ME 406 Control systems
Laboratory 1

Parametric Time Domain System Identification of a Mass-Spring-Damper System

We will investigate the effects of varying the parameters of a physical spring mass damper system, and see how its behavior is different from the lumped parameter model.

Objectives

The objectives of this lab are to:

- Estimate mass, stiffness, damping, and static gain of four systems
- Collect experimental data of system step response, and compare with a theoretical simulation
- Explain some of the limitations of lumped parameter models.

First a few notes:

- Each of the systems is quite expensive, **BE CAREFUL!**
- There is a hardware shut off button on the box. One member of the group must be ready to push this button whenever the device is being used.
- Do not touch the system when it is operating.
- We will only be using the first cart for this lab. The other two masses should remain locked in position with a nut between the stop tabs.
- The damper must not be completely shut. It should be opened at least two turns.

Procedure

1. Set up the environment.
   - Log on to the computer and start the ECP software under: Start-Programs-ECP-ECP32
   - Configure the system in 1 DOF mode with one spring (preferably medium), two 500g brass masses, and no damping.
2. For each response, you will need to do the following:
   - Select Command-Trajectory. Make sure the Step radio button is selected, and press the Setup button. Select the Open Loop Step radio button, set the Step Size to 0.5 volts, the dwell time to 2000 ms, and the number of reps to 2. Press OK and OK again.

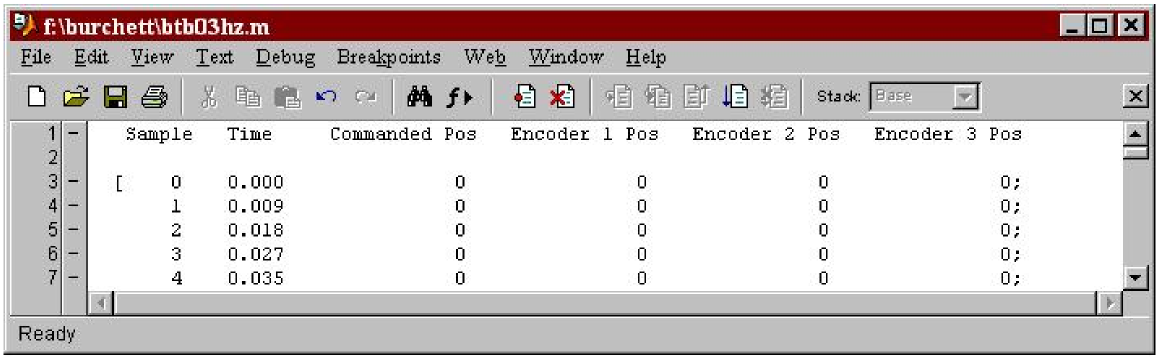

Figure 1: ECP Data Export Format

- Select Utility-Zero Position. Push the black power on button on the ECP interface (big box to your right). Now Select Command-Execute, and push Run. Watch the response. Press OK when you get the 'samples uploaded' dialog.
- (Optional) View your data in the ECP plotting window. Select Plotting-Setup Plot. Remove Encoder 3 Position from the left axis, and add Encoder 1 Position. Push Plot data.
- Export your data. For each response, you will export a unique data file. Select Data-Export Raw Data. Assign a filename with meaning, such as 'nodamp1000gstiff.m', or 'lab4data1.m' and be sure to record the parameters of each response. Save the data to the desktop or a floppy disk. Be sure to copy all of your files to your afs space or a floppy before leaving the lab, and erase your files from the lab computer.

3. Next, connect the damper, but remove the adjustable plug. How should the response be different (under ideal lumped parameter assumption)? How well does the response match your assumption, and why are there differences?

4. You should try all of the following configurations for comparison:

- Vary the damping:
  1. m=1000g, Damper connected with plug inserted 1/2 turn, medium spring (use this as a benchmark for comparison).
  2. m=1000g, Damper connected with plug inserted 4 full turns, medium spring
  3. m=1000g, no damper, medium spring (described above).
- Vary the Mass:
  4. m=0g (no masses added to cart), Damper connected with plug inserted 1/2 turn, medium spring
  5. m=500g, Damper connected with plug inserted 1/2 turn, medium spring
  6. m=1500g, Damper connected with plug inserted 1/2 turn, medium spring
  7. m=2000g, Damper connected with plug inserted 1/2 turn, medium spring

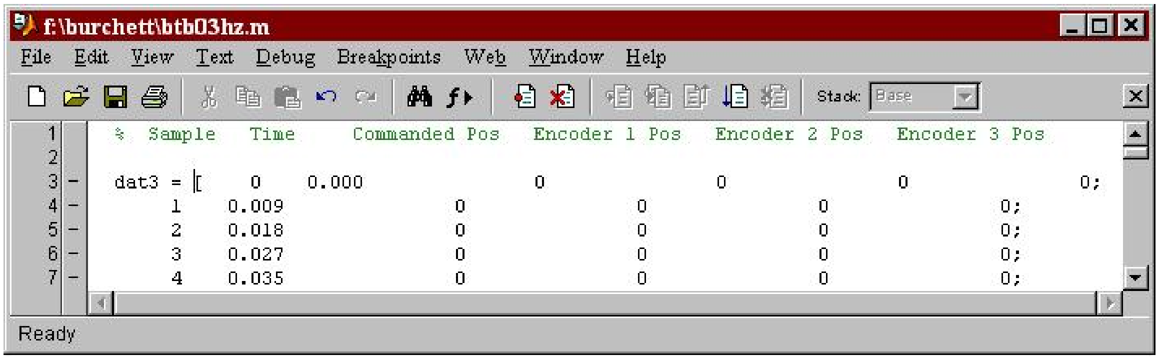

Figure 2: Data file after Editing

- Vary the stiffness: (this might require some help from your instructor)

  8. m=1000g, Damper connected with plug inserted 1/2 turn, stiff spring
  9. m=1000g, Damper connected with plug inserted 1/2 turn, light spring.

Is there a way to increase the stiffness even further? Time permitting, try implementing as many additional stiffnesses as you can configure.

Analysis

Each exported data file will have the format shown in Figure 1. Each row of the array is a sample and the columns are, respectively: Sample, Time, Commanded Pos, Encoder 1 Pos, Encoder 2 Pos, Encoder 3 Pos. Change the file as shown in Figure 2 so that you have an array assignment.

For each reponse, estimate the system static gain, damping ratio, and damped natural frequency. This can be done by:

- Measure the peak time from the experimental response and apply the peak time equation to determine damped natural frequency (ω_d)

$$t_p = \frac{\pi}{\omega_d}$$

- Measure the peak response (y_{peak}) and steady state response (y_{ss}) and calculate percent overshoot (po)

$$po = \frac{y_{peak} - y_{ss}}{y_{ss}}$$

- Apply the percent overshoot equation to find the damping ratio

$$\zeta = \sqrt{\frac{(\ln(po))^2}{(\ln(po))^2 + \pi^2}}$$

- Consider the static gain to be the steady state value (y_{ss}) in counts divided by the step magnitude (0.5)

(Hint, log decrement should provide a better estimate for very lightly damped responses.) You can treat the output units as either 'counts' or cm, however models based on 'counts' will work better for designing controllers in subsequent labs. The conversion is 2196counts = 1 cm.

Use the results of cases 4,5,6, and 7 to estimate the actual cart and damper mass. You can use a procedure as follows:

- Generate four equations by applying the known values of damped natural frequency and damping ratio to the following equation

$$\omega_{di} = \sqrt{\frac{k}{m_i}}\sqrt{1-\zeta_i^2}$$

- These eqns can be manipulated to the following form which is linear in the unknowns (k and m_i)

$$m_i\omega_{di}^2 - k\left(1-\zeta_i^2\right) = 0 \qquad (1)$$

- Generate three more eqns from the known difference (500g) between masses

$$\begin{aligned} m_{i+1} - m_i &= 0.5, \quad i = 1,3 \\ m_{i+1} - m_i &= 1.0, \quad i = 2 \end{aligned} \qquad (2)$$

- Set up and solve the overdetermined system including four permutations of Eq. 1 and three of Eq. 2 above

$$\begin{bmatrix} \omega_{d1}^2 & 0 & \cdots & \zeta_1^2 - 1 \\ -1 & 1 & \cdots & 0 \\ 0 & \omega_{d2}^2 & \cdots & \zeta_2^2 - 1 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{Bmatrix} m_1 \\ m_2 \\ \vdots \\ k \end{Bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ \vdots \end{bmatrix} \qquad (3)$$

The estimates of mass, and stiffness can be obtained using Eq. 3 above. The damping constant c can be found by matching coefficients between the expected system model and standard forms of the model. That is, matching coefficients between the parametric model

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{1}{m}f(t)$$

and the form based on second-order response characterstics

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = K\omega_n^2 f(t)$$

results in the following formula for the damping constant c.

$$c = 2\zeta m \frac{\omega_d}{\sqrt{1-\zeta^2}}$$

Matching coefficients between the parametric form

$$\frac{m}{k}\ddot{x} + \frac{c}{k}\dot{x} + x = Kf(t)$$

and the Bode form of the second order ODE

$$\frac{\ddot{x}}{\omega_n^2} + \frac{2\zeta}{\omega_n}\dot{x} + x = Kf(t)$$

results in the formula

$$c = \frac{2\zeta k\sqrt{1-\zeta^2}}{\omega_d}$$

For cases 4,5,6, and 7, estimate the system parameters (m, c, k and K) use Matlab to plot a theoretical step response, and compare to your experimental step response.

Reporting the Lab

Submit one FORMAL lab report per team. Each individual will receive the team grade for the adventure. Be sure your report includes, but is not limited to the following,

- Plots, Matlab code, and any Simulink models used (do not include hardcopies of your experimental data (numbers), this would be far too lengthy.
- Plots of all system responses grouped logically–that is, put up to four responses per axis, the groups above (vary the mass, etc.) are strongly suggested
- Answer all questions in this handout thoroughly (make sure you read through this assignment carefully).
- List any suggestions for improving the adventure.