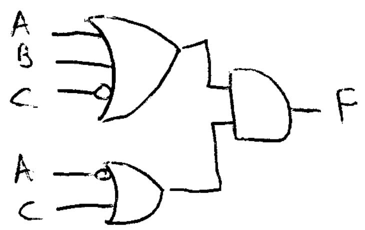
Homework 7

(1) Implement the following Boolean expression in a CMOS circuit. Remember that a CMOS circuit contains equal numbers of p-FETs and n-FETs. Also, in this circuit you do not have access to the complements of variables – only the non-complemented variables are inputs.

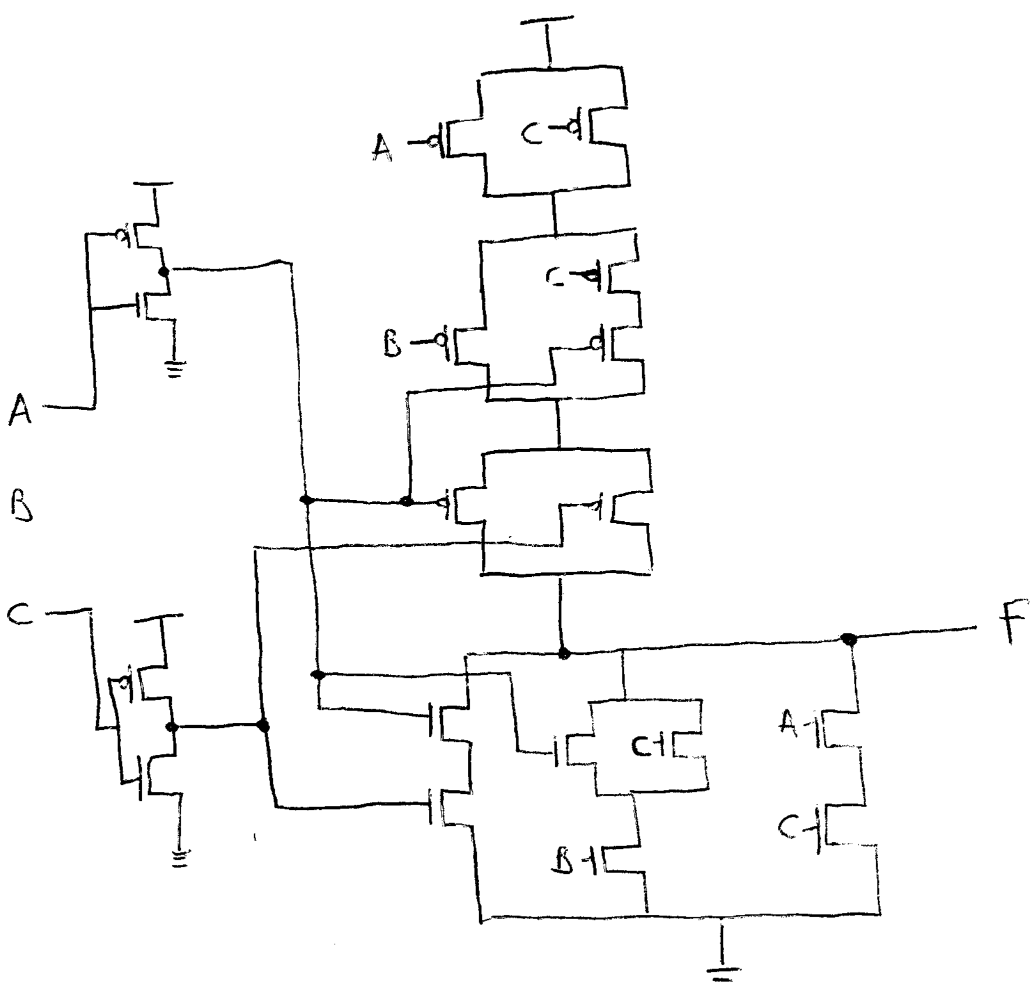
$$F = (A + B + C')(A' + C)$$



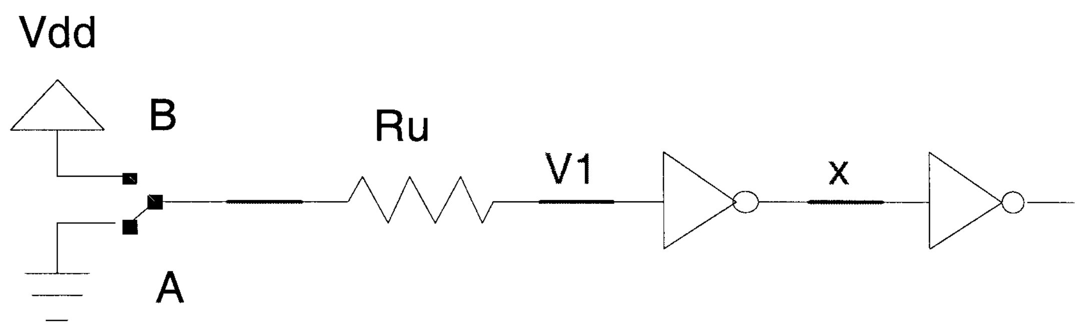
$$= A\bar{A} + AC + B\bar{A} + BC + \bar{C}\bar{A} + \bar{C}C$$

$$= AC + \bar{A}\bar{C} + \bar{A}B + BC$$

$$= AC + \bar{A}\bar{C} + B(\bar{A} + C)$$

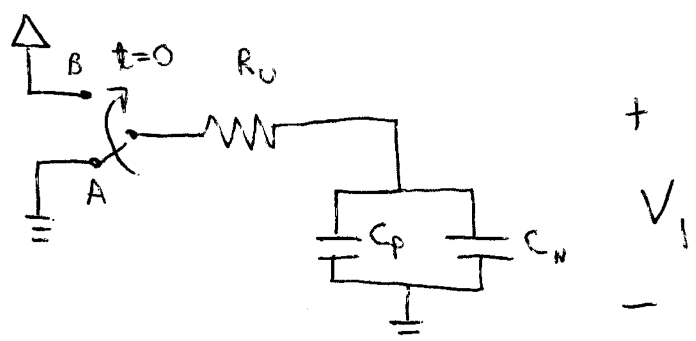


(2) Consider the following circuit:



The switch changes from A to B at time t = 0.

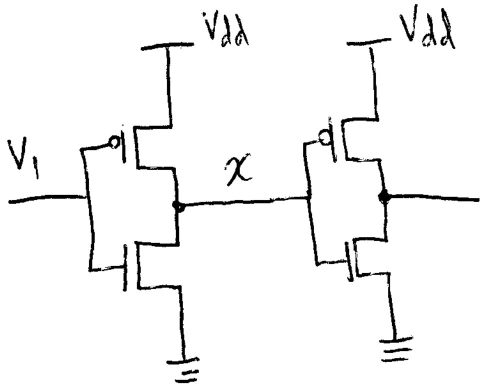
Draw a circuit diagram that illustrates the connection of the switch to the input of the first inverter. Assume that pFETs have an input capacitance of Cp and nFETs have an input capacitance of Cn.



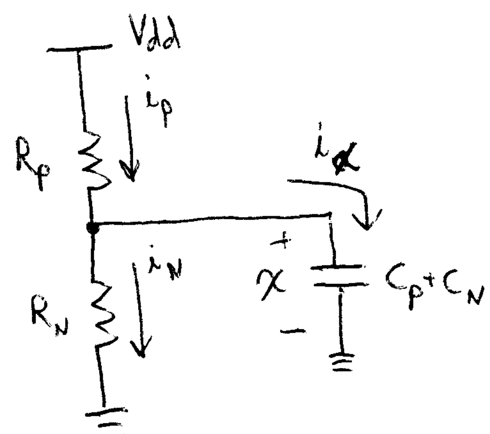
Solve your circuit for the input voltage, V1, as a function of time. Use the constants Vdd, Ru, Cn, and Cp in your equation.

$$V_1 = V_{dd}\left(1 - e^{-\frac{t}{R_u(C_N + C_P)}}\right)$$

Draw a circuit (transistor) level diagram for the two inverters using nFETs, pFETs, source voltage (Vdd), and ground.



Replace the pFET transistors in the first inverter with resistors with resistance Rp. Replace the nFET transistors in the first inverter with resistors with resistance Rn. Finally, replace the pFET transistors in the second inverter with capcitances Cp and the nFET transistors in the second inverter with capacitances Cn. Draw the resulting circuit below.



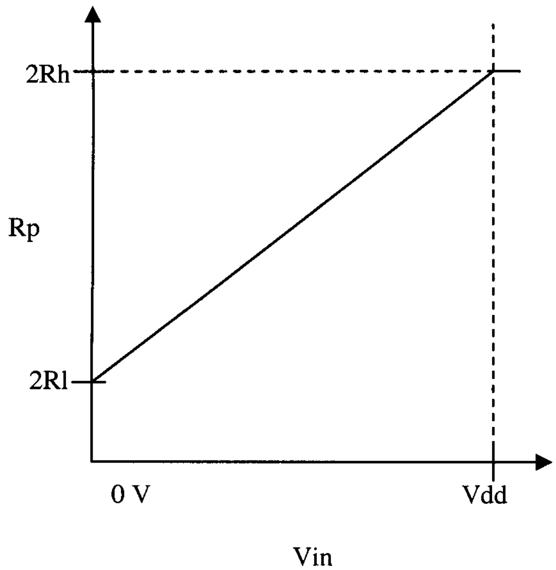
From your circuit, write a differential equation for x, the output voltage, as a function of Vdd, Rp, Rn, Cn, and Cp. **Do not solve this equation.**

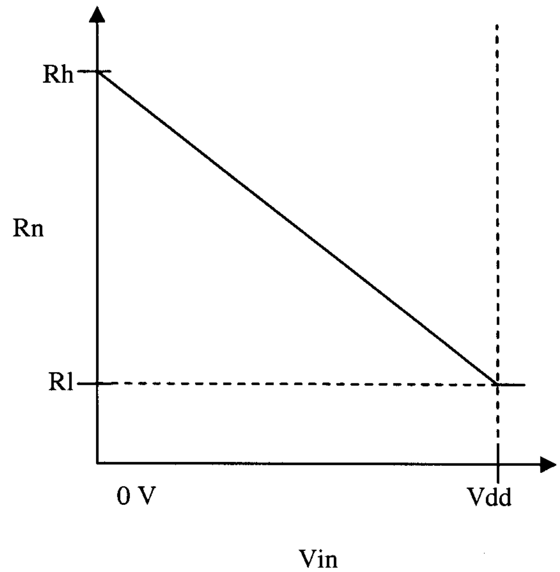
$i_p = i_x + i_N \qquad i_x = (C_p + C_N)\frac{dx}{dt} \qquad i_N = \frac{x}{R_N} \qquad i_p = \frac{V_{dd} - x}{R_p}$

$$\frac{V_{dd} - x}{R_p} = (C_p + C_N)\frac{dx}{dt} + \frac{x}{R_N}$$

$$\boxed{(C_p + C_N)\frac{dx}{dt} + x\left(\frac{1}{R_N} + \frac{1}{R_p}\right) = \frac{V_{dd}}{R_p}}$$

Assume a linear model for the transistor resistances based on the input voltage:





Derive equations for Rp and Rn in terms of Vin, Vdd, Rh, and Rl.

$$R_P = 2R_L + \frac{2R_H - 2R_L}{V_{dd}} V_{IN}$$

$$R_N = R_H + \frac{R_L - R_H}{V_{dd}} V_{IN}$$

Substitute your equation for V1 from above as the input voltage to your equations for Rp and Rn. Now you have a model of the resistance as a function of time based on the initial switching event.

$$R_P = 2R_L + 2(R_H - R_L)\left(1 - e^{-\frac{t}{R_U(C_N + C_P)}}\right)$$

$$R_N = R_H + (R_L - R_H)\left(1 - e^{-\frac{t}{R_U(C_N + C_P)}}\right)$$

Use your equations for Rp and Rn as functions of time in conjunction with your above differential equation to produce PLOTS of the output voltage x and the associated output current. Also generate a plot of instantaneous power consumption, the product of voltage and current.

For these plots use the following constants:

Vdd = 5V
Rh = 1 MΩ
Rl = 10 Ω
Ru = 200 Ω
Cn = Cp = 200 pF

<u>You do not need to solve the equations to produce these plots</u>. Be creative and use whatever software you like.

Matlab code:

```
% ECE 333, Winter 2004
% Homework #7 Problem #2

clear all;
close all;

Vdd = 5;
Rh = 1e6;
Rl = 10;
Ru = 200;
Cn = 200e-12;
Cp = Cn;

T = 1e-6;
N = 1e4;
t = linspace( 0, T, N );

V1 = Vdd * ( 1 - exp( - t / ( Ru * ( Cn + Cp ) ) ) );

figure;
plot( t, V1 );
title( 'input voltage' );
xlabel( 'time (s)' );
ylabel( 'voltage (V)' );

Rp = 2*Rl + 2*(Rh-Rl)/Vdd*V1;
Rn = Rh + (Rl-Rh)/Vdd*V1;

figure;
plot( t, Rp, t, Rn );
title( 'effective FET resistances' );
xlabel( 'time (s)' );
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```
ylabel( 'resistance (\omega)' );
legend( 'R_p', 'R_N' );

x = zeros( size(t) );
dx = zeros( size(t) );
x(1) = Vdd;

for m = 2:N
    dx(m) = (Vdd/Rp(m-1) - x(m-1)*(1/Rn(m-1)+1/Rp(m-1)))/(Cp+Cn)*T/N;
    x(m) = x(m-1) + dx(m);
end

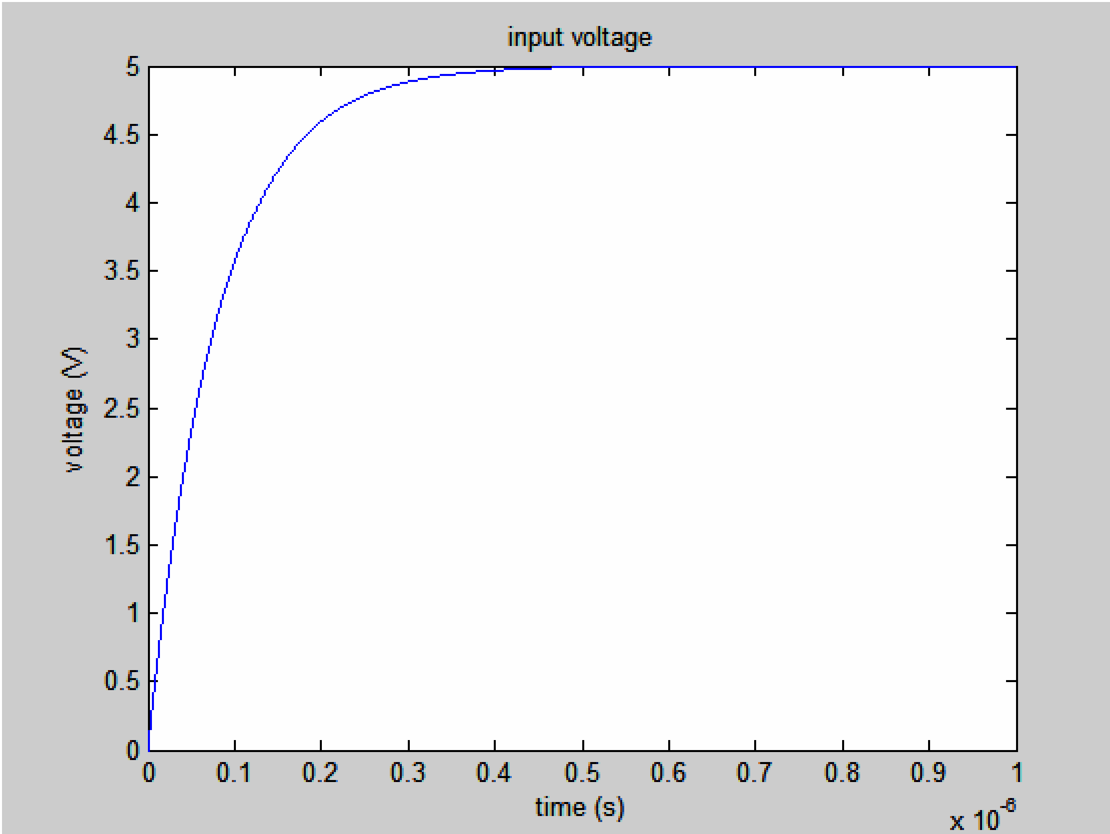
figure;
plot( t, x );
title( 'output voltage, x' );
xlabel( 'time (s)' );
ylabel( 'voltage (V)' );

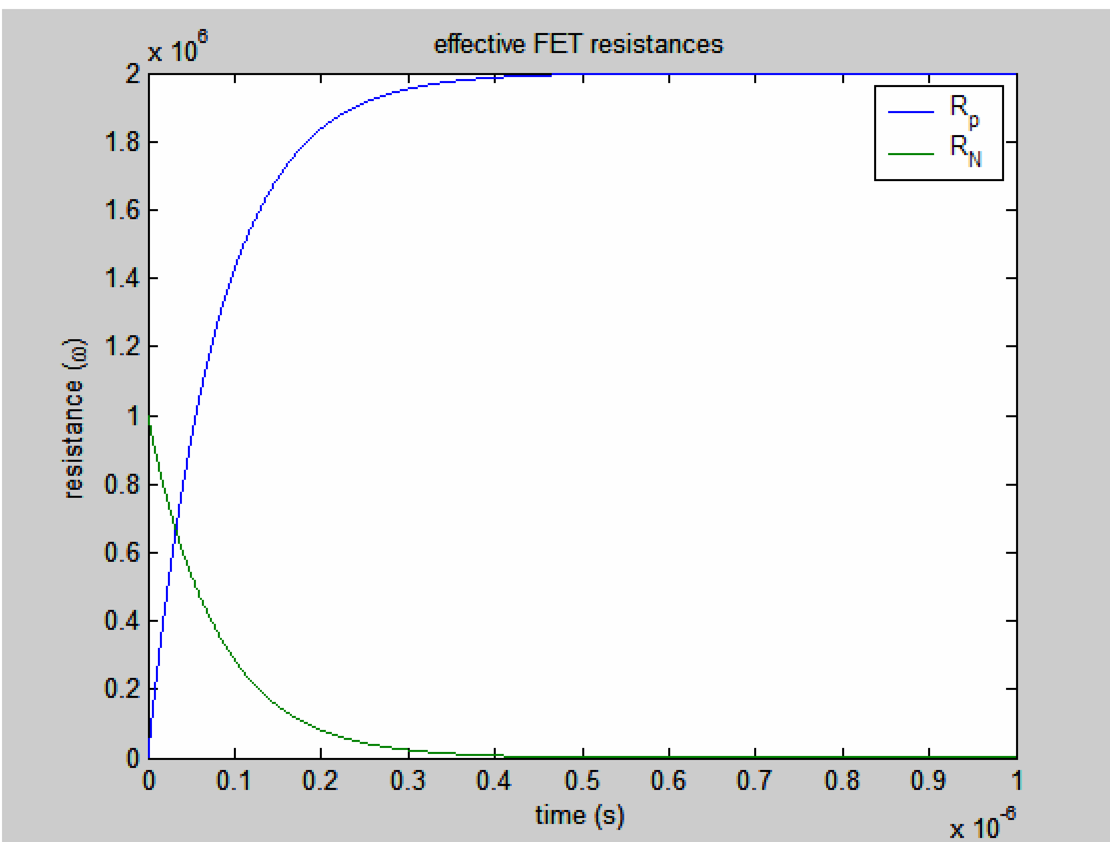
figure;
plot( t, -dx*(Cp+Cn)/T*N );
title( 'negative output current, C dx/dt' );
xlabel( 'time (s)' );
ylabel( 'current (A)' );

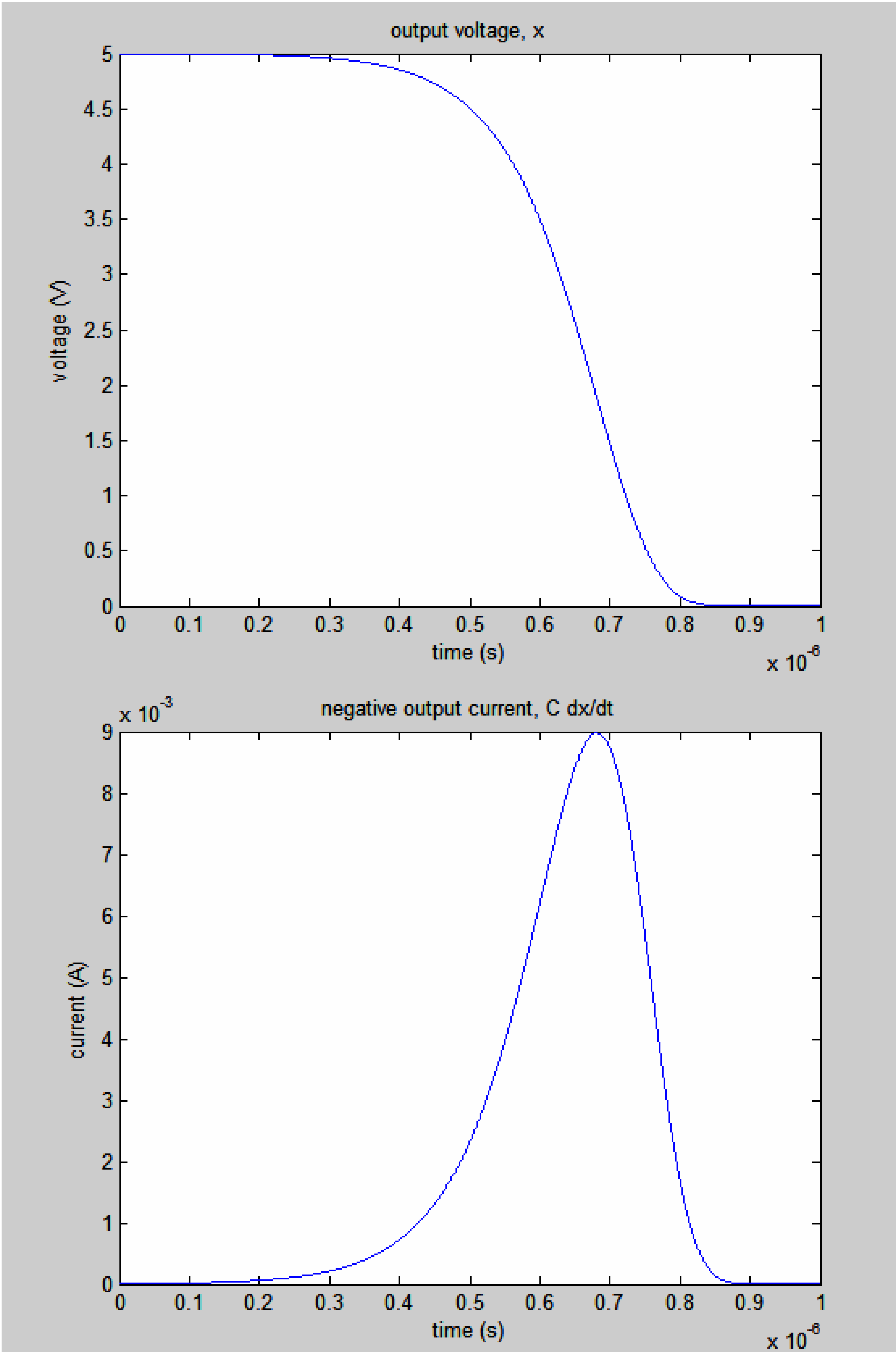
figure;
plot( t, -dx*(Cp+Cn).*x/T*N );
title( 'instantaneous power output' );
xlabel( 'time (s)' );
ylabel( 'power (W)' );

iN = x./Rn;
iP = (Vdd-x)./Rp;

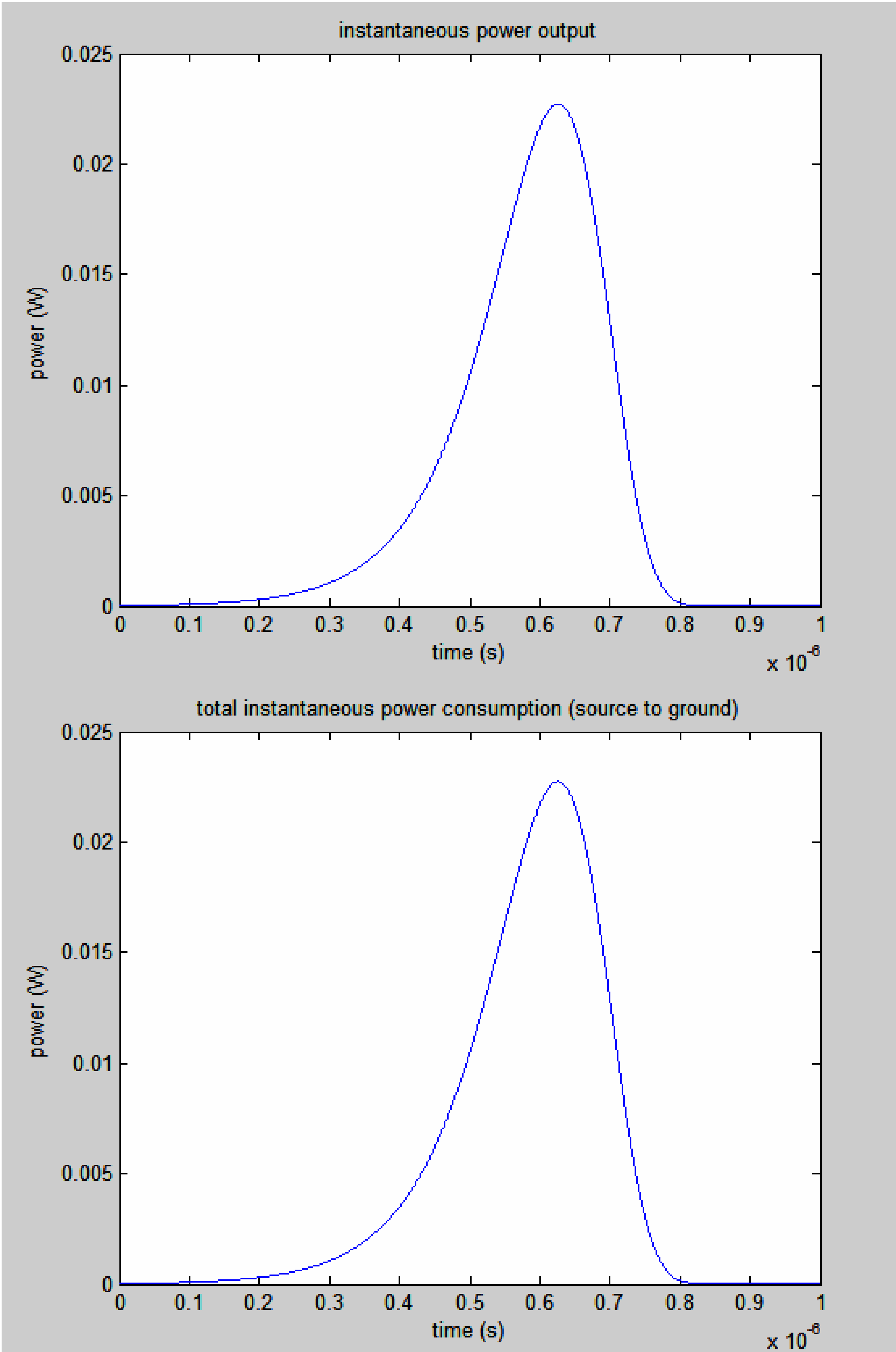
figure;
plot( t, abs(iN.*x)+abs(iP.*(Vdd-x)) );
title( 'total instantaneous power consumption (source to ground)' );
xlabel( 'time (s)' );
ylabel( 'power (W)' );
```


input voltage
voltage (V)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
time (s)
x 10^-6


effective FET resistances
x 10^6
resistance (ω)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
time (s)
x 10^-6
Rp
RN



output voltage, x
voltage (V)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
time (s)
x 10^-6
negative output current, C dx/dt
x 10^-3
current (A)
9
8
7
6
5
4
3
2
1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
time (s)
x 10^-6


instantaneous power output
power (W)
0.025
0.02
0.015
0.01
0.005
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
time (s)
x 10^{-6}
total instantaneous power consumption (source to ground)
power (W)
0.025
0.02
0.015
0.01
0.005
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
time (s)
x 10^{-6}