Lab 2

EXPERIMENTALLY DETERMINING THE TRANSFER FUNCTION OF A SPRING-MASS SYSTEM

OBJECTIVES

At the conclusion of this experiment, students should be able to:

Experimentally determine the best set of fourth order transfer function models for the 2 DOF system, and the best system state space model.

Collect experimental frequency response data.

Understand the significance of the Bode plot for predicting system behavior, and determining non-parametric system models.

DELIVERABLES

The deliverables of this experiment are:

A memo style report including, but not limited to the following:

-introduction, results/discussion, conclusion, and appropriate appendices

-Show your best fit transfer functions, and the locations of their poles.

-Show your values for the undetermined parameters.

-A Bode magnitude plot showing the experimental data, and best fit. An example is shown on the last page of this handout, your results may vary significantly.

-List any suggestions for improving the lab.

THEORY

By frequency response, we mean the response of a system to a harmonic input. A linear system cannot change the frequency from input to output. Thus, as we have seen earlier, the output will be the a sinusoid of frequency identical to that of the input, only amplified or attenuated, and shifted in phase:

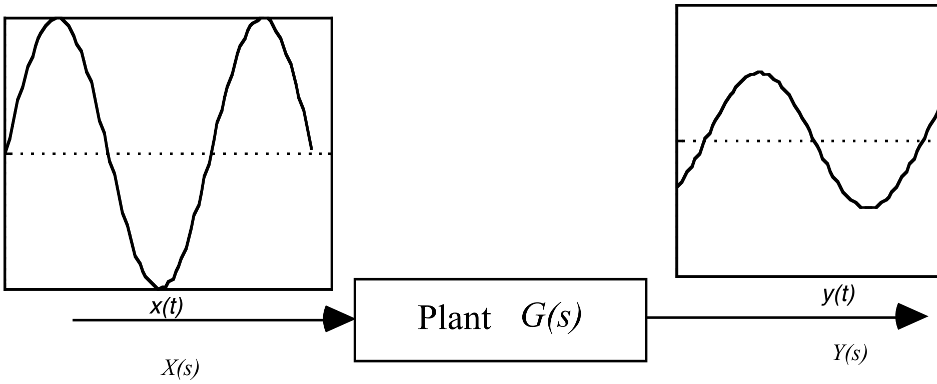


Figure 1, Physical Meaning of Frequency Response

$$x(t) = X\sin \omega t$$

$$y(t) = \left|G(j\omega)\right| X \sin\left(\omega t + \angle G(j\omega)\right)$$

A Pole-Zero Map is the easiest way to visualize the magnitude and phase of a Transfer Function. The figure below shows the pole zero map of the transfer function:

$$G(s) = \frac{s^2 + 2.1s + 0.2}{s\left(s^3 + 3s^2 + 93.25s + 91.25\right)}$$

We have drawn the vectors from each pole and zero to the point $s = j9.5$, which is the 'resonant' frequency of the system.

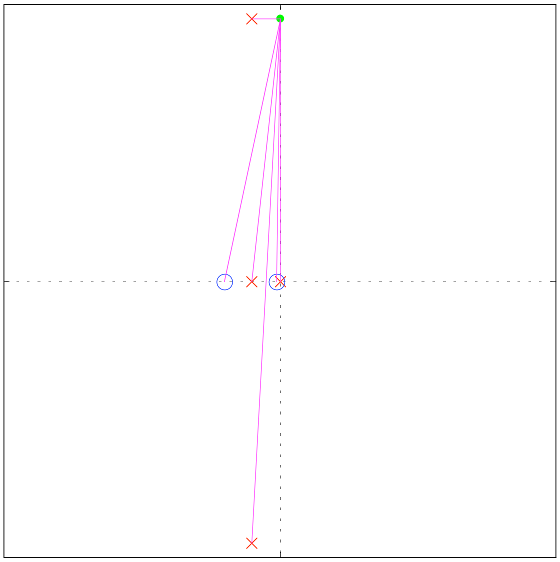


Figure 2, Graphical Evaluation of Frequency Response

Note that the magnitude of the transfer function at this point is given as the product of the lengths of the vectors from the zeros over the product of the lengths of the vectors from the poles:

$$\left|G(j\omega)\right| = \frac{\prod_i \left|z_i\right|}{\prod_j \left|p_j\right|} \tag{1}$$

The transfer function argument (angle) is the sum of the zero vector angles minus the sum of the pole vector angles. In each case the angles are defined as counter-clockwise from a segment parallel to the real axis to the vector.

$$\angle G(j\omega) = \sum_i \angle z_i - \sum_j \angle p_j$$

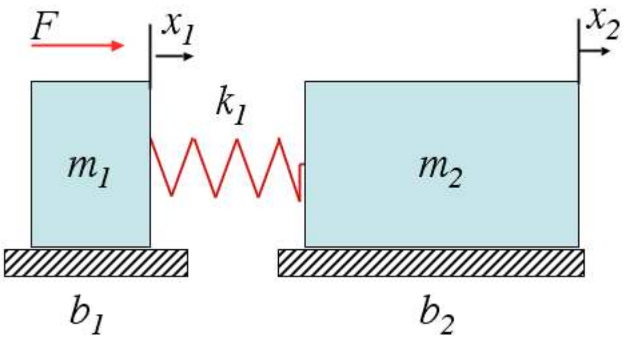
The closer a complex pole is to the $j\omega$ axis, (less system damping) the higher a resonant peak will be. Likewise, the closer a zero, the lower the 'notch'.

The Bode plot is a mapping of the entire $+j\omega$ axis (the entire positive frequency spectrum) through the system transfer function to Gain and Phase plots. We typically plot Gain and Phase together on a semilog axis. The abscissa (x-axis) for the Bode plot is log frequency. The distance between a frequencies 1 and 10, 0.1 and 1, 10 and 100, etc. is termed a 'decade'. Gain is plotted in 'decibels' (dB) where the conversion from magnitude to dB is given by

$$\left|G(j\omega)\right| (\mathrm{dB}) = 20\log_{10}\left|G(j\omega)\right|$$

This week in class we are investigating how to determine the Bode plot knowing the system transfer function. In this lab we will explore the *inverse* problem, that is, knowing the frequency response, can we determine an appropriate transfer function? We will base our transfer function estimate on the experimental Bode magnitude plot only. As illustrated in Fig 2, and Eqn 1, without considering the Bode phase plot, the poles and zeros could lie on either side of the $j\omega$ axis (right or left half plane) and we would observe the same Bode magnitude plot. With this in mind, you may find it necessary to arbitrarily change unstable system poles to their stable equivalents after tuning your model.

The system we are trying to identify is a two mass, one spring system as shown to the right. The input is a voltage to a DC motor connected to a rack and pinion, then directly connected to the first mass. We will neglect the motor dynamics. This system is very similar to the one in textbook problem 2.8.



Since we have some insight into the physical characteristics, this is a *gray box* problem, rather than a *black box* problem, where we have no inkling as to what connects input to output. We will consider the outputs to be the positions of the first mass x_1 and the second mass x_2. From first principles, we can write the system state space description as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}F$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{k_1}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & 0 & -\frac{k_1}{m_2} & -\frac{b_2}{m_2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{K}{m_1} \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

With the output equation

$$\mathbf{y} = \mathbf{C}\mathbf{x} + DF$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{D} = [0]$$

The two system transfer functions are then found from the state space description. From F to y_1 we should have:

$$\frac{Y_1}{F} = \frac{K\left(m_2 s^2 + b_2 s + k_1\right)}{s\left(m_1 m_2 s^3 + \left(b_1 m_2 + b_2 m_1\right)s^2 + \left(k_1\left(m_1 + m_2\right) + b_1 b_2\right)s + k_1\left(b_1 + b_2\right)\right)} \tag{2}$$

From F to y_2 the transfer function is:

$$\frac{Y_2}{F} = \frac{K k_1}{s\left(m_1 m_2 s^3 + \left(b_1 m_2 + b_2 m_1\right)s^2 + \left(k_1\left(m_1 + m_2\right) + b_1 b_2\right)s + k_1\left(b_1 + b_2\right)\right)} \tag{3}$$

We already know the value of m_1 from the previous lab. In some cases you will find it much easier to find a non-parametric model to fit the data first. That is, instead of finding m_2, b_i, K, and k_1, we will match the following transfer function

$$G(s) = \frac{K|p_2|\omega_{n3}}{s(s - p_2)(s^2 + 2\zeta_3\omega_{n3}s + \omega_{n3}^2)} \tag{4}$$

Where the unknown parameters are K, ζ_3, ω_{n3}, and p_2. In fact when reducing the data, you might find it easier to first identify the values in Eq. 4, then Eqs 3 and 2, then finally Eq. 1. That is, run an optimization to find the correct K value, then re-run to narrow down the other values, and so on.

THE ADVENTURE BEGINS

During the adventure you will set out to accomplish the following:

1. Set up the environment.

a. Each station should be set up in 2 DOF mode with two 500g brass masses on the first carriage and four 500g masses on the second. A stiff spring should connect the two masses. The damper should be disconnected. If your station is not configured properly, ask your instructor to fix it.

b. Log on to the computer (username=student, password=student) and start the ECP executive program under Programs/ECP.

2. Recording Frequency response data: For each frequency of interest, do the following steps:

a. Select Command/Trajectory... In the dialog select the Sinusoidal radio button and press Setup. Now select Open Loop Move. The Amplitude should be set to 0.5v. Select a frequency of interest. You will need to run the following set of frequencies: [1 2 3 4 5 6 7 8 9 10] Hz
Set the number of reps equal to 10 times the frequency (in Hz) and click OK. Click OK on the Trajectory Configuration dialog.

b. Move the first mass one cm from neutral toward the motor. (The rigid body mode will cause the carts to initially drift to your right, this will help prevent exceeding the limits.)

c. Under Utility, select Zero Position

d. Select Command/Execute and press Run.

e. Watch the response. After the Upload successful dialog completes, click OK. If the system should exceed its physical limits, divide the input amplitude by 2 and try running again. Be sure to record amplitude for those runs which differ from 0.5v.

f. (Optional) To look at an individual data set, select Plotting/Setup Plot. In the dialog, add Commanded Position and Encoder 2 Position to the Left Axis, the click Plot Data. You might want to do this for some of the higher frequencies to insure that the system has reached steady state.

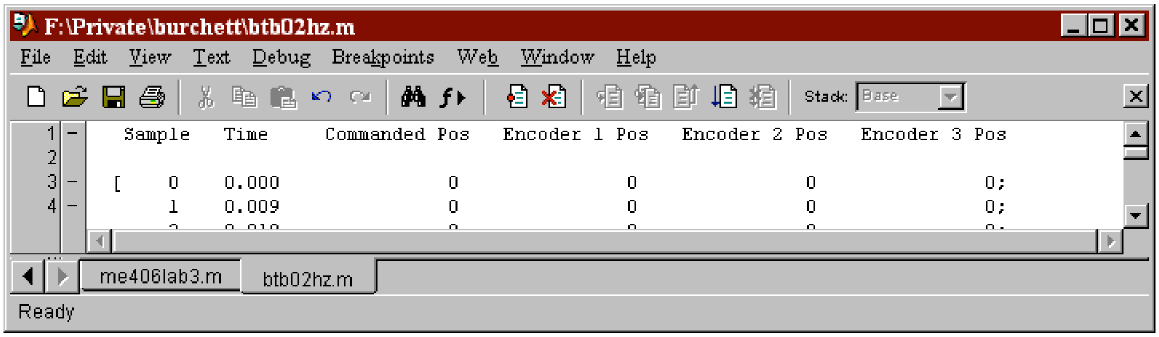
g. Export your data. Select Data/Export Raw Data... Browse to a convenient directory, floppy disks are recommended for storing this data. Save as type All files (*.*). Choose a name with meaning, like 'inixx.m' where ini is a group member's initials, xx is the frequency in Hz.

h. Repeat for the remaining frequencies of interest. After your initial analysis, you **must** return to the lab, and collect data for what you think is the resonant frequency.

i. You can use Secure FX to move your data to your afs space. A shortcut is provided on the lab computer desktop.

3. Analysis:

a. Each exported data file will have the following format.

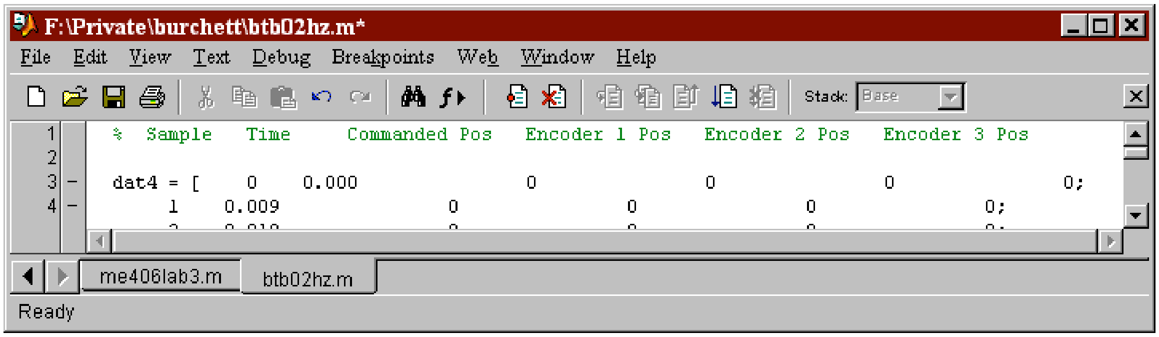

F:\Private\burchett\btb02hz.m
File Edit View Text Debug Breakpoints Web Window Help
Stack: Base
me406lab3.m
btb02hz.m
Ready

```
   Sample    Time      Commanded Pos   Encoder 1 Pos   Encoder 2 Pos   Encoder 3 Pos

[      0     0.000                0               0               0               0;
       1     0.009                0               0               0               0;
```

Each row of the array is a sample and the columns are, respectively: Sample, Time, Commanded Pos, Encoder 1 Pos, Encoder 2 Pos, Encoder 3 Pos.

Change the file so that you have an array assignment.


F:\Private\burchett\btb02hz.m*
File Edit View Text Debug Breakpoints Web Window Help
Stack: Base
me406lab3.m
btb02hz.m
Ready

```
% Sample    Time      Commanded Pos   Encoder 1 Pos   Encoder 2 Pos   Encoder 3 Pos

dat4 = [     0    0.000               0               0               0               0;
       1     0.009                0               0               0               0;
```

Extract the Encoder 2 and Encoder 1 Position data and subtract the mean:

```
enc2cm = dat4(:,5)-mean(dat4(500:1000,5));
enc1cm = dat4(:,4)-mean(dat4(500:1000,4));
```

Now determine the ratio of output to input amplitude. You might want to plot the data to determine where the oscillations have reached steady state. Output amplitude can then be determined by taking the maximum value from several steady state oscillations. Since the input amplitude was specified to be 0.5 volts, we divide by 0.5 to get the ratio of output magnitude over input magnitude. For example:

```
Magy(10) = max(en2cm(500:1000))/0.5;
Magx(10) = max(en1cm(500:1000))/0.5;
```

Note, the array indices in this statement need to be adjusted to that you consider only oscillations after the system has reached steady state. It is probably easiest to do these calculations inside each data file.

b. Write a top-level script that 1) executes each data file. 2) Converts the Magy and Magx vectors to dB, and plots the experimental data on a semilog plot. 3) Uses fminsearch to find the fourth order transfer functions, state space description, or parametric transfer functions that best fit the data. (My code provides a switch between the various model forms—yours need not be quite so fancy—you could write three separate scripts). You should field an initial guess based on where the resonant peaks appear to be in the experimental data. The system is very lightly damped, so guess each damping ratio to be 0.1. Reduce the transfer function of Eqn (3) to the following form, and assign the unknown parameters.to a vector x0.

$$G(s) = \frac{K|p_2|\omega_{n3}}{s(s - p_2)(s^2 + 2\zeta_3\omega_{n3}s + \omega_{n3}^2)}$$

Then put the coefficients of your guess into a column vector to serve as the initial guess for **`fminsearch`**. Use the following syntax to run your optimization:

```
»options = optimset(@fminsearch)
»options = optimset(options,'Display','iter');
»coeffs = fminsearch(@lab2,x0,options)
```

You will need to write a function 'lab2' that computes the sum squared error between experimental magnitudes and theoretical magnitudes along the Bode plot. You are encouraged to use the features of the control toolbox, including, in particular, the bode command. Here are a few lines of code to get you started:

```
function J = lab2(x)
% function used to compute cost function for optimizing TF model
% the basic steps are shown for a non-parametric TF model
% the same general steps should be used in forming SS or
% parametric models
s = tf('s');
p2 = x(1);   % extract parameter values from the input vector
wn3 = x(2);
zeta3 = x(3);
```

```
K = x(4);
% form the system—be it SS, non-param TF or parametric TF
G = K*abs(p2)*wn3^2/(s*(s-p2)*(s^2+2*zeta3*wn3*s+wn3^2));
% assign the spectrum for comparison
ww = 2*pi*[1: 10]; % be sure to include your resonant freq later
magdB = [% insert your experimental data here%]'; % don’t forget
                                                  % the apostrophe
% use the Bode function to get magnitude data
% be careful with dimensions of output when you have a system with
% more than one output.
maggie = bode(G,ww);  % the (:) on the next line will need to be
maggie = 20*log10(maggie(:)); % replaced with a ‘reshape’ command
J = norm(magdB - maggie); % for systems with more than one output
% END of lab2.m %
```

Plot the magnitude of your best fit transfer function, and determine the resonant frequencies. (You will need to re-run the bode function with the ‘best’ transfer function coefficients, and use a fine frequency vector like `ww=logspace(0,2,100))`. Many of these steps are repeated from the function above.

```
s = tf('s');
p2 = coeffs(1);  % extract the unknown params from the coeffs
wn3 = coeffs(2); % vector
zeta3 = coeffs(3);
K = coeffs(4);
G = K*abs(p2)*wn3^2/(s*(s-p2)*(s^2+2*zeta3*wn3*s+wn3^2));
ww = logspace(0,2,100); % choose the ‘fine’ frequency vector
maggie = bode(sys,ww);  % compute Bode magnitude data
hold on                 % hold the figure (assumes
                        % experimental data is already plotted)
semilogx(ww,20*log10(maggie(:))) % plot theoretical curve
```

From the theoretical Bode plot, estimate the frequency of the resonant peak, and ‘notch’. Return to the lab, and collect data at these frequencies. You will probably need to reduce the input amplitude to 0.25 volts to avoid exceeding the travel limits of the device. Incorporate this data into your analysis, and re-do the numerical fit analysis.

Check the stability of your tuned transfer function. Forcing the search to use absolute values of m_2, b_1, and b_2 should result in a stable model. Be sure to report *stable* models for your final answer.

A sample plot of experimental and theoretical best fit Bode magnitude data is provided below, your results may be significantly different depending on which set-up you use for the analysis.

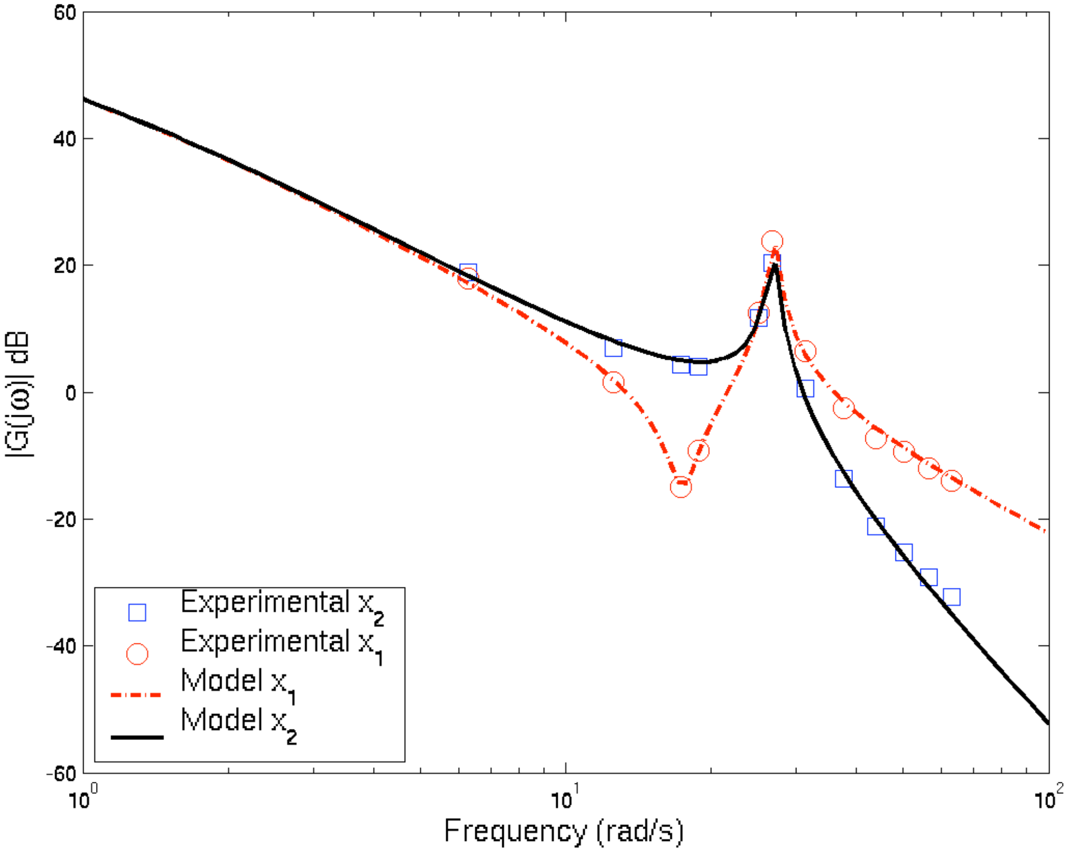


Figure 3, Experimental Bode Magnitude Data with best fit theoretical transfer function.

Here is the state space version of lab2:

```
function J = lab2(x)
m2 = abs(x(1));  % abs forces the function to use positive values for m2
  m1 =  1.9649;

  b1 = abs(x(2));  % b1, and b2
  b2 = abs(x(3));

  k1 = x(4);
  K = x(5);
  A = [zeros(2,2), eye(2);
      [-k1/m1 k1/m1; k1/m2 -k1/m2], [-b1/m1 0; 0 -b2/m2]];
B = [0; 0; K/m1; 0];
C = [eye(2), zeros(2,2)];
D = 0;

Gss = ss(A,B,C,D);
ww = 2*pi*[1: 10];  % experimental spectrum, adjust as needed

maggie = bode(Gss,ww); % change dimensions in the following reshape command
maggie = 20*log10(reshape(maggie,2,10)); % when you have more than 10 data
magxdB = [ % put your experimental data here %]';
magydB = [% ditto %]';  % don’t forget the apostrophes!!!

J = norm(maggie(1,:)' - magxdB) + norm(maggie(2,:)' - magydB);
```