ECE-521

Lab 4: Modeling, Simulation, and Control of a 2 Degree of Freedom Inverted Pendulum System

Overview In this lab you will do the following:

1) Experimentally determine the frequency response of a system and use this to identify the system (find the transfer function or state space model)

2) Simulate the model with your controller using Simulink.

3) Implement the controller on the ECP system by modifying your Simulink model

4) Controlling the ECP system with Simulink

For the two degree of freedom inverted pendulum system configuration discussed in class, with $q_1 = x$, $q_2 = \dot{x}$, $q_3 = \theta$, and $q_4 = \dot{\theta}$, we get the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\left(\frac{\omega_1^2}{\Delta}\right) & -\left(\frac{2\zeta_1\omega_1}{\Delta}\right) & \left(\frac{K_1\omega_1^2\omega_\theta^2}{\Delta}\right) & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{\omega_1^2\omega_\theta^2}{g\Delta}\right) & \left(\frac{2\zeta_1\omega_1\omega_\theta^2}{g\Delta}\right) & -\left(\frac{\omega_\theta^2}{\Delta}\right) & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{K_2\omega_1^2}{\Delta}\right) \\ 0 \\ -\left(\frac{\omega_1^2\omega_\theta^2 K_2}{g\Delta}\right) \end{bmatrix} F$$

We need to identify all of these quantities to get the A and B matrices for the state variable description. For our system $D = 0$ and C is determined by whatever we want the output to be.

<u>Step 0: Set Up the System</u>. Only the first cart should be able to move. In addition:

- There should be at least two masses on the first cart.
- There should be at least one spring in the system. If there are two springs the stiffer spring should be between the cart and the motor.
- The pendulum should be securely fastened to the first cart. It should rest on top of the masses and be securely tightened.
- The mass on the pendulum should be within about 2 or three inches from the pivot. Remember that the cart must be able to get under the center of mass of the pendulum in order to right it, so if the center of mass of the pendulum is too far away the cart will never be able to get under it. Be sure to record the length of exposed bar from the pivot to the edge of the center of mass. It is probably easiest to measure this distance using small pieces of paper.
- The ECP system should be moved to the edge of the bench, so that the pendulum is completely free to swing without hitting the bench.
- The wire to measure the position of the pendulum position encoder should be securely attached (with the screws) and the cart and pendulum should be free to move.

You will be using this configuration throughout most of the course so be sure you write down all of the information you need to duplicate this configuration.

<u>Step 1a) Estimate of</u> ω_θ

From the equations of motion, if we assume the cart is fixed, then $\ddot{x} = 0$ and we have

$$\ddot{\theta} + \omega_\theta^2 \theta = 0$$

This is the equation for a simple pendulum. If the pendulum is deflected a small angle and released, it will oscillate with frequency ω_θ.

To measure this:

- Set the input in **Model210_Openloop.mdl** to 0
- Set the X-Y graph in **Model210_Openloop.mdl** to measure the position of the pendulum. You may want to change the y-min and y-max values in the X-Y graph. We are measuring angles in radians, not degrees.
- Displace the pendulum and let it go. Since we are using a small angle assumption, the pendulum should not be displaced too far.
- Using Matlab, plot the displacement of the pendulum versus time, and determine the period of the pendulum, and determine ω_θ. Include this (well labeled) plot in your memo.

Step 1b) Estimation of ω_1 *and* ζ_1

If we assume there is not input ($F = 0$) and the pendulum does not move very much ($\ddot{\theta} \approx 0$) then we have

$$\frac{1}{\omega_1^2}\ddot{x} + \frac{2\zeta_1}{\omega_1}x + x = 0$$

Use the log-decrement method to get estimates of ω_1 and ζ_1. Not that we don't really have the case of $\ddot{\theta} \approx 0$, but this approximation is not too far off. You will need to include these log-decrement results in your memo.

Step 1c) Estimation of K_2

Applying a step input of amplitude A to the cart, estimate

$$K_2 = \frac{x_{ss}}{A}$$

Step 1d) Fitting the Estimated Frequency Response to the Measured Frequency Response

The transfer function between the input and the position of the first cart is given by

$$\frac{X(s)}{F(s)} = \frac{\omega_1^2 K_2 (s^2 + \omega_\theta^2)}{\left(1 - K_1\omega_1^2 \frac{\omega_\theta^2}{g}\right)s^4 + \left(2\zeta_1\omega_1\right)s^3 + (\omega_1^2 + \omega_\theta^2)s^2 + (2\zeta_1\omega_1\omega_\theta^2)s + \omega_1^2\omega_\theta^2}$$

We will use this expression to determine K_1 and get better estimates of ω_1 and ζ_1, then we will have all of the parameters we need for our state variable model. We will be constructing the magnitude portion of the Bode plot and fitting this measured frequency response to the frequency response of the expected transfer function to determine these parameters. For each frequency $\omega = 2\pi f$ we have as input $F(t) = A\cos(\omega t)$ where, for out systems, A is measured in centimeters. After a transition period, the steady state output will be $x(t) = B\cos(\omega t + \theta)$ for the position of the first cart

Since we will be looking only at the magnitude portion of the Bode plot, we will ignore the phase angles.

You will go through the following steps

For frequencies $f = 0.5, 1, 1.5 ... 7.5$ Hz

- Make sure the first cart is free to move.
- Modify **Model210_Openloop.mdl** so the input is a sinusoid. To make any changes to **Model210_Openloop**.mdl, the mode must be **Normal**.
- Set the frequency and amplitude of the sinusoid. Try a small amplitude to start, like 0.01
- Compile **Model210_Openloop.mdl**
- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model210_Openloop.mdl.** If the cart does not seem to move much, increase the amplitude of the input sinusoid. If the cart moves too much, decrease the amplitude of the input sinusoid. Note that if the cart hits the stops you will probably need to adjust the pendulum.
- Record the input frequency (f), the amplitude of the input (A), and the amplitude of the output (B) when the system is in steady state. In Matlab you can just type **plot(time,x1); grid**; once the system has stopped.

You will probably notice that the output does not look as sinusoidal as usual. This is because we are not really giving the pendulum enough time to reach steady state. Enter the values of f , A, and B into the program **process_data_pendulum.m** (you need to edit the file)

At the Matlab prompt, type **data = process_data_pendulum**;

<u>Step 2a) Modelling the Regular Pendlum</u>

Run the program **model_pendulum_full.m.** There are 5 input arguments to this program:

- **data**, the measured data as determined by **process_data_pendulum.m**
- the estimated value of K_2
- ω_θ, the estimated frequency of the pednulum, in radians/sec
- ζ_1, the estimated damping ratio of the cart.
- ω_1, the estimated natural frequency of the cart, in radians/sec

The program **model_pendulum_full.m** will produce the following:

- A graph indicating the fit of the transfer function from the input to the position of the cart to the measured frequency response data. (You need to include the *<u>final</u>* graph of this fit in your memo.)
- The optimal estimates of all parameters (written at the top of the graphs)

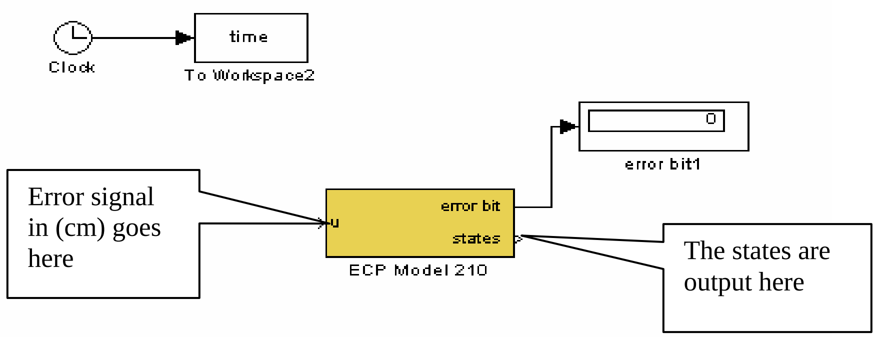
- A file **state_model.mat** in your directory**.** This file contains the A, B, C, and D matrices for the state variable model of the system. If you subsequently type **load state_model** you will load these matrices into your workspace.
- A list of the poles and zeros of the estimated transfer function. This allows you to see how close to a pole/zero cancellation you have.

You need to be sure you have 4 points close to the resonant peaks of the transfer functions. This is particularly true if you have very small values of ζ (which correspond to very sharp peaks.) In addition, you should add points near the frequency of the pole/zero cancellation to clean things up. You may also want to simulate the system at the natural frequency of the pendulum.

You should also compare the final estimates of the parameters with your initial estimates. The values for the all of the frequencies and gains should be fairly close to the final values. The damping ratios may be quite different. You should make a table comparing these values in your report (and include a brief discussion)

Step 2b) Connecting the ECP system to your Simulink file.

First you need to get the Simulink model **ECP210_Template.mdl**, as shown below.



The next step is to remove the model of the system and the output from your Simulink model of the system, and merge the two Simulink files

Step 2c) Controlling the ECP System with Simulink for the Regular Pendulum

a) We want first of all to try and control a regular pendulum. This is a *regulator* problem in that we will be trying to maintain a set point (i.e., to keep the pendulum pointing down.) Simulate the system with zero input, and all initial conditions set to zero except for the initial pendulum displacement. Set this to something like 0.01 rad. Utilize state variable feedback so the **first cart** moves to keep the pendulum pointing down (the pendulum angle is zero degrees) with a settling time of less than 1.0 seconds. Try to utilize the LQR algorithm to determine the feedback gain values. You also need to be sure the control effort is less then 0.4 and the cart does not move more than about 2.5 cm. You will have to simulate your **model** a number of times to get the performance you want before you try it on the ECP system. Try to keep the first two gains (k_1 and k_2) less than 0.15 if you can.

b) In the Matlab workspace, set the final time to 20 seconds by typing Tf = 20;

c) Now try to compile your Simulink that will be driving the ECP system with these same parameters.

d) Be sure the pendulum is pointing straight down and is at rest. Rest the system using **ECPDSPReset** (this is how the ECP finds out where zero is).

e) Connect to the system and start the system. Since there is no input the system should do nothing. Gently nudge the pendulum and the cart should move so the pendulum returns to an angle of zero degrees. If your system does not seem to work very well, do not go on to the next part until you've figured out what is wrong.

If the ECP system does not work (or buzzes), first try resetting the system. Then try making the closed loop poles closer to the origin. Be sure to rerun the **model** of the system to get all the necessary parameters in the workspace before compiling the ECP system.

3a) Modelling the Inverted Pendlum

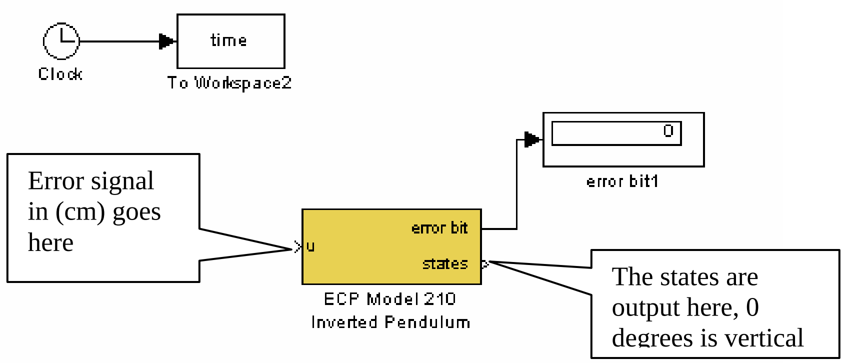
Run the program **model_inverted_pendulum_full.m.** *(Note that this is a different program than you used before!)* There are 5 input arguments to this program:

- **data**, the measured data as determined by **process_data_pendulum.m**
- the estimated value of K_2
- ω_θ, the estimated frequency of the pednulum, in radians/sec
- ζ_1, the estimated damping ratio of the cart.
- ω_1, the estimated natural frequency of the cart, in radians/sec

The program **model_inverted_pendulum_full.m** will produce the same output as the program **model_pendulum_full.m** produced. The only difference will be the state variable model.

Step3b) Connecting the ECP system to your Simulink file.

First you need to get the Simulink model **ECP210_Inverted_Pendulum_Template.mdl**, as shown below.



It is very important that you use this model for the system, since it contains the weird mapping you need so the system believes that 0 degrees is straight up. The next step is to remove the model of the system and the output from your Simulink model of the system, and prepare to merge the two Simulink files

Step 3c) Controlling the ECP System with Simulink for the Inverted Pendlum

This is much more difficult than controlling the regular pendulum, since the system is inherently unstable. You should expect failure until you get the hang of it. The biggest problem is that you must start holding the pendulum nearly vertical (note that in the homework you started with an initial angle of 0.02 radians and it was difficult to control the motion.) If you are off even slightly in holding the pendulum vertical the system will not work. In addition, it is very important to follow the following steps

Read through the following steps before you try and run the inverted pendulum system!

a) Simulate the system with zero input, and all initial conditions set to zero except for the initial pendulum displacement. Set this to something like 0.01 rad. Utilize state variable feedback so the **first cart** moves to keep the pendulum pointing up with a settling time of less than 1.0 seconds. A really good idea is to use the same LQR algorithm (same weights) as you used for the regular pendulum.

b) Be sure the pendulum is free to swing without hitting the workbench.

c) In the Matlab workspace, set the final time to 20 seconds by typing Tf = 20;

d) Now try to compile your Simulink that will be driving the ECP system with these same parameters.

e) Be sure the pendulum is pointing straight down and is a rest. Rest the system using **ECPDSPReset** (this is how the ECP finds out where zero is). ***You must always do this step before you try and run the inverted pendulum!***

f) Connect to the ECP system, but do not start the Simulink. Gently rotate the pendulum up to vertical and hold it loosely between your fingers. Start the Simulink while still holding the pendulum loosely. ***If the system moves rapidly when started you need to let go of the pendulum and stop the system.*** If the system does not move much (you'll know this), gently let go of the pendulum.

f) While the pendulum is vertical, gently blow on it. This should provide some external disturbance which the system should be able to overcome (***yeah feedback!***). You need to plot the position of the pendulum versus time for this part and include it in your memo.

If the ECP system does not work (or buzzes), first try resetting the system. Then try making the closed loop poles closer to the origin. Be sure to rerun the **model** of the system to get all the necessary parameters in the workspace before compiling the ECP system.

You memo should include the following plots

- the graph used to determine the period (and angular frequency) of the pendulum
- the log-decrement results
- the fit of the model (from **model_pendulum_full**)
- the position of the pendulum and it's response to disturbances.

Be sure to include figure numbers and captions (included in the document). Do not just attach graphs, they must be imbedded into the document. The body of the memo should have a brief discussion of the results and any suggestions for improving the lab.