ECE-320 Linear Control Systems
Laboratory 1

Time domain methods for estimating ω_n and ζ

In this lab we will practice making second order models in a couple of different ways, and then comparing the models with the real system. In future labs we will have to make models of the systems before we can do any controls, so you need to be able to do this quickly. *We will estimate the system parameters for five different systems in this lab. You should analyze systems with three different spring configurations with and without an active damper.* This analysis method will hopefully become very routine by the end of the lab, we will be using it for the remainder of the course.
First a few notes:

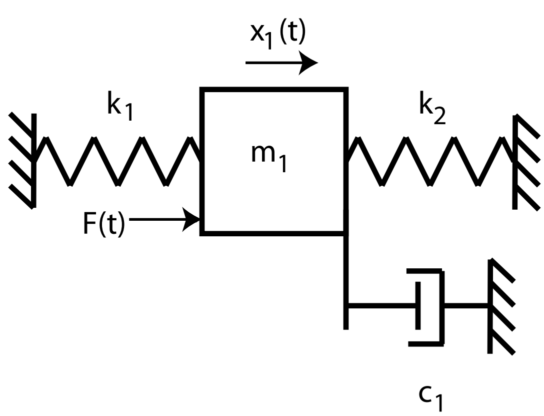
- Each of the systems is quite expensive, **BE CAREFUL!**
- There is a hardware shut off button on the box. One member of the group must be ready to push this button whenever the device is being used.
- Do not touch the system when it is operating.
- We will only be using the first cart for this lab. The other two masses should remain locked in position with a nut between the stop tabs.
- The damper must not be completely shut. It should be opened at least two turns.

The last page of this lab indicates the types of things that need to be collected. Be sure the nuts and tools are put away before you leave the lab!

Pre-Lab

1) Print out this lab and **read it**

Consider the following model of the one DOF (degree of freedom) system we will be using.



2) Draw a freebody diagram of the forces on the mass.

3) Show that the equations of motion can be written

$$m_1\ddot{x}_1(t) + c_1\dot{x}_1(t) + (k_1 + k_2)x_1(t) \quad = \quad F(t)$$

or

$$\frac{1}{\omega_n^2}\ddot{x}_1(t) + \frac{2\zeta}{\omega_n}\dot{x}_1(t) + x_1(t) \quad = \quad \frac{1}{k_1 + k_2}F(t) \; \equiv \; K_{static}u(t)$$

where $u(t)$ is the motor input in volts, and K_{static} is the static gain for the system. Note that this gain also includes the open loop motor gain. What are ω_n and ζ in terms of m_1, k_1, k_2, and c_1?

4) Taking Laplace transforms, show that we get

$$G(s) \quad = \quad \frac{X_1(s)}{U(s)} = \frac{K_{static}}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1}$$

We will be using two different methods, the *log decrement* method and fitting the data to a step response, to estimate ζ and ω_n. We will then estimate K_{static} by comparing the step response of our system model with the real system.

Log Decrement Method

We will assume our mass/spring/damper system is an ideal second order system. If the system is at rest and we provide the mass with an initial displacement away from equilibrium, the response due to this displacement is

$$x(t) \quad = Ae^{-\zeta\omega_n t}\cos(\omega_d t + \theta)$$

where

$$\begin{aligned} x(t) &= \text{the displacement of the mass as a function of time} \\ \zeta &= \text{the damping ratio} \\ \omega_n &= \text{the natural frequency} \\ \omega_d &= \text{the damped frequency} = \omega_n\sqrt{1-\zeta^2} \end{aligned}$$

After the mass is released, the mass will oscillate back and forth with period given by $T_d = 2\pi/\omega_d$, so if we measure the period of the oscillation (T_d) we can determine ω_d.

Let's assume t_0 is the time of one peak of the cosine. Subsequent peaks will occur at times given by $t_n = t_0 + nT_d$.

5) Now examine the ratio of the response of the two times, and show that

$$\frac{x(t_0)}{x(t_n)} \quad = \quad e^{\zeta\omega_n T_d n}$$

6) Define the log decrement as $\delta = \ln\left[\frac{x(t_0)}{x(t_n)}\right]$ and show that

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 n^2 + \delta^2}}$$

From our estimate of ζ and ω_d we can estimate ω_n. From these estimates we know all of the parameters except K_{static}. We will be using a Matlab program **log_decrement_step** to automate the log decrement method for computing these parameters.

Step Response Method

For this method, we look at the step response of our model and the step response of the real system and try and adjust the parameters to get as good a fit as we can. If we estimate the damping ratio ζ and have estimated the damped frequency ω_d, then then $\omega_n = \omega_d/\sqrt{1-\zeta^2}$.

We will be using a Matlab program **fit** to automat this method for estimating the system parameters.

In general, we will be doing the same things to estimate the transfer function parameters for *five different systems*. The details will be explained in more detail on subsequent pages.

1. set up a system (adjusting masses/springs/dampers) (*record the system configuration*)
2. reset the zero position of the system
3. with the (open loop) step input set at zero volts, displace the mass and hold it
4. release the mass while running the system
5. look at the data on the screen
6. if the data is good, export the data to a file
7. set the input (open loop) step response to a voltage between 0.5 and 2 volts.
8. determine the step response of the system
9. look at the data on the screen
10. if the data is good, export it
11. edit both file so Matlab can read them
12. using the **log_decrement_step** program, determine a second order system representation
13. compare the step response of the estimated system with the true step response
14. using **fit** program, estimate the second order system based only on the step response.

In what follows, I think I've listed all of the commands you will need to complete this Lab, in the order you will need to use them. It looks bad, but it will be easy after a few tries.

If your system starts to sound funny or oscillate wildly, either shut it off or click on the ABORT button!!

Running The ECP Systems

0. Starting the software

From Windows, go to **Programs** → **ECP** → **ECP32**

1. Setting up the system

With the control system turned off (push the button on the white and black box, the green **pwr** light should be off), set up the device for you group. You may change the number of masses, add/subtract/change springs, and add/subtract the damper. Be sure all masses/springs are tightened down. If you use the dashpot, be sure the screw on the dashpot (damper) is at least two full turns away from its closed position. If the dashpot exerts too much damping your system will not oscillate.

2. Setting the mechanical zero position

By turning the thumbscrews, set the mechanical zero position indicator. This will help you determine the size of the initial displacement.

3. Turn on the system

Push the button on the white and black box to enable the control system.

4. Set the electrical zero position

Select **Utility** → **Zero Position** to set the current position to zero. You may have to click on this a few times. Look at the **Following Error** readouts, if they are zero or near it you can continue..

5. Set the units

Select **Setup** → **User Units** and set the units to **counts**.

6. Set the trajectory for an initial condition response

Select **Command** → **Trajectory**. Select **Step** and click on **Setup**. Select **Open Loop Step** and set **Step Size** to **0 (zero)** volts. This is important, we do not want the system trying to move the cart! Set the **Dwell Time** to something like 2000 ms, this is the time the system will be recording data. Finally click **OK**, then **OK** and you should be back to the main menu.

7. Prepare to collect data

Select **Data** → **Setup Data Aquisition**. Set the **Sample Period** to every 1 servo cycle. Be sure you are recording from all of the encoders (if you need to change this, see me). Click **OK** to get back to the main menu.

8. Prepare to plot the data

Select **Plotting** → **Setup Plot** You'll want to remove **Encoder 3 Position** and add **Encoder 1 Position**. The click **OK** and get back to the main menu.

9. Collecting initial condition data for log-decrement analysis

Select **Command Execute**. A menu box will come up with a number of options, and a big green **Run** button. At this point one person should displace the first cart and try and hold it still (so there is no initial velocity, only an initial position). One partner should then click on the **Run** button, and a short time later the person holding the cart should release it. You want to record the initial position and the subsequent motion of the cart. **If the motor is on, release the mass at once!** When the system has finished collecting data, a box will appear indicating the how many sample points of data have been collected. (If you have hit a stop, the system stops recording data.) Click on **OK** to get back to the main menu.

10. Plotting the data

Select **Plotting** → **Setup Plot**,then **Plot Data**. You should look at the data before you export it.

11. Exporting the data

Select **Data** → **Export Raw Data**. When asked where to put the data, put it into the **ECE 320** folder or any folder you want to in the **ECE 320** folder. (Remember, there may be multiple sections of ECE-320!)

12. Set the trajectory for a step response

Select **Command** → **Trajectory**. Select **Step** and click on **Setup**. Select **Open Loop Step** and set **Step Size** to a voltage level below 3 volts. You may want to try various voltages. Be sure to record this voltage! Set the **Dwell Time** to something like 2000 ms, this is the time the system will be recording data. Finally click **OK**, then **OK** and you should be back to the main menu.

13. Collecting step response data

Select **Command Execute**. A menu box will come up with a number of options, and a big green **Run** button. Click on the **Run** button. When the system has finished collecting data, a box will appear indicating the how many sample points of data have been collected. (If you have hit a stop, the system stops recording data. This usually means you're input amplitude in step 12 was too large. Got back to step 12 and choose a smaller voltage.) Click on **OK** to get back to the main menu.

14. Plotting the data

Select **Plotting** → **Setup Plot**, or just **Plotting Data** → **Plot Data**. You should look at the data before you export it.

15. Exporting the data

Select **Data** → **Export Raw Data**. When asked where to put the data, put it into the **ECE 320** folder or any folder you want to in the **ECE 320** folder. You should give this file a name similar to the name you gave to the corresponding initial condition response so you will remember they go together.

16. Preparing the data for analysis

At this point you need to locate the files you have exported, and edit out the first line and the '[' at the beginning of the second line. Save the files as type '.dat'. If you screw up you'll still have the original files, and the GUI's expect files to have the suffix '.dat'.

17. Log-Decrement Analysis

Start Matlab and set the default folder to the **ECE 320** folder (or wherever you put your data). Type log_dec_step to start the log-decrement analysis. You will need to compare the estimated transfer function with the measured step response. Be sure to print out a figure of your final estimate of the step response.(See subsequent page for a description of this program)

18. Step Response Analysis

From Matlab, type **fit**, which starts a routine to help you find a second order estimate of a transfer function using the settling time and the damping ratio directly. Be sure to print out a figure of your final estimate of the step response.
(See subsequent page for a description of this program)

NOTE The *log-decrement* and *fit* programs are to be use *independently*. Do not just take the estimates from the *log-decrement program* and type them into the *fit* program. You may want to have one partner operate one program and the other operate the other program and then compare the answers.

log_dec_step Program

We are trying to estimate the parameters of a second order model

$$G(s) = \frac{K_{static}}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1}$$

This program basically duplicates what you did in the prelab. It allows you to determine the peaks of the initial condition response, and then using the locations and amplitudes of these peaks estimate the damping ratio ζ and the damped frequency ω_d and then the natural frequency ω_n. You need to guess the static gain K_{static} so the model and the real system have the same steady state values.

You should generally go through the following steps in the following order:

0. Select Cart Select cart 1 for this lab.

1. Load IC Response Click here to load the file with the initial condition response.

2. Final Time This is initially set to the final time of the data. You can choose a different final time to display the IC response. You may want to change this and then plot the IC response again since you are interested in only the first few cycles.

3. Plot IC Response This will plot the initial condition response from the file you chose. If you don't get anything you may have entered the filename wrong or chose the wrong encoder. Other possibilities are that you did not record all of the encoders in the default order, or that you did not edit the data file properly.

4. Compute + Peaks/ Compute - Peaks The log-decrement looks at what happens from one set of peaks to another. You have the choice of looking at positive peaks or negative peaks. In some instances, where there is alot of damping, you may not get much oscillation. Then you may want to use the negative peaks to do the log-decrement. See also (**8. Samples Between Peaks**)

5. Peak X(n) and Peak X(n+N) Once the peaks have been identified, you need to choose the peaks to use for the log-decrement analysis. Here you enter the lowest number (for the starting peak, X(n)) and the highest number (for the ending peak X(n+N)). Note that it is generally a bad idea to use all of the peaks. *For these systems, you will get a better estimate using the first few peaks and ignoring the others.*

6. Estimate Parameters Click on this box to estimate the transfer function parameters (ζ and ω_n) based on the log-decrement analysis and your chosen peaks. If you choose different peaks you will need to click on this again to get the new estimates.

7. Make Log-Decrement Figure This will print out a figure with the peaks numbered and the estimated parameters in the title. This is useful for your lab reports.

8. Samples Between Peaks This is a parameter that determines the minimum number of samples between peaks. The program finds a peak, and then waits this many samples before it finds another peak. It is important that peaks be numbered consecutively (at least the peaks after the initial displacement). You may need to change this parameter if the program is not identifying all of the peaks correctly.

9. Load Step Response Click here to load the step response file into the program.

10. Final Time Determine the final time for the step response plot. Note that since the system step response is both "on" then "off" you usually choose a final time where the step is still "on".

11. Gain The program makes an initial guess for the gain K_{static}. You probably have to modify this value, try and get the steady state values of the measured and estimated step responses to be the same.

12. Plot Setp Response/Log Dec Estimate Click on this to see the step response of the true system and the step response of your model. Note that these may not be very close since we didn't model the motor/rack/pinion very well. You may have to iterate between this step and step 11 to get the gain correct.

13. Make Step/Log Dec Figure This will produce a figure of the step response of the real system and your model. The important system parameters will be printed out. This is a good figure to have in your report.

fit Program

This program allows us to choose values of ζ, the settling time T_s (and hence ω_n) and the static gain K_{static} so our model

$$G(s) = \frac{K_{static}}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1}$$

matches the actual step response as closely as possible. Note that we want to match the final value and then the initial response.

1. Load Step Response Click here to load the step response file into the program.

2. Final Time Determine the final time for the step response plot. Note that since the system step response is both "on" then "off" you usually choose a final time where the step is still "on".

You need to make guesses for the following three: *gain*, *damping ratio.* and *damped frequency.*

- **first** adjust the gain so the steady state values between the estimated and measured step responses are the same. If the systems are still oscillating, try to be sure the final values will be the same.
- **second** adjust the damping ratio so the peak values are the same. Smaller damping ratios mean larger overshoots (higher peaks)
- **third** adjust the damped frequency to try and match the early part of the step response.

3. Gain Here you guess at the system gain. At this point we have a model of the system but we don't know the system gain. It won't take long to iterate and get it pretty close.

4. Damping Ratio Enter the damping ratio. It is most important to match the early part of the step response.

5. Damped Frequency Enter the estimated damped frequency. It is most important to match the early part of the step response.

6. Plot Mesaured/Estimated Step Response Plot the estimated and measured step response of the system with the current estimates for ζ, ω_n, and K_{static}.

7. Make Figure This will produce a figure of the step response of the real system and your model. The important system parameters will be printed out. This is a good figure to have in your report.

Memo

Your memo for this lab should indicate (briefly)

- if the two different methods of estimating the transfer function seemed to match.
- a list any improvements to the lab you might suggest.

You should have as attachments to your memo (though all in the same document file) plots showing the response of the estimated transfer function using the *log-decrement* program and plots of the estimated transfer function using the *fit* program. Hence there should be two plots for each system analyzed. The captions to these figures should indicate the configuration of the system (weights, springs, damper settings). Your pre-lab does not need to be computer generated.

You should try to identify the transfer function parameters for five different systems

- with three different spring configurations
- with and without an active damper