<u>ECE-521</u>

Lab 3: Modeling, Simulation, and Control of a 3 Degree of Freedom System

<u>Overview</u> In this lab you will do the following:

1) Experimentally determine the frequency response of a system and use this to identify the system (find the transfer function or state space model)

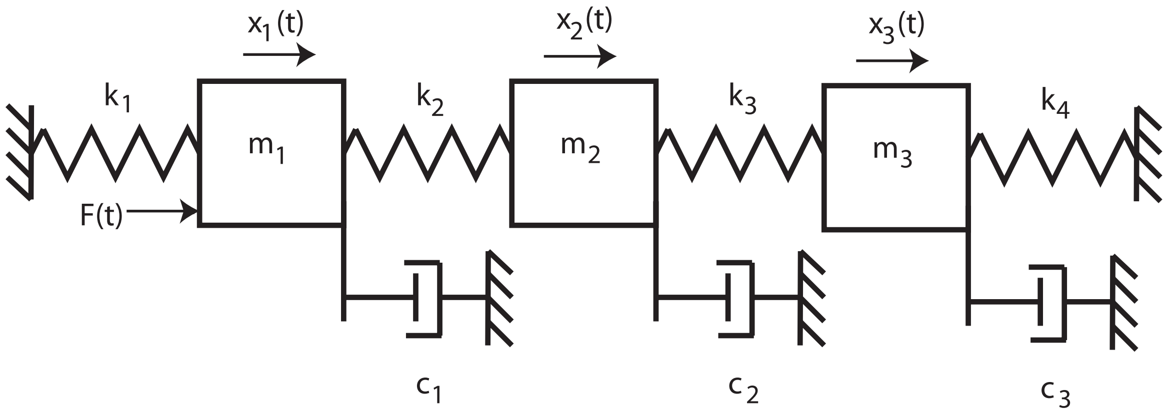
2) Simulate the model (with your controller and/or observer system) using Simulink.

3) Implement the controller on the ECP system by modifying your Simulink model

4) Controlling the ECP system with Simulink

5) Comparing the predicted response (from the model) with the actual response (from the ECP system)

For the following generic three degree of freedom configuration (one of the springs many be missing



we get the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\left(\frac{k_1+k_2}{m_1}\right) & -\left(\frac{c_1}{m_1}\right) & \left(\frac{k_2}{m_1}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \left(\frac{k_2}{m_2}\right) & 0 & -\left(\frac{k_2+k_3}{m_2}\right) & -\left(\frac{c_2}{m_2}\right) & \left(\frac{k_3}{m_2}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \left(\frac{k_3}{m_3}\right) & 0 & -\left(\frac{k_3+k_4}{m_3}\right) & -\left(\frac{c_3}{m_3}\right) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{1}{m_1}\right) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F$$

We need to identify all of these quantities to get the A and B matrices for the state variable description. For our system $D = 0$ and C is determined by whatever we want the output to be.

Step 0: Set Up the System. All three carts should be able to move. In addition:

- The mass on each cart should be the same as that of the previous cart, or the mass should decrease from left to right. You need at least one mass on each cart.
- Either all springs connecting carts should have equal stiffness, or the springs should get less stiff from left to right. You need to use at least three springs.
- If you want to use the active damper, unscrew the screw in the damper.

You will be using this configuration throughout most of the course so be sure you write down all of the information you need to duplicate this configuration.

Step 1a) Initial Estimates of ω_1, ζ_1, ω_2, ζ_2, ω_3, *and* ζ_3

From the equations of motion we have

$$
\begin{aligned}
\ddot{x}_1 + 2\zeta_1\omega_1\dot{x}_1 + \omega_1^2 x_1 &= \frac{k_2}{m_1}x_2 + \frac{1}{m_1}F \\
\ddot{x}_2 + 2\zeta_2\omega_2\dot{x}_2 + \omega_2^2 x_2 &= \frac{k_2}{m_2}x_1 + \frac{k_3}{m_2}x_3 \\
\ddot{x}_3 + 2\zeta_3\omega_3\dot{x}_3 + \omega_3^2 x_3 &= \frac{k_3}{m_3}x_2
\end{aligned}
$$

a) If there is no applied force ($F = 0$) and the second cart is fixed in place($x_2 = 0$), we have

$$s^2 + 2\zeta_1\omega_1 s + \omega_1^2 = 0$$

Use the log decrement method to get our initial estimate of ω_1 and ζ_1.

b) If the third cart is free to move and the second cart is fixed in place ($x_2 = 0$), we have

$$s^2 + 2\zeta_3\omega_3 s + \omega_3^2 = 0$$

Use the log decrement method to get our initial estimate of ω_3 and ζ_3.

c) If the second cart is free to move, and both the first cart and third carts are fixed in place ($x_1 = 0$ and $x_3 = 0$), we have

$$s^2 + 2\zeta_2\omega_2 s + \omega_2^2 = 0$$

Use the log decrement method to get our initial estimate of ω_2 and ζ_2.

For the log-decrement analysis you will go through the following steps for each cart:

- Reset the system using **ECPDSPresetmdl.mdl.**
- Modify **Model210_Openloop.mdl** so the input has zero amplitude. To make any changes to **Model210_Openloop**.mdl, the mode must be **Normal**.
- Compile **Model210_Openloop.mdl**
- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)

- Displace the first mass (or the second or third mass) and hold it.
- Start **(play) Model210_Openloop.mdl** and let the mass go.
- Run the m-file **Log_Dec.m.** This should be in the same directory as **Model210_Openloop.mdl** and **Log_Dec.fig**. This routine assumes the position of the first cart is labeled *x1*, the position of the second cart is labeled *x2*, the position of the third cart is labeled *x3*, and the time is labeled *time*. (These are the defaults in **Model210_Openloop.mdl**.)

You will need to include these log-decrement results in your memo.

Step 1b) Estimating the Gains K_1, K_2, *and* K_3

You will go through the following steps:

- Be sure both carts are free to move
- Reset the system using **ECPDSPresetmdl.mdl.**
- Modify **Model210_Openloop.mdl** so the input is a step. To make any changes to **Model210_Openloop**.mdl, the mode must be **Normal**.
- Set the amplitude to something small, like 0.01 or 0.02.
- Compile **Model210_Openloop.mdl**
- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model210_Openloop.mdl.** If the carts do not seem to move much, increase the amplitude of the step. If the carts moves too much, decrease the amplitude of the step. You must also recompile after any changes.
- Estimate the gains as

$$K_1 = \frac{x_{1,ss}}{A} \quad K_2 = \frac{x_{2,ss}}{A} \quad K_3 = \frac{x_{3,ss}}{A}$$

where x_{ss} is the steady state value of the cart position, and A is the input amplitude.

Step 1c) Fitting the Estimated Frequency Response to the Measured Frequency Response

We will be constructing the magnitude portion of the Bode plot and fitting this measured frequency response to the frequency response of the expected transfer function to determine the parameters we need. For each frequency $\omega = 2\pi f$ we have as input $u(t) = A\cos(\omega t)$ where, for out systems, A is measured in centimeters. After a transition period, the steady state output will be $x_1(t) = B_1 \cos(\omega t + \theta_1)$ for the first cart, $x_2(t) = B_2 \cos(\omega t + \theta_2)$ for the second cart, and $x_3(t) = B_3 \cos(\omega t + \theta_3)$ for the third cart. B_1, B_2, and B_3 are measured in cm. Since we will be looking only at the magnitude portion of the Bode plot, we will ignore the phase angles.

You will go through the following steps

For frequencies $f = 0.5, 1, 1.5 ... 7.5$ Hz

- Make sure all carts are free to move.
- Modify **Model210_Openloop.mdl** so the input is a sinusoid. To make any changes to **Model210_Openloop**.mdl, the mode must be **Normal**.
- Set the frequency and amplitude of the sinusoid. Try a small amplitude to start, like 0.01
- Compile **Model210_Openloop.mdl**
- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model210_Openloop.mdl.** If the carts do not seem to move much, increase the amplitude of the input sinusoid.. If the carts move too much, decrease the amplitude of the input sinusoid.
- Record the input frequency (f), the amplitude of the input (A), and the amplitude of the output (B_1, B_2, and B_3) when the system is in steady state. In Matlab you can just type **plot(time,x1,'-',time,x2,'--',time,x3,':'); grid**; **legend('x_1','x_2','x_3');** once the system has stopped.

Enter the values of f, A, B_1, B_2 and B_3 into the program **process_data_3carts.m** (you need to edit the file)

At the Matlab prompt, type **data = process_data_3carts**;

Run the program **model_3carts.m.** There are 14 input arguments to this program:

- **data**, the measured data as determined by **process_data_3carts.m**
- the estimated value of K_3
- ω_a, the estimated frequency of the first resonance of the third cart, when all carts are moving, in radians/sec
- ζ_a, the estimated first damping ratio when all carts are moving. Assume $\zeta_a = 0.1$.
- ω_b, the estimated frequency of the second resonance of the third cart, when all carts are moving, in radians/sec
- ζ_b, the estimated second damping ratio when all carts are moving. Assume $\zeta_b = 0.1$.
- ω_c the estimated frequency of the third resonance of the third cart, when all carts are moving, in radians/sec
- ζ_c, the estimated third damping ratio when all carts are moving. Assume $\zeta_c = 0.1$.
- ω_1 the estimated natural frequency of the first cart when it is the only cart moving (from the log decrement analysis)
- ζ_1 the estimated damping ratio of the first cart when it is the only cart moving (from the log decrement analysis)
- ω_2 the estimated natural frequency of the second cart when it is the only cart moving (from the log decrement analysis)
- ζ_2 the estimated damping ratio of the second cart when it is the only cart moving (from the log decrement analysis)
- ω_3 the estimated natural frequency of the third cart it is the only cart moving (from the log decrement analysis)
- ζ_3 the estimated damping ratio of the third cart when it is the only cart moving (from the log decrement analysis)

The program **model_3carts.**m will produce the following:

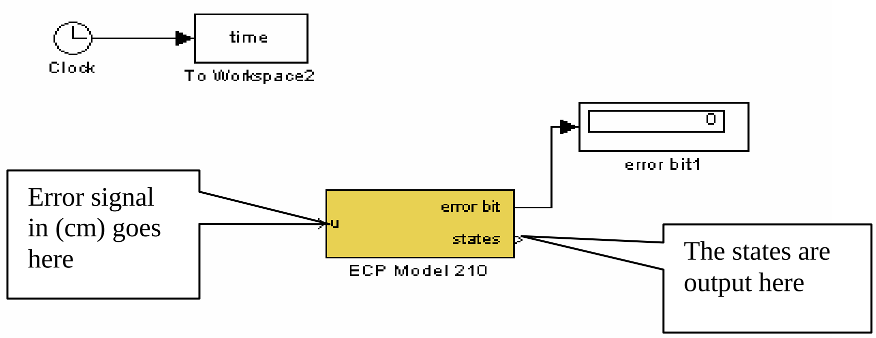
- A graph indicating the fit of the identified transfer function to the measured data for the third cart (You need to include the <u>*final*</u> graph of this fit in your memo.)
- A graph indicating the fit of the identified transfer function to the measured data for the second cart (You need to include the <u>*final*</u> graph of this fit in your memo.)
- A graph indicating the fit of the identified transfer function to the measured data for the first cart (You need to include the <u>*final*</u> graph of this fit in your memo.)
- The optimal estimates of all parameters (written at the top of the graphs)
- A file **state_model.mat** in your directory**.** This file contains the A, B, C, and D matrices for the state variable model of the system. If you subsequently type **load state_model** you will load these matrices into your workspace.

You need to be sure you have 4 points close to the resonant peaks of the transfer functions. This is particularly true if you have very small values of ζ (which correspond to very sharp peaks) At this point you probably should go back and add a few points near the resonant peaks and nulls.

You should also compare the final estimates of the parameters with your initial estimates. The values for the all of the frequencies and gains should be fairly close to the final values. The damping ratios may be quite different. You should make a table comparing these values in your report (and include a brief discussion)

Step 2) Connecting the ECP system to your Simulink file.

First you need to get the Simulink model **ECP210_Template.mdl**, as shown below.



The next step is to remove the model of the system and the output from your Simulink model of the system, and prepare to merge the two Simulink files

Note that we need to use different names in the Simulink files for

- the time (time or m_time)
- the reference input (r or m_r)
- the position of the first cart (x1 or m_x1)
- the velocity of the first cart (x1_dot or m_x1_dot)
- the position of the second cart (x2 or m_x2)
- the velocity of the second cart (x2_dot or m_x2_dot)
- the position of the third cart (x3 or m_x3)
- the velocity of the third cart (x3_dot or m_x3_dot)

This is so we can compare the response of the model with the response of the real system.

3) Controlling the ECP System with Simulink

a) Utilize the linear quadratic regulator algorithm to determine the gains so the **first cart** tracks a 1 cm input with a settling time of less than 0.4 seconds. You will have to simulate your **model** a number of times to get the performance you want before you try it on the ECP system. Record the weights you used. Include plots of both the estimated and actual positions of the carts and the estimated and actual velocities of the carts in your memo.

Now try to compile your Simulink that will be driving the ECP system with these same parameters.

Run the Simulink driving the ECP system. If the system runs acceptably, produce plots comparing the positions of the carts as a function of time with the predicted positions of the carts as a function of time. Similarly, compare the velocities of the carts as a function of time with the predicted velocities of the carts as a function of time. All of these plots should be well labeled using legends and different line types and included in your memo. How close is the predicted response to the real (measured) response?

If the ECP system does not work (or buzzes), first try resetting the system. Then try making the closed loop poles closer to the origin. Be sure to rerun the **model** of the system to get all the necessary parameters in the workspace before compiling the ECP system.

b) Utilize the linear quadratic regulator algorithm to determine the gains so the **second cart** tracks a 1 cm input with a settling time of less than 0.4 seconds. You will have to simulate your **model** a number of times to get the performance you want before you try it on the ECP system. Record the weights you used. Include plots of both the estimated and actual positions of the carts and the estimated and actual velocities of the carts in your memo.

c) Utilize the linear quadratic regulator algorithm to determine the gains so the **third cart** tracks a 1 cm input with a settling time of less than 0.4 seconds. You will have to simulate your **model** a number of times to get the performance you want before you try it on the ECP system. Record the weights you used. Include plots of both the estimated and actual positions of the carts and the estimated and actual velocities of the carts in your memo.

You memo should include all of these plots (with legend and different line types), with figure numbers and captions (included in the document). Do not just attach graphs, they must be imbedded into the document. The body of the memo should have a brief discussion of the results and any suggestions for improving the lab.