Midterm Exam 3

# ECE205 Dynamical Systems

## Midterm Exam 3
## 5/12/11

NAME: ___________________________ CM: ___________

- You must **show work** to receive partial and full credit.
- Put a box around your final answer and it must include units, if necessary.
- Time allowed : 50 minutes.

| Question # | Possible Points | Awarded Points |
|---|---|---|
| 1 | 10 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |
| 6 | 10 | |
| Total | 100 | |

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**1)** **(10 points)** For the following multiple choice questions circle the letter next to the correct answer.

The following transfer function is for questions i, ii, and iii.

$$H(s) = \frac{1}{(s+4)(s^2+4s+4)(s^2+2s+5)}$$

i) Which of the following is **not** a **characteristic mode** of the system?

a) $e^{-4t}$ b) $te^{-2t}$ c) $e^{-2t}$ d) $e^{t}\cos(2t)$ e) $e^{-t}\sin(2t)$

ii) The best estimate of the **settling time** for this system is

a) 4 seconds b) 2 seconds c) 1 second d) 0.2 seconds e) 8 seconds

iii) The **dominant pole(s)** of this system are

a) -2 and -2 b) -1+2j and -1-2j c) -4 d) -20 e) 0

iv) How many of the following impulse responses represent **unstable systems**?

$$h_1(t) = [t + e^{-t}]u(t)$$
$$h_2(t) = e^{-2t}u(t)$$
$$h_3(t) = [2 + \sin(t)]u(t)$$
$$h_4(t) = [1 - t^3 e^{-0.1t}]u(t)$$
$$h_5(t) = [1 + t + e^{-t}]u(t)$$
$$h_6(t) = [te^{-t}\cos(5t) + e^{-2t}\sin(3t)]u(t)$$

a) 0 b) 1 c) 2 d) 3 e)5

v) Which of the following transfer functions represents a **stable** system?

$$G_a(s) = \frac{s-1}{s+1} \qquad G_b(s) = \frac{1}{s(s+1)} \qquad G_c(s) = \frac{s}{s^2-1}$$

$$G_d(s) = \frac{s+1}{(s+1+j)(s+1-j)} \qquad G_e(s) = \frac{(s-1-j)(s-1+j)}{s} \qquad G_f(s) = \frac{(s-1-j)(s-1+j)}{(s+1-j)(s+1+j)}$$

a) all but $G_c$ b) only $G_a$, $G_b$, and $G_d$ c) only $G_a$, $G_d$, and $G_f$

d) only $G_d$ and $G_f$ e) only $G_a$ and $G_d$

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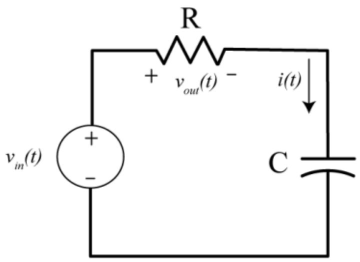
**2)** **(20 points)** For the following ciruit,




Write the output, $V_{out}(s)$ in terms of $V_{in}(s)$, R, C and $v(0^-)$. Identify the ZSR (zero state reponse) and the ZIR (zero input response). (You can leave your answer in the s-domain)

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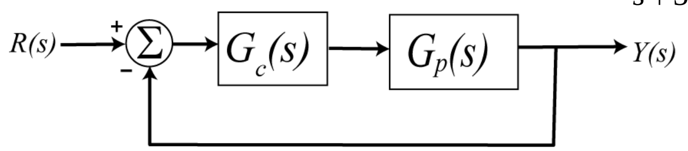
**3)** **(20 points)** Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \frac{3}{s+5}$.




a) Determine the settling time of the plant alone (assuming there is no feedback).

b) Determine the steady-state error due to a unit step input of the plant alone (assuming there is no feedback).

c) For a proportional controller, $G_c(s) = k_p$,

i) Determine the closed loop transfer function $G_o(s)$.

ii) What is the settling time in terms of $k_p$?

iii) What is the steady state error due to a unit step input, in terms of $k_p$?

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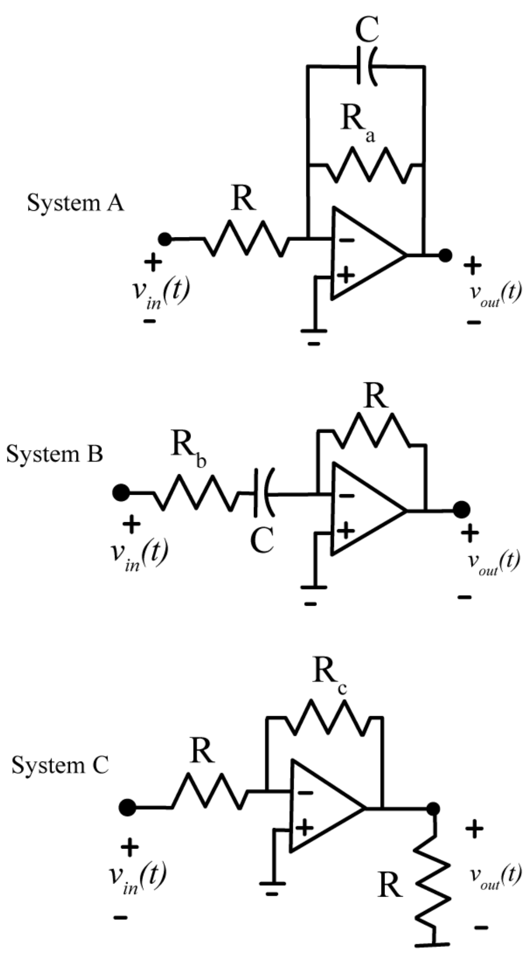
**4)** **(20 points)** The following figure shows three different circuits, which are subsystems for a larger system. We can write the transfer functions for these systems as

$$G_a(s) = \frac{-K_{low}\omega_{low}}{s+\omega_{low}} \quad G_b(s) = \frac{-K_{high}s}{s+\omega_{high}} \quad G_c(s) = -K_{ap}$$

Determine the parameters $K_{low}, \omega_{low}, K_{high}, \omega_{high}$, and $K_{ap}$ in terms of the parameters given (the resistors and capacitors).



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**5)** **(20 points)** For the following transfer functions, determine the **impulse response** of the system. Do not forget any necessary unit step functions.

a) $H(s)=\dfrac{e^{-s}}{s+2}$

For the following transfer functions, determine the **unit step response** of the system. Do not forget any necessary unit step functions.

b) $H(s) = \frac{1}{(s+1)^2}$

c) $H(s) = \frac{1}{s^2+4s+20}$

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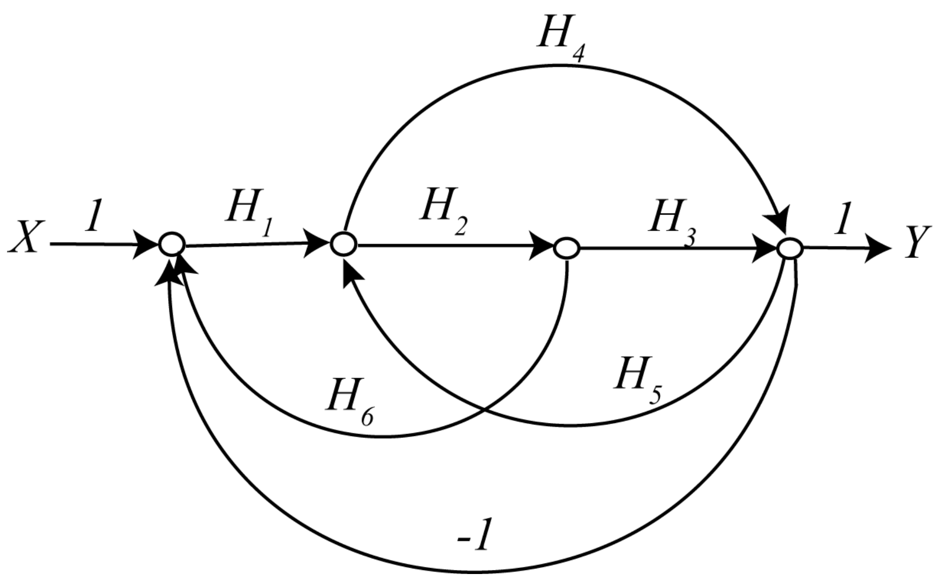
**6**) **(10 points)** For the following signal flow graph, determine the transfer function between the input and output using Mason's gain formula. You do not need to simplify your final answer.



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**EQUATION SHEET**

$$\mathcal{L}\{\delta(t)\} = 1$$

$$\mathcal{L}\{u(t)\} = \frac{1}{s}$$

$$\mathcal{L}\{tu(t)\} = \frac{1}{s^2}$$

$$\mathcal{L}\left\{\frac{t^{m-1}}{(m-1)!}u(t)\right\} = \frac{1}{s^m}$$

$$\mathcal{L}\{e^{-at}u(t)\} = \frac{1}{s+a}$$

$$\mathcal{L}\{te^{-at}u(t)\} = \frac{1}{(s+a)^2}$$

$$\mathcal{L}\left\{\frac{t^{(m-1)}}{(m-1)!}e^{-at}u(t)\right\} = \frac{1}{(s+a)^m}$$

$$\mathcal{L}\{\cos(\omega_0 t)u(t)\} = \frac{s}{s^2+\omega_0^2}$$

$$\mathcal{L}\{\sin(\omega_0 t)u(t)\} = \frac{\omega_0}{s^2+\omega_0^2}$$

$$\mathcal{L}\{e^{-\alpha t}\cos(\omega_0 t)u(t)\} = \frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$$

$$\mathcal{L}\{e^{-\alpha t}\sin(\omega_0 t)u(t)\} = \frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$$

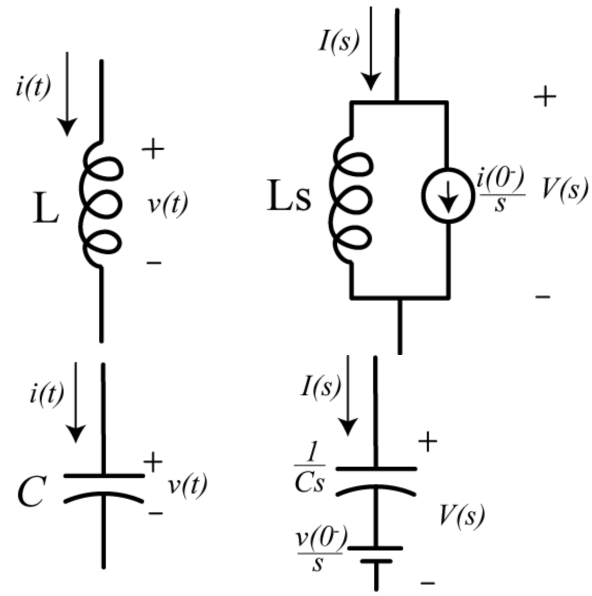
$$\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0^-)$$

$$\mathcal{L}\left\{\frac{d^2x(t)}{dt^2}\right\} = s^2X(s) - sx(0^-) - \dot{x}(0^-)$$

$$\mathcal{L}\{x(t-a)\} = e^{-as}X(s)$$

$$\mathcal{L}\{e^{-at}x(t)\} = X(s+a)$$

$$\mathcal{L}\left\{x\left(\frac{t}{a}\right), a>0\right\} = aX(as)$$




***Second Order System Properties***

Percent Overshoot: $P.O. = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$

If $\beta = \frac{PO^{max}}{100}$ then $\zeta = \frac{\frac{-\ln(\beta)}{\pi}}{\sqrt{1+\left(\frac{-\ln(\beta)}{\pi}\right)^2}}$ ,

$\theta = cos^{-1}(\zeta)$ Time to Peak:

$$T_p = \frac{\pi}{\omega_d}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

2% Settling Time: $T_s = \frac{4}{\zeta\omega_n} = 4\tau$

**Initial Value Theorem:** If $x(t) \leftrightarrow X(s)$ $\lim_{t\to 0^+} x(t) = \lim_{s\to\infty} sX(s)$

**Final Value Theorem:** If $x(t) \leftrightarrow X(s)$ $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$