

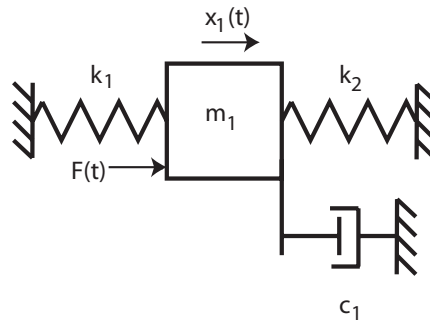
## Lab 2: Time Domain Modeling of One Degree of Freedom Systems

### Overview

In this lab you will be modeling two rectilinear and two rotational one degree of freedom systems using time-domain analysis. The goal is to review the log-decrement method and step response analysis for modeling 2nd order systems.

### Background

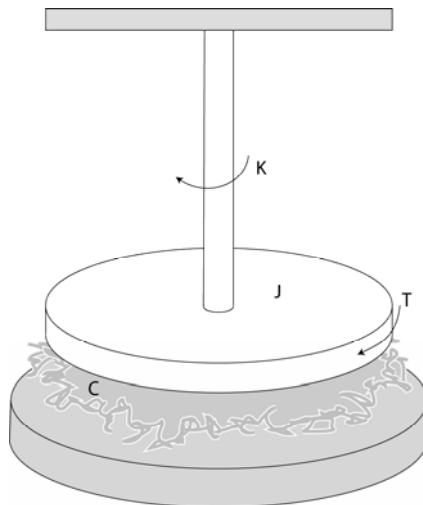
A one degree of freedom rectilinear mass-spring-damper system can be modeled as



By drawing a free body diagram and balancing forces, we get the equation of motion:

$$m_1 \ddot{x}_1(t) + c_1 \dot{x}_1(t) + (k_1 + k_2)x_1(t) = F(t)$$

A one degree of freedom rotational mass-spring-damper system can be modeled as



By drawing a free body diagram and balancing torques, we get the equation of motion

$$J \ddot{\theta}(t) + c \dot{\theta}(t) + k \theta(t) = T(t)$$

*Despite the fact that the systems appear quite different, the transfer functions for both of the one degree of freedom systems can be put into the standard form*

$$G(s) = \frac{K}{\frac{1}{\omega_n^2} s^2 + \frac{2\zeta}{\omega_n} s + 1}$$

*Here  $K$  is the static gain,  $\omega_n$  is the natural frequency, and  $\zeta$  is the damping ratio. These are the parameters we need to determine for these models.*

*You will need to set up a folder for Lab 2 and copy all of the file from the **basic\_files** folder into this new folder.*

### **Part A: One degree of Freedom Rectilinear Systems**

You will need to go through the following steps for *two different* configurations (different masses and/or springs).

*Be sure to load the correct controller personality file for the ECP system (and reset the controller)!!!*

*Step 1: Set Up the System. Only the first cart should move, all other carts should be fixed. You need to have at least one spring connected to the cart and at least one mass on the cart. Do not use the damper. **Be sure you write down all of the information you need to duplicate this configuration.***

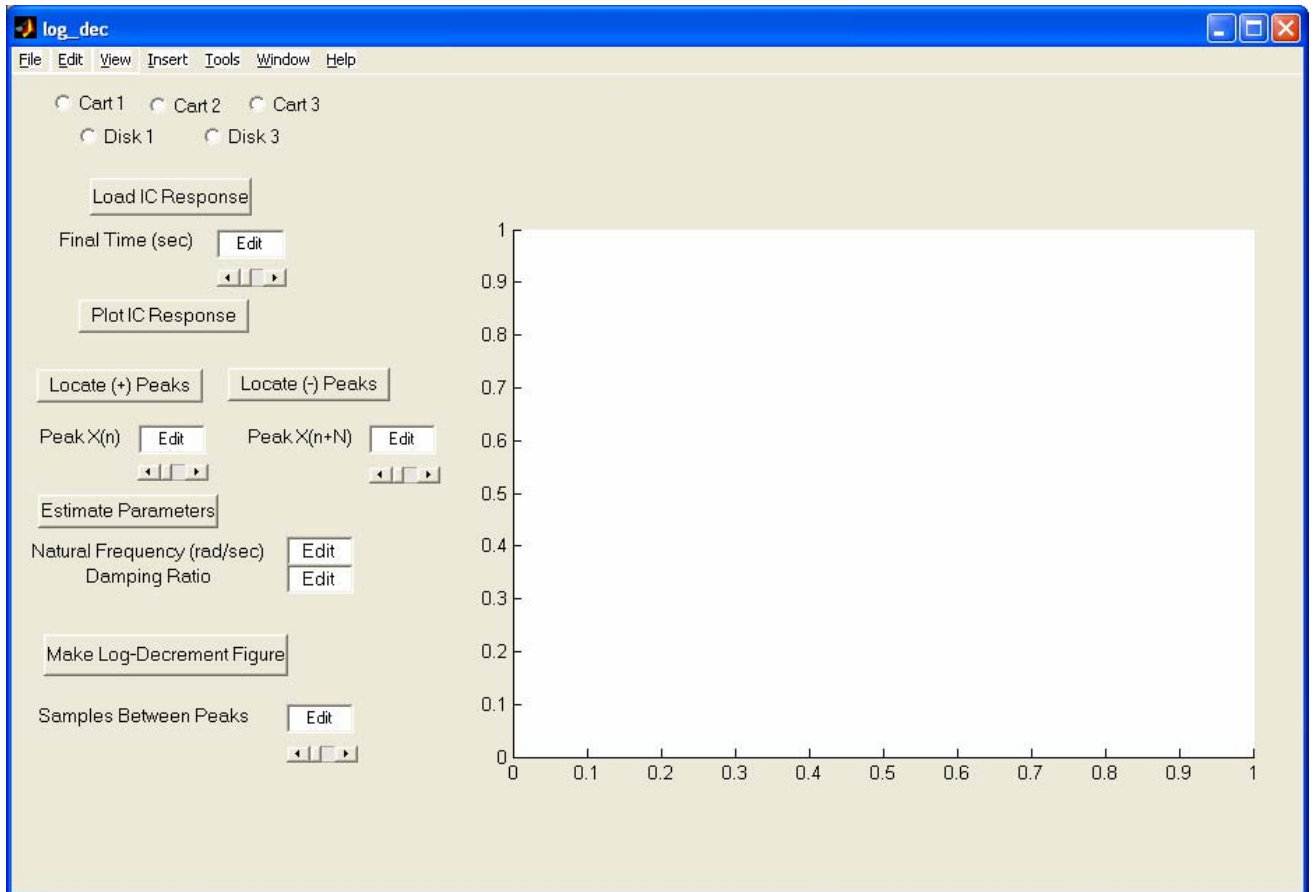
*Step 2: Log Decrement Estimate of  $\zeta$  and  $\omega_n$*

As you recall, the log decrement method is a way of estimating the natural frequency  $\omega_n$  and damping ratio  $\zeta$  of a second order system. However, this method does not determine the static gain.

You will go through the following steps:

- Reset the system using **ECPDSPresetmdl.mdl**.
- Modify **Model210\_Openloop.mdl** so the input has zero amplitude.
- Compile **Model210\_Openloop.mdl** if necessary.
- Connect **Model210\_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Displace the first mass, and hold it.
- Start (**play**) **Model210\_Openloop.mdl** and let the mass go.
- Run the m-file **Log\_Dec.m**. This should be in the same directory as **Model210\_Openloop.mdl** and **Log\_Dec.fig**. This routine assumes the position of the first cart is labeled *x1* and the time is labeled *time*. (These are the defaults in **Model210\_Openloop.mdl**.)

The program **Log\_Dec** comes up with the following GUI:



You need to

- Select the **cart** to be analyzed (cart one in this case)
- Select **Load IC (initial condition) Response** (the variables *time* and *xI* will be loaded from the workspace). At this point some initial estimates will be made.
- Set/modify the **Final Time**
- Select **Plot IC Response** to plot the initial condition response
- Choose to identify the positive peaks (**Locate + Peaks**) or negative peaks (**Locate - Peaks**). If the peaks are not numbered consecutively, you need to decrease the **Samples Between Peaks** and try again until all peaks have been identified.
- Choose the initial peak (**Peak x(n)**) and final peak (**Peak x(n+N)**) to use in the log-decrement analysis. These should be fairly close to the beginning of the initial condition response. Don't try and use more than a few peaks.
- Select **Estimate Parameters** to get the initial estimates of  $\zeta$  and  $\omega_n$
- Select **Make Log-Decrement Figure** to get a plot and summary of the results. You need to put this figure in your memo.

### Step 3: Second Order Step Response

We will now try to identify the required system parameters by determining the step response of the real (ECP) system, simulating an ideal second order system, and comparing the two responses. If you cannot make the ideal system match the real system, make the ideal system match the real system as well as possible at the beginning of the step.

This part of the lab should be done *independently* of Part 2. That is, ***do not*** just use the values for the natural frequency  $\omega_n$  and damping ratio  $\zeta$  you obtained from the log-decrement method. Part of the reason for doing this is so you can develop a better feel for how changing these parameters will change the step response.

You will go through the following steps:

- Reset the system using **ECPDSPresetmdl.mdl**.
- Modify **Model210\_Openloop.mdl** so the input is a step. You may have to set the mode to **Normal**.
- Set the amplitude to something small, like 0.01 or 0.02 cm.
- Compile **Model210\_Openloop.mdl**, if necessary.
- Connect **Model210\_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model210\_Openloop.mdl**. If the cart does not seem to move much, increase the amplitude of the step. If the cart moves too much, decrease the amplitude of the step. You may have to recompile after the change.
- You only need to run the system until it comes to steady state, then stop it.
- Estimate the static gain as

$$K = \frac{x_{ss}}{A}$$

where  $x_{ss}$  is the steady state value of the cart position, and  $A$  is the input amplitude. You should do this in Matlab, don't use the X-Y Graph. The variables  $x1$  and  $time$  should be in your workspace.

Once the static gain has been estimated, use your modified **openloop\_driver.m** and **compare1.m** (from homework 1) to simulate the ideal second order system and then compare the response of the model with the response of the real system. Iterate on the values of  $\omega_n$  and  $\zeta$  to get the best fit between the model and the real system. Remember that fitting during the early part of the step is most important. Record the values of static gain  $K$ , natural frequency  $\omega_n$ , and damping ratio  $\zeta$  that results in the best fit. Include the plot of the fit of your best model to the real system in your memo. The values of the static gain should be similar for both methods.

## **Part B: One degree of Freedom Rotational Systems**

*Be sure to load the correct controller personality file for the ECP system (and reset the controller)!!!*

You and your partner should change functions/positions now, i.e. whoever was running Simulink and the ECP system for the first part should not do this for the second part.

You will need to go through the following steps for two different configurations (different mass positions). Note that for the rotational systems, the mass positions should be placed symmetrically about the rotational axis. There are lines on the disks to help you accomplish this.

*Step 1: Set Up the System. Only the first disk should move, all other disks should be fixed. **Be sure you write down all of the information you need to duplicate this configuration.***

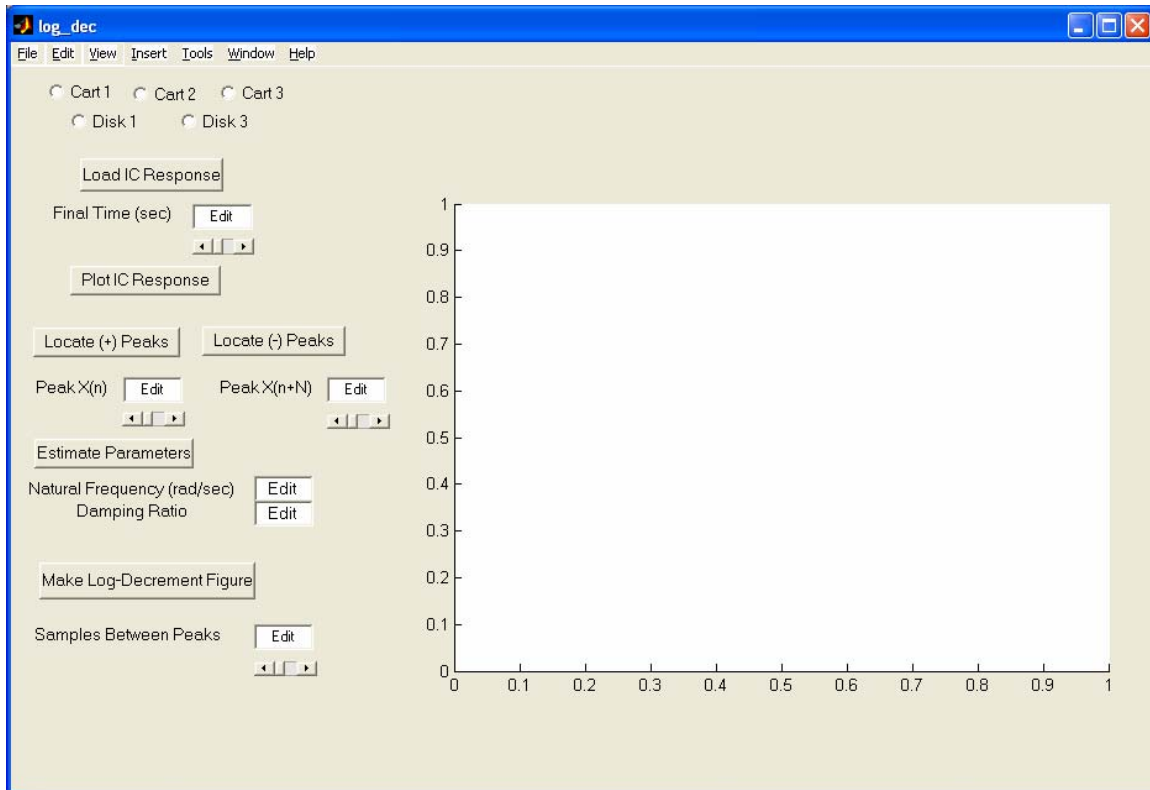
*Step 2: Log Decrement Estimate of  $\zeta$  and  $\omega_n$*

As you recall, the log decrement method is a way of estimating the natural frequency  $\omega_n$  and damping ratio  $\zeta$  of a second order system. However, this method does not determine the static gain.

You will go through the following steps:

- Reset the system using **ECPDSPresetmdl.mdl**.
- Modify **Model205\_Openloop.mdl** so the input has zero amplitude.
- Compile **Model205\_Openloop.mdl** if necessary.
- Connect **Model205\_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Displace the first disk, and hold it. Don't displace it more than about 15 degrees.
- Start (**play**) **Model205\_Openloop.mdl** and let the mass go.
- Run the m-file **Log\_Dec.m**. This should be in the same directory as **Model205\_Openloop.mdl** and **Log\_Dec.fig**. This routine assumes the position of the first disk is labeled *theta1* and the time is labeled *time*. (These are the defaults in **Model205\_Openloop.mdl**.)

The program **Log\_Dec** comes up with the following GUI:



You need to

- Select the **disk** to be analyzed (disk one in this case)
- Select **Load IC (initial condition) Response** (the variables *time* and *theta1* will be loaded from the workspace). At this point some initial estimates will be made.
- Set/modify the **Final Time**
- Select **Plot IC Response** to plot the initial condition response
- Choose to identify the positive peaks (**Locate + Peaks**) or negative peaks (**Locate - Peaks**). If the peaks are not numbered consecutively, you need to decrease the **Samples Between Peaks** and try again until all peaks have been identified.
- Choose the initial peak (**Peak x(n)**) and final peak (**Peak x(n+N)**) to use in the log-decrement analysis. These should be fairly close to the beginning of the initial condition response. Don't try and use more than a few peaks.
- Select **Estimate Parameters** to get the initial estimates of  $\zeta$  and  $\omega_n$
- Select **Make Log-Decrement Figure** to get a plot and summary of the results. You need to put this figure in your memo.

### Step 3: Second Order Step Response

We will now try to identify the required system parameters by determining the step response of the real (ECP) system, simulating an ideal second order system, and comparing the two responses. If you cannot make the ideal system match the real system, make the ideal system match the real system as well as possible at the beginning of the step.

For this part, *you need to be sure you are using input and output units of radians, not degrees*. You may want to save a new variable from **Model205\_Openloop.mdl** that saves the position of the disk in radians (*theta1* is in degrees). Since the program **compare1.m** expects the output of the ECP system to be called *x1*, this may be a good choice. Note also that your output plots must be in either *radians* or *degrees* for this part, not in cm!

This part of the lab should be done independently of Part 2. That is, **do not** just use the values for the natural frequency  $\omega_n$  and damping ratio  $\zeta$  you obtained from the log-decrement method. Part of the reason for doing this is so you can develop a better feel for how changing these parameters will change the step response.

You will go through the following steps:

- Reset the system using **ECPDSPresetmdl.mdl**.
- Modify **Model205\_Openloop.mdl** so the input is a step. To make any changes to **Model205\_Openloop.mdl**, the mode must be **Normal**.
- Set the amplitude to something small, like 5 degrees (note the ECP system works in *radians*).
- Compile **Model205\_Openloop.mdl**, if necessary.
- Connect **Model205\_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model205\_Openloop.mdl**. If the disk does not seem to move much, increase the amplitude of the step. If the cart moves too much, decrease the amplitude of the step. You must also recompile after any changes.
- You only need to run the system until it comes to steady state, then stop it.
- Estimate the static gain as

$$K = \frac{\theta_{ss}}{A}$$

where  $\theta_{ss}$  is the steady state value of the cart position, and  $A$  is the input amplitude.

You should do this in Matlab, don't use the X-Y Graph. The variables *theta1* and *time* should be in your workspace. Be sure you use the *same units* for  $\theta_{ss}$  and  $A$ .

Once the static gain has been estimated, use your modified **openloop\_driver.m** and **compare1.m** (from homework 1) to simulate the ideal second order system and then

compare the response of the model with the response of the real system. Iterate on the values of  $\omega_n$  and  $\zeta$  to get the best fit between the model and the real system. Remember that fitting during the early part of the step is most important. Record the values of static gain  $K$ , natural frequency  $\omega_n$ , and damping ratio  $\zeta$  that results in the best fit. Include the plot of the fit of your best model to the real system in your memo. The values of the static gain should be similar for both methods.

*Your memo should include descriptions of each system (so you can set them up again), a table comparing the estimated values of  $K$ ,  $\omega_n$  and  $\zeta$  (two different methods for each parameter). You should include as attachments 8 graphs (log-decrement and step response graphs for two rectilinear and two rotational systems), each with a Figure number and caption.*