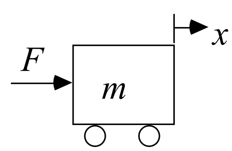
The objective of this adventure is to:

+ Become familiar with how feedback control is implemented on the ECP system
+ Determine an appropriate set of feedback gains using 2nd order response characteristics
+ collect experimental data of system step response, and compare with a theoretical prediction
+ Describe the individual effects of Proportional, Derivative, and Integral control
+ Experiment with Ziegler-Nichols tuning of P and PID controllers
+ Recognize that systems without integral control generally have non-zero steady state error.

A. PRE-LAB

Read this handout, and complete the calculations in parts 1, 2 and 3, below. Record your answers on the worksheet provided on pages 5 and 6.

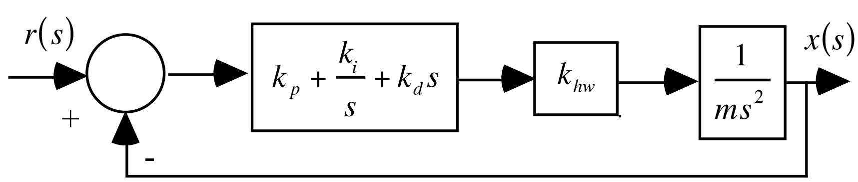
For this lab, we will use only the first cart and mass with 'no springs attached'. A lumped parameter model might look like the following:



Applying first principles, the diferential equation of motion is: $m\ddot{x} = F$, which makes the transfer function:

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2} \tag{1}$$

Use the value of mass *m* obtained in Lab 1 with four 500g masses loaded. For the purpose of this lab, we will ignore the motor dynamics, which is equivalent to treating the motor input voltage as *F*. By implementing a PID controller in the forward path, we arrive at the following block diagram:



Applying the feedback law, the closed-loop transfer function is:

$$T_{cl}(s) = \frac{k_d s^2 + k_{hw}(k_p s + k_i)}{ms^3 + k_{hw}(k_d s^2 + k_p s + k_i)}$$

Which is third order, and thus a bit cumbersome to use for direct design. By direct design, I mean determining the controller gains by matching coefficients with a desired 2nd order characteristic equation. In order to simplify, we will set the integral gain (k_i) to zero. This results in the closed-loop transfer function:

$$T_{cl}(s) = \frac{k_{hw}(k_d s + k_p)}{ms^2 + k_{hw}(k_d s + k_p)} \qquad (2)$$

From which it is clearly seen that

$$\omega_n = \sqrt{\frac{k_p k_{hw}}{m}} \qquad (3)$$

and

$$2\zeta\omega_n = \frac{k_{hw} k_d}{m} \Rightarrow \zeta = \frac{k_{hw} k_d}{2m\omega_n} \qquad (4)$$

The hardware gain can be computed by

$$k_{hw} = k_c k_a k_t k_{mp} k_e k_{ep} k_s \qquad (5)$$

Where the numerical values in consistent units are

$$k_c = 10/32768$$
$$k_a = 2$$
$$k_t = 0.1$$
$$k_{mp} = 26.25$$
$$k_e = 16000/2\pi$$
$$k_{ep} = 89$$
$$k_s = 32$$

1. Calculations for P control

Using equations 3 and 5, compute the proportional gain to give the system a natural frequency of $\sqrt{2}$ Hz. Also, double this value and predict what will happen to the system natural frequency if the larger gain is used?

2. Calculations of PD control

a. Compute the value of k_d needed to make $k_d k_{hw} = 50$ N / (m/s), also compute five times this value.

b. Using equations 3 and 5, compute the proportional gain to give the system a natural frequency of 4 Hz. Use Equation 4 to compute the values of k_d needed to give the closed-loop system damping ratios of 0.2, 1, and 2 respectively. Record all of these values for use in the lab.

3. Calculations for PID control

Compute the value of k_i needed to make $k_i k_{hw} = 7500$. Record this value and twice this value.

B. THE ADVENTURE CONTINUES - IN THE LAB
During the adventure you will set out to accomplish the following:

1. Set up the environment.

a. Use the same station you used for Lab 1. It should be set up in 1 DOF mode with four 500g brass masses on the first carriage. Remove all springs from the first carriage. Connect the damper and remove its plug. If you need help configuring your station, ask your instructor.

b. Position the mass at zero on the position ruler. Under Utility, select Zero Position.

c. Log on to the computer and start the ECP executive program under Programs/ECP.

d. Check the encoder polarity. This is an important step that should be done each time before implementing a feedback controller. Some systems have 'reverse polarity' meaning that the sensor says the mass is displaced to the left when it is really displaced to the right. I have even seen a system change its polarity overnight! Select Setup/Control Algorithm... In the dialog box, select the PID radio button, and press 'Setup Algorithm'. Enter a small value for proportional control such as 0.06. Set the derivative and integral gains to zero. Press Implement Algorithm. Press OK. Push the black button of the ECP control box. Set up the trajectory for a closed-loop step with an amplitude of 1000 counts, and a dwell time of 3000ms. Select Command/Execute and press Run. Select Plotting/Setup Plot. In the dialog, add Commanded Position and Encoder 1 Position to the Left Axis, the click Plot Data. If the Encoder 1 position moves in the same direction as the Commanded position, your system has normal polarity. If Encoder 1 position moves in the opposite direction, you have reverse polarity. This means you must multiply your contoller gains by -1 throughout the rest of this experiment.

e. Set up your first proportional controller. Enter the first value of proportional control found in part A1. Set the derivative and integral gains to zero. Press Implement Algorithm. Press OK. Push the black button of the ECP control box. You might hear the system rattle a bit due to sensor noise in the feedback loop.

f. While the controller is active, try pushing the mass one way or the other. Do not manually displace the mass more than 2cm, or you will 'break' the control loop due to internal thermal protection on the motor. If you accidentally break the control loop, you will need to repeat parts b and c above. With P control only, it should feel like there is a spring trying to restore the mass to its neutral position.

2. Recording Step response data:

a. Select Command/Trajectory... In the dialog select the Step radio button and press Setup. Now select Closed Loop Move. The Amplitude should be set to 1000, set the dwell time to 3000 ms. Click OK. Click OK on the Trajectory Configuration dialog.

b. Select Command/Execute and press Run.

d. Watch the response. After the Upload successful dialog completes, click OK.

e. To look at an individual data set, select Plotting/Setup Plot. In the dialog, add Commanded Position and Encoder 1 Position to the Left Axis, the click Plot Data.

f. Export your data. Select Data/Export Raw Data... Browse to a convenient directory, floppy disks are recommended for storing this data. Save as type All files (*.*). Choose a name with meaning, like 'inixxx123.m' where ini is a group member's initials, xxx is P, PD or PID, and 123 is the response number.

g. Repeat steps 1b,c and 2 a-f for you second proportional gain value. Repeat steps 1b-d for the first two derivative gains found in part A2a. Set the proportional and integral gains to zero. With D control only, it should feel like there is an invisible damper resisting fast motions. Repeat steps 1b,c and 2a-f for the proportional and derivative gains computed in part A2b. Set the integral gain to zero. Repeat steps 1b-d for the integral gain computed in part A3. Try to move the mass about 1mm from its neutral position, and hold it. You should feel the control effort build up with time with integral control only. Repeat steps 1b,c and 2a-f using the proportional and derivative gains computed in part A2b for $\zeta = 1$, and the integral gains computed in part A3.

h. You can use Secure FX to move your data to your afs space. A shortcut is provided on the lab computer desktop.

D. REPORTING THE ADVENTURE **Team Members:**______________________

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Submit this worksheet along with 3 Plots of closed loop step response data, comparing P, PD and PID controllers, and your hand calculations. For extra credit, see that last page of this handout.

1. Calculations for P control

Using equations 3 and 5, compute the proportional gain to give the system a natural frequency of $\sqrt{2}$ Hz. Also, double this value and predict what will happen to the system natural frequency if the larger gain is used:

k_p=________________________________ $\omega_n = \sqrt{2}$ Hz.

$2k_p$=________________________________ ω_n=________________________________

Plot the two P step responses on the same axis. Use the peak time to determine the damped natural frequency of step responses obtained with P control. How do these values compare to the predictions made in the pre-lab?

Step 1: t_p = ________ ω_d = ____________ Step 2: t_p = ________ ω_d = ____________

2. Calculations of PD control

a. Compute the value of k_d needed to make $k_d k_{hw} = 50$ N / (m/s), also compute five times this value.

k_d = ________________________________ $5k_d$ = ________________________________

b. Using equations 3 and 5, compute the proportional gain to give the system a natural frequency of 4 Hz.

k_p=________________________________ ω_n = 4 Hz.

Use Equation 4 to compute the values of k_d needed to give the closed-loop system damping ratios of 0.2, 1, and 2 respectively.

ζ = 0.2 k_d = ________________________________

ζ = 0.2 k_d = ________________________________

ζ = 0.2 k_d = ________________________________

Record all of these values for use in the lab.

Plot the three PD control responses on the same axis. Use the percent overshoot to determine the damping ratio of step responses obtained with PD control. How do these values compare to the intended design values? Comment on the steady state error seen when using PD control.

Step 1: M_p = _________ ζ= ______________ Step 2: M_p = _________ ζ= ______________

Step 3: M_p = _________ ζ= ______________

Comments:

3. Calculations for PID control

Compute the value of k_i needed to make $k_i k_{hw} = 7500$. Record this value and twice this value.

k_i = ___________________________________ $2k_i$ = ____________________________________

Plot the two PID step responses on the same axis. Does integral control eliminate the steady state error? How does integral control affect the system transient responses?

E. EXTRA CREDIT, ZIEGLER-NICHOLS TUNING

1. Configure the system for a step response with proportional control as described in sections A1 and A2. Try progressively larger gains until the system is marginally stable or slightly unstable (oscillations are persistent or slowly increasing in amplitude). The proportional gain that causes marginal stability is considered the ultimate gain k_u. Export the step response, and determine the actual period of oscillation. (The period of oscilllation can be determined as twice the peak time, $P_u = 2t_p$. For reference, the highest gain I tried was 0.16.

k_u = ___________________________________ P_u = ____________________________________

2. Compute the Ziegler-Nichols gains for P and PID control using Table 4.2 from the course textbook. Realize that the parameters T_I and T_D as defined by Franklin and Powell are not the integral and derivative gains respectively. After you have determined T_I and T_D , use the following to compute the integral and derivative gains. Franklin's PID convention is:

$$D_c(s) = k_{p\square}\left(1 + \frac{1}{T_I s} + T_D s\right)$$

Where I have used the extra $\square$ subscript to distinguish Franklin's P gain. The ECP software uses:

$$D_c(s) = k_{p\square} + \frac{k_i}{s} + k_d s$$

By equating terms of the two transfer functions, we get the conversions from F&P's parameters, to those you should use with the hardware, shown in the first column of the table.

Parameter / Conversion	P control	PID control
$k_{p\square} = k_{p\square}$	=	=
$k_i = \dfrac{k_{p\square}}{T_I}$	=	=
$k_d = k_{p\square} T_D$	=	=

3. Obtain the step response using the Ziegler-Nichols P and PID gains. Plot the responses on a single axis. According to the course textbook, p. 222, Ziegler-Nichols tuning should result in a closed-loop damping ratio of approximately 0.21. Check the percent overshoot for each response to see whether this is true.