3.7 Logarithm and Anti-Logarithm Op Amp Circuits

Based upon nonlinear logarithmic volt-ampere relationship that exists between the collector current, Ic, and the base-emitter voltage, Vbe, in a silicon planar transistor:

$$Vbe = 0.060 \cdot \log\left(\frac{Ic}{Io}\right) \qquad (3.10)$$

(The log function is a base-10 log)

Io = Reverse Saturation Current = 10^{-13} A at 27 deg C

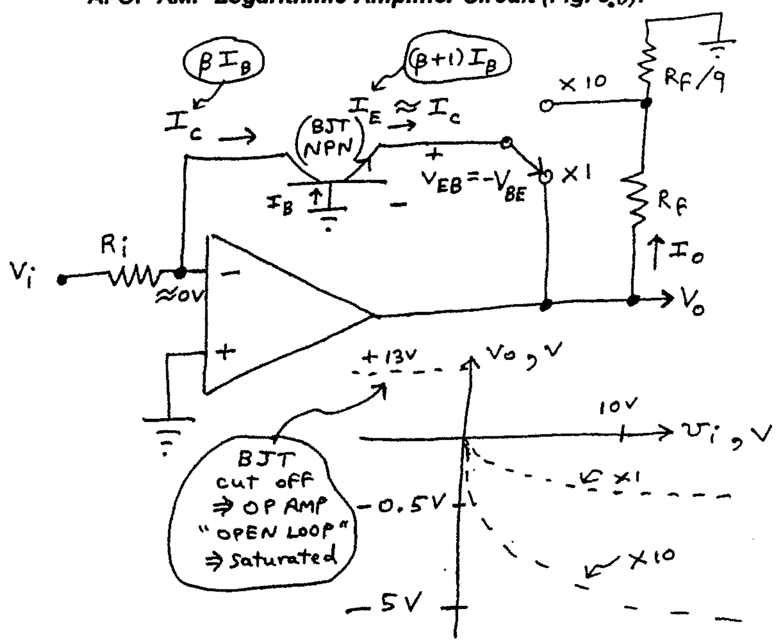
Equation (3.10) is approximately valid over the following range of Ic values:

$$10^{-7} < Ic < 10^{-2} \quad \text{Amperes}$$

Which correponds to the following range of Vbe values:

$$0.36 < Vbe < 0.66 \quad \text{Volts}$$

A. OP AMP Logarithmic Amplifier Circuit (Fig. 3.8):



1. With the switch down (X1) position:

Due to the negative feedback loop, the voltage at the inverting input of the op amp is virtually 0, thus the base and collector of the transistor are at ground potential, hence

$$Vo = Veb = Vec = -Vbe = -0.060 \cdot \log\left(\frac{Ic}{Io}\right)$$

Because the (-) input is at virtual ground, $Ic = \frac{Vi}{Ri}$

Therefore $Vo = -0.060 \cdot \log\left(\frac{Vi}{Ri \cdot Io}\right)$ *Thus a logarithmic relationship exists between Vo and Vi.*

Recall $-0.66 < Vo < -0.36$ V

In other words, because of the negative feedback loop, the OP AMP adjusts its output voltage source Vo to a value that equals the -Vbe value that is established by the collector current Ic according to Eqn (3.10), which in turn, is set by the input voltage Vi. This is because the negative feedback loop ***always attempts*** to drive the (-) input of the OP AMP to virtually the same voltage as the (+) input.

2. With the switch in the UP (X10) position

We shall choose Ri >> Rf, thus Ic is quite small compared to the current flowing through the output resistors Rf and Rf/9. But because the β of the transistor is assumed to be large (100 or more), Ic is approximately the same as Ie, and therefore Ie can be neglected compared to the current through Rf. Thus the current through Rf and Rf/9 is approximately Vo/(Rf+Rf/9), and the voltage across Rf/9, or -Vbe, is approximately

$$-Vbe = Vo \cdot \frac{\frac{Rf}{9}}{Rf + \frac{Rf}{9}} \qquad \textit{For } Ri >> Rf$$

Solving for Vo yields

$$Vo = -10 \cdot Vbe$$

Or substituting in the result from the previous section,

$$Vo = -0.6 \cdot \log\left(\frac{Vi}{Ri \cdot Io}\right)$$

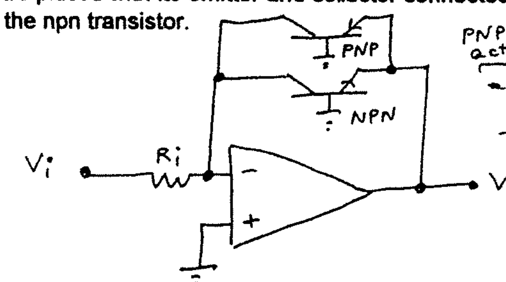
Thus, in the UP position, the circuit has a gain of 10, and thus Vo can range between -6.6 and -3.6 Volts over the permissible range the operation of the logarithmic amplifier. If Ri = 100 Kilohms, the permissible range of input voltages can be found:

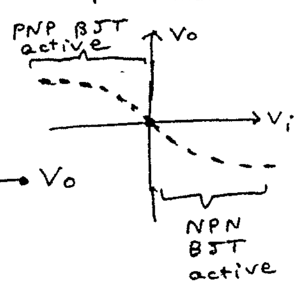
$Ic\_low := 10^{-7}$ A $\quad Ic\_high := 10^{-2}$ A $\quad Ri := 100 \cdot 10^{3}$ Ohms

$Vi\_low := Ic\_low \cdot Ri \qquad Vi\_low = 0.01$ Volts

$Vi\_high := Ic\_high \cdot Ri \qquad Vi\_high = 1 \cdot 10^{3}$ Volts

Therefore we note that this logarithmic amplifier can compress a wide-ranging waveform (that varies from 0.01 volts to 1000 volts) into a much smaller range (from -0.36 to -0.66 volts). For this reason, the circuit is sometimes called a ***signal compression*** circuit. In order to compress an AC waveform that goes both positive and negative, a pnp transistor with a grounded base may be placed with its emitter and collector connected in parallel with the npn transistor.





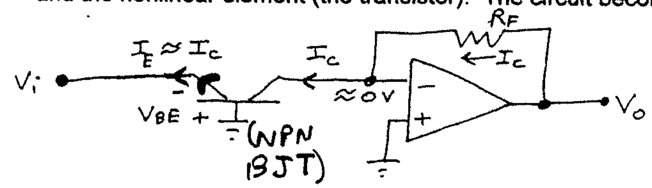
This logarithmic amplifier contains a dc offset. We can see this by using the fact that log(a/b) = log(a)-log(b)

$$Vo = -0.060 \cdot \log\left(\frac{Vi}{Ri \cdot Io}\right) = -0.060 \cdot \log(Vi) + 0.060 \cdot \log(Ri \cdot Io)$$

Thus we see that an additive constant offset (0.060*log(Ri*Io)) appears at the output of the logarithmic amplifier. If this constant offset is a problem, it may be removed by using an OP AMP summing amplifier at the output of the logarithmic amplifier.

B. OP AMP Antilogarithmic Amplifier

An anti-logarithmic amplifier (***signal decompression circuit***) can be built by interchanging the position of the input resistor and the nonlinear element (the transistor). The circuit becomes:



In order for Eqn (3.10) to remain valid, we still must require that Vi be restricted to the range of -0.66 < Vi < -0.36 Volts. As long as this restriction is obeyed, Eqn (3.10) is valid, and solving it for Ic,

$$V_{be} = 0.06 \log\left(\frac{I_C}{I_O}\right) \Rightarrow I_C = I_O\, 10^{(V_{be}/0.06)}$$

But $Vi = -Vbe$

Therefore $\quad Ic = Io \cdot 10^{-\frac{Vi}{0.06}}$

And therefore the amplifier output voltage is

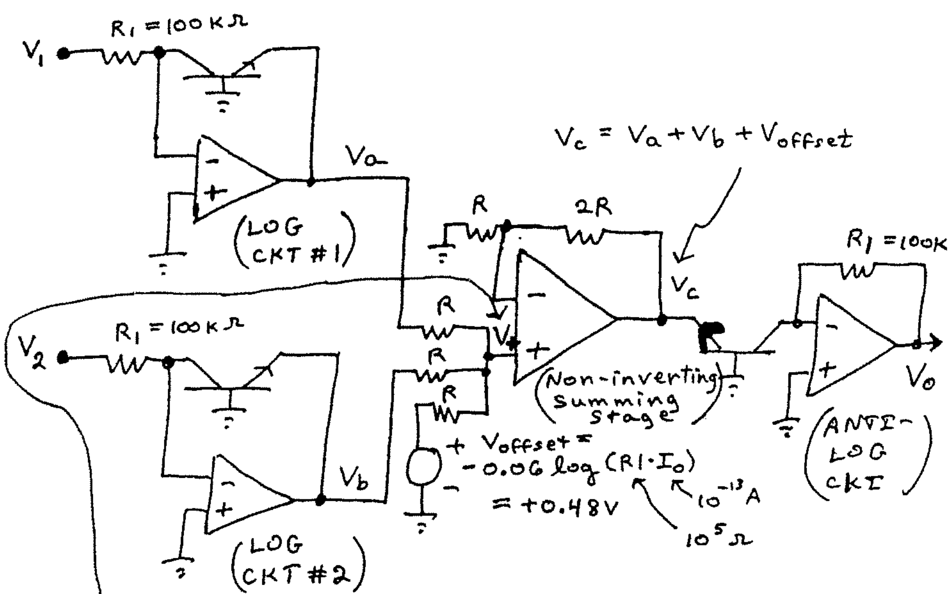
$$Vo = Rf \cdot Ic$$

$$Vo = (Rf \cdot Io) \cdot 10^{-\frac{Vi}{0.06}}$$

Thus an exponential relationship exists between Vo and Vi.

C. *Analog Multiplier Design*

Consider the following circuit that uses two logarithmic amplifiers, a summing amplifier, and an anti-log amplifier to perform analog multiplication "using logarithms":



By Linear Superposition

$$V_+ = \left(\frac{R/2}{R/2+R}\right) v_a + \left(\frac{R/2}{R/2+R}\right) v_b + \left(\frac{R/2}{R/2+R}\right) v_{offset} = \frac{1}{3}\left(v_a + v_b + v_{offset}\right)$$

Analysis of analog multiplier circuit:

$$Va = -0.06 \cdot \log\left(\frac{V1}{R1 \cdot Io}\right) \qquad Vb = -0.06 \cdot \log\left(\frac{V2}{R1 \cdot Io}\right)$$

$$Vc = -0.06 \cdot \left(\log\left(\frac{V1}{R1 \cdot Io}\right) + \log\left(\frac{V2}{R1 \cdot Io}\right)\right) - 0.06 \cdot \log(R1 \cdot Io)$$

$$Vc = -0.06 \cdot (\log(V1) + \log(V2) - 2 \cdot \log(R1 \cdot Io)) - 0.06 \cdot \log(R1 \cdot Io)$$

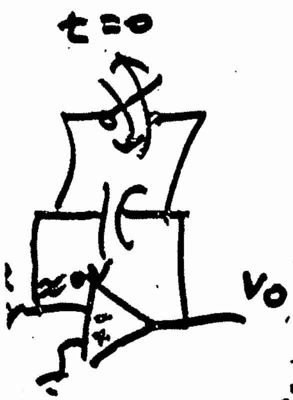
$$Vc = -0.06 \cdot (\log(V1) + \log(V2) - \log(R1 \cdot Io))$$

$$Vo = (Rf \cdot Io) \cdot 10^{-\frac{Vc}{0.06}} = R1 \cdot Io \cdot 10^{(\log(V1) - \log(V2) - \log(R1 \cdot Io))}$$

$$Vo = V1 \cdot V2$$

Note that the "dc offset" term generated by one of the log circuits, log(R1*Io), must be canceled out in the summing amplifier, since the anti-log circuit will cancel only one such dc offset term.

3.8 Integrators



So far in this chapter we have considered only circuits with a flat gain-versus-frequency characteristic. Now let us consider circuits that have a deliberate change in gain with frequency. The first such circuit is the *integrator*. Figure 3.10 shows the circuit for an integrator, which is obtained by closing switch S_1. The voltage across an initially uncharged capacitor is given by

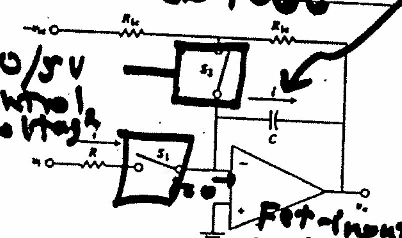
$$v_c = \frac{1}{C}\int_0^{t_1} i\,dt + v_c(0) \tag{3.11}$$

where i is the current through C and t_1 is the integration time. For the integrator, for v_i positive, the input current $\boxed{i = v_i/R}$ flows through C in a direction to cause v_o to move in a negative direction. Thus

$$v_o = -\frac{1}{RC}\int_0^{t_1} v_i\,dt + v_{ic} \tag{3.12}$$

This shows that v_o is equal to the negative integral of v_i, scaled by the factor $1/RC$ and added to v_{ic} [...]

Low-leakage cap.
polypropylene dielectric

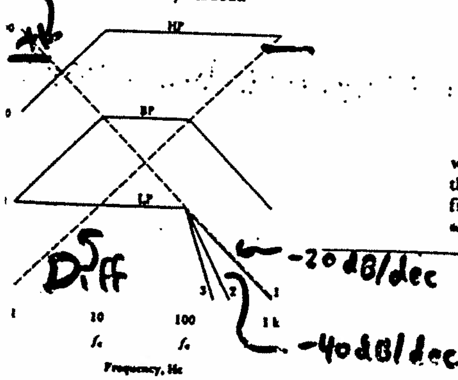


integrate mode
S1 closed, S2 open
Hold Mode
S1 open, S2 open
initialize Mode
S2 closed
S1 open

Figure 3.10 A three-mode integrator. With S_1 open and S_2 closed, the dc circuit behaves as an inverter. [...] can be set to any desired initial condition. With S_1 closed and S_2 open, the circuit integrates. With both switches open, the circuit holds v_o constant, making possible a leisurely readout.

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{1/j\omega C}{R} = -\frac{1}{j\omega RC} = -\frac{1}{j\omega\tau} \tag{3.13}$$

where $\tau = RC$, $\omega = 2\pi f$, and f = frequency. Equation (3.13) shows that the circuit gain decreases as f increases; Figure 3.11 shows the frequency response; and (3.13) shows that the circuit gain is 1 when $\omega\tau = 1$.



3.11 Bode plot (gain versus frequency) for various filters. Integrator (I); differentiator (D); low pass (LP), 1, 2, 3 section (pole); high pass (HP); bandpass (BP). Corner frequencies f_c for high-pass, low-pass, and [...] filters.

"Charge Amplifier"

Example 3.2 The output of the piezoelectric transducer shown in Figure 2.11(b) may be fed directly into the negative input of the integrator shown in Figure 3.10, as shown in Figure E3.2. Analyze the circuit of this charge amplifier and discuss its advantages.

Answer Because the FET-op-amp negative input is a virtual ground, $i_{tc} = i_{tR} = 0$. Hence long cables may be used without changing transducer sensitivity or time constant, as is the case with voltage amplifiers. From Figure E3.2, current generated by the transducer, $i_t = K\,dx/dt$, all flows into C, so, using (3.11), we find that v_o is

$$v_o = -v_c = -\frac{1}{C}\int_0^{t_1}\frac{K\,dx}{dt}\,dt = -\frac{Kx}{C}$$

which shows that v_o is proportional to x, even down to dc. Like the integrator, the charge amplifier slowly drifts with time, because of bias currents required by the op-amp input. A large feedback resistance R must therefore be added to prevent saturation. This causes the circuit to behave as a high-pass filter, with a time constant $\tau = RC$. It then responds only to frequencies above $f_c = 1/2\pi RC$ and has no frequency-response improvement over the voltage amplifier.

added to stop long-term drift ⇒ piezo xducer still can't be used for d.c. displacements.

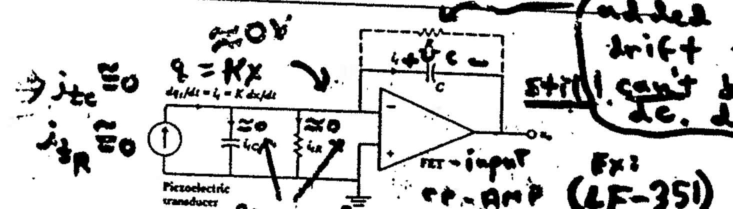
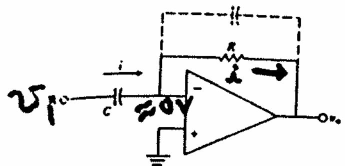


Figure E3.2 The charge amplifier transfers charge generated from a piezoelectric transducer to the op-amp feedback capacitor C.

⇒ Long cables may be used w/o degrading transducer sensitivity or Freq.-response (time constant).

Differentiator

$$i = C\frac{dv_i}{dt}$$



$$v_o = -iR$$

$$v_o = -RC\frac{dv_i}{dt}$$

Figure 3.12 A differentiator. Dashed lines indicate that a small capacitor must usually be added across the feedback resistor to prevent oscillation.

If dv_i/dt is positive, i flows through R in a direction such that it yields a negative v_o. Thus

$$v_o = -RC\frac{dv_i}{dt} \quad (3.15)$$

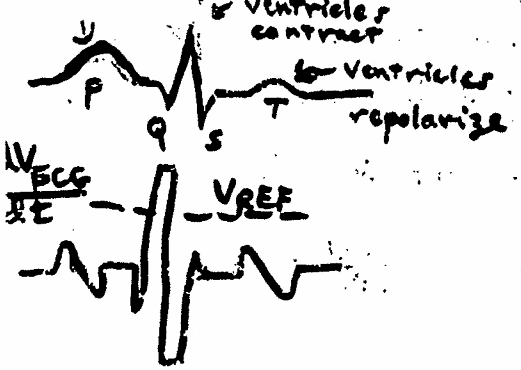
The frequency response of a differentiator is given by the ratio of feedback to input impedance:

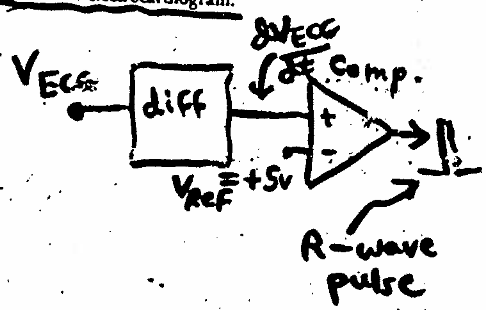
$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{R}{1/j\omega C} = -j\omega RC = -j\omega\tau \quad (3.16)$$

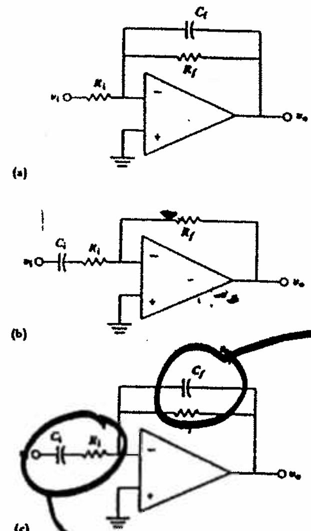
infinite gain @ infinitely high freq.

Equation (3.16) shows that the circuit gain increases as f increases, and is equal to unity when $\omega\tau = 1$. Figure 3.11 shows the frequency response.

Unless specific preventive steps are taken, the circuit tends to oscillate. The output also tends to be noisy, since the circuit emphasizes high frequencies. A differentiator followed by a comparator is useful for detecting an event whose slope exceeds a given value; for example, the detection of the R wave in an electrocardiogram.







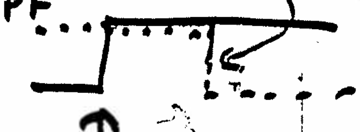


Figure 3.13 Active filters. (a) Low-pass filter attenuates high frequencies. (b) High-pass filter attenuates low frequencies and blocks dc. (c) Bandpass filter attenuates both low and high frequencies.

(1st order) Low Pass Filter

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{(R_f/j\omega C_f)/[(1/j\omega C_f) + R_f]}{R_i} = -\frac{R_f}{(1 + j\omega R_f C_f)R_i} = -\frac{R_f}{R_i}\frac{1}{1 + j\omega\tau} \quad (3.17)$$

(1st order)

High-pass filter

Figure 3.13(b) shows a one-op-amp high-pass filter. This is useful for amplifying a small ac voltage that rides on top of a large dc voltage, because C_i blocks the dc. The frequency-response equation is

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{R_f}{1/j\omega C_i + R_i} = -\frac{j\omega R_f C_i}{1 + j\omega C_i R_i} = -\frac{R_f}{R_i}\frac{j\omega\tau}{1 + j\omega\tau} \quad (3.18)$$

where $\tau = R_i C_i$. Figure 3.11 shows the frequency response. For $\omega << 1/\tau$, the circuit behaves as a differentiator (Figure 3.12), because C_i is the dominant input impedance. For $\omega >> 1/\tau$, the circuit behaves as an inverter, because the impedance of R_i is large compared with that of C_i. The corner frequency f_c, which is defined by the intersection of the two asymptotes shown, is given by the relation $\omega\tau = 2\pi f_c \tau = 1$.

Impedance Magnitude Scaling → For more practical R values, we desire to scale R values fm 1Ω up to "R" ohms. We must proportionally increase Z_C (and Z_L) values by the factor R, so ckt still performs as before, except with higher input + output impedance...

for C

$$Z_C' = RZ_C = R\left(\frac{1}{j\omega C}\right) = \frac{1}{j\omega(C/R)} = \frac{1}{j\omega C'}$$

$$\Rightarrow \boxed{C' = C/R}$$

where C was original C value
and C' is the new magnitude-scaled C value

for L

$$Z_L' = RZ_L = R(j\omega L) = j\omega(RL) = j\omega L'$$

$$\Rightarrow \boxed{L' = RL}$$

Frequency Scaling

For more practical cutoff frequency, we desire to change the elements values again so they exhibit the impedance they now have at 1 r/s at a higher frequency, "ω_c" r/s.

First, note that R is freq. independent

$$\therefore \quad R' = R$$

for C

$$Z_C\Big|_{\omega=1} = \frac{1}{j(1)C} = \frac{1}{j\omega_c\left(\frac{C}{\omega_c}\right)} = \frac{1}{j\omega_c C'}$$

$$\Rightarrow \boxed{C' = C/\omega_c} \boxed{= \frac{C}{2\pi f_c}}$$

for L

$$Z_L\Big|_{\omega=1} = j(1)L = j\omega_c\left(\frac{L}{\omega_c}\right) = j\omega_c L'$$

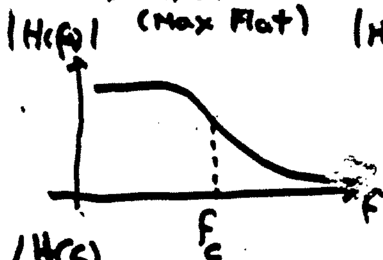
$$\Rightarrow \boxed{L' = L/\omega_c = \frac{L}{2\pi f_c}}$$

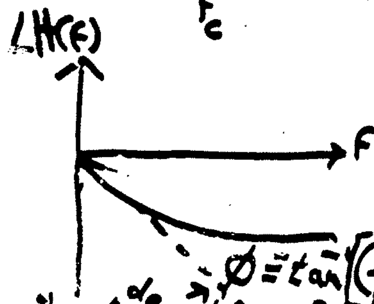
Combined result allows us to replace 1Ω resistors with RΩ resistors and "C" Fd capacitors with $\frac{C}{2\pi f_c R}$ Fd capacitors with the cutoff freq shifted fm 1r/s to f_c Hz

3

Practical LPF's

Butterworth (Max Flat)



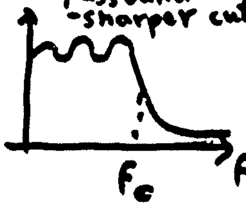


$\phi = \tan^{-1}\left[\left(\frac{f}{f_c}\right)^N\right]$?

"Linear Phase" = desired for no waveform distortion

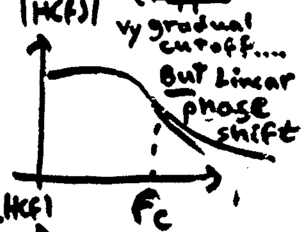
all comp. freq. delayed same amt.

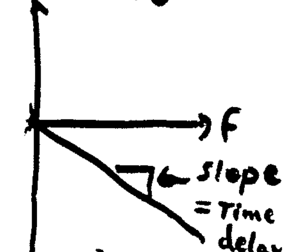
Chebyshev (ripple in passband - sharper cutoff)



∠H(f) even More Nonlinear Than Butterworth

Bessel (+ vy gradual cutoff.... But Linear phase shift





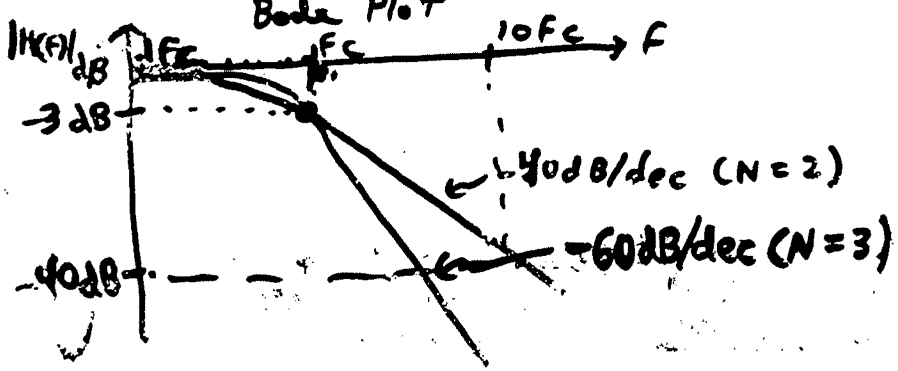
⇒ No "delay distortion"

Butterworth LPF (N^{th} order) Freq. Response:

$$|H(f)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^{2N}}}$$

Bode Freq. Resp. Plots ⇒ falls off at $-20N$ dB/decade (for $f \gg f_c$)

Bode Plot



LPF/HPF Filters*

section are designed for more critical requirements, such as a constant gain for all frequencies in the passband, a rapid falloff from the passband to the stop band, a low gain for frequencies in the stop band, and the ability to transmit a pulse with little change in shape.

The **Butterworth** filter has a flat frequency response below the characteristic frequency f_0, but responds poorly to transients because the phase–frequency relationship is nonlinear. It is commonly used for antialiasing in circuits that sample analog waveforms (described in Chapters 3 and 5) because it transmits signal amplitudes faithfully. The **Bessel** filter has a linear phase variation for frequencies below f_0, and, hence, has a constant delay in this range. Since each Fourier component is shifted by the same time, the signal is transmitted without a change in shape but a constant delay is introduced. The **transitional**, or **Paynter** filter (also called the "Besselworth") has properties intermediate between those of either the Butterworth or Bessel filter. The **Chebyshev** filter maximizes the sharpness of the frequency roll-off, but introduces ripples in the passband. This filter is actually a family of filters classified by the amplitude of the ripples (in dB). Achieving the intended response requires accurate component values (typically 1 to 5%) and low-leakage capacitors. Inductors are rarely used as they are bulky and not very ideal.

The basic circuit realizations are the unity-gain Sallen–Key filter (Figures 2.19 and 2.20, Table 2.3) and the equal-component-value (or VCVS, voltage-controlled voltage source) Sallen–Key filter (Figures 2.21 and 2.22, Table 2.4). Each of these circuits provides two poles of low-pass or two poles of high-pass filtering. Higher-order filters use cascaded stages. The equal-component design has the advantage of providing gain as the bandwidth is reduced, which reduces the effect of amplifier noise.

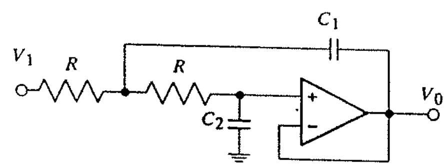


Figure 2.19 Unity-gain Sallen–Key low-pass two-pole filter. $RC_1\omega_0 = k_1$ and $RC_2\omega_0 = k_2$. Higher-order filters use cascaded stages. See Table 2.3 for values of k_1 and k_2.

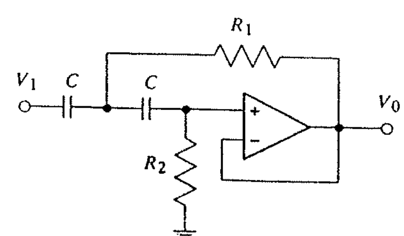


Figure 2.20 Unity-gain Sallen–Key high-pass two-pole filter. $R_1C\omega_0 = 1/k_1$ and $R_2C\omega_0 = 1/k_2$. Higher-order filters use cascaded stages. See Table 2.3 for values of k_1 and k_2.

* From "Interfacing", by Stephen DeRenzo, Prentice-Hall

TABLE 2.3 UNITY-GAIN SALLEN–KEY LOW-PASS AND HIGH-PASS FILTERS. REFER TO FIGURES 2.19 AND 2.20 FOR CIRCUIT DIAGRAMS.

Poles	Butterworth		Transitional		Bessel		Chebyshev (0.5 dB)	
	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2
2	1.414	0.707	1.287	0.777	0.907	0.680	1.949	0.653
4	1.082	0.924	1.090	0.960	0.735	0.675	2.582	1.298
	2.613	0.383	2.206	0.472	1.012	0.390	6.233	0.180
6	1.035	0.966	1.060	1.001	0.635	0.610	3.592	1.921
	1.414	0.707	1.338	0.761	0.723	0.484	4.907	0.374
	3.863	0.259	2.721	0.340	1.073	0.256	13.40	0.079
8	1.019	0.981	1.051	1.017	0.567	0.554	4.665	2.547
	1.202	0.832	1.191	0.876	0.609	0.486	5.502	0.530
	1.800	0.556	1.613	0.615	0.726	0.359	8.237	0.171
	5.125	0.195	3.373	0.268	1.116	0.186	23.45	0.044

Source: Brian K. Jones, *Electronics for Experimentation and Research*. By permission of Prentice-Hall International (UK), Ltd., London.

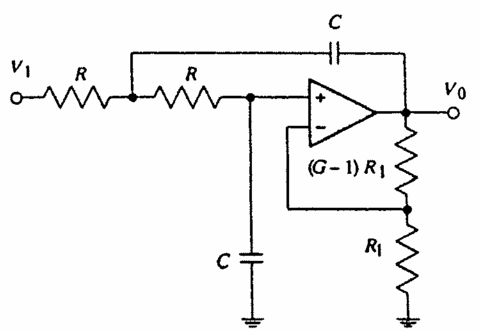


Figure 2.21 Equal-component-value Sallen–Key low-pass two-pole filter. $RC\omega_0 = k_3$ and R_1 is chosen for convenience. See Table 2.4 for values of gain G and k_3.

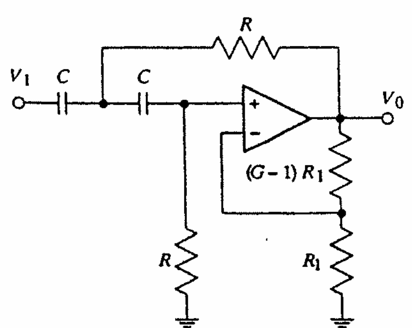


Figure 2.22 Equal-component-value Sallen–Key high-pass two-pole filter. $RC\omega_0 = 1/k_3$ and R_1 is chosen for convenience. See Table 2.4 for values of gain G and k_3.

TABLE 2.4 EQUAL-COMPONENT-VALUE SALLEN–KEY LOW-PASS AND HIGH-PASS FILTERS. REFER TO FIGURES 2.21 AND 2.22 FOR CIRCUIT DIAGRAMS.

Poles	Butterworth		Transitional		Bessel		Chebyshev (0.5 dB)	
	k_3	G	k_3	G	k_3	G	k_3	G
2	1.000	1.586	1.000	1.446	0.785	1.268	1.129	1.842
4	1.000	1.152	1.023	1.123	0.704	1.084	1.831	1.582
	1.000	2.235	0.977	2.035	0.628	1.759	1.060	2.660
6	1.000	1.068	1.030	1.056	0.622	1.040	1.332	2.627
	1.000	1.586	1.009	1.492	0.591	1.364	1.355	2.448
	1.000	2.483	0.962	2.293	0.524	2.023	1.029	2.846
8	1.000	1.038	1.034	1.032	0.561	1.024	3.447	1.522
	1.000	1.337	1.021	1.284	0.544	1.213	1.708	2.379
	1.000	1.889	0.996	1.765	0.510	1.593	1.188	2.711
	1.000	2.610	0.951	2.436	0.455	2.184	1.017	2.913

Source: Brian K. Jones, *Electronics for Experimentation and Research*. By permission of Prentice-Hall International (UK), Ltd., London.

For a given order, the various filters differ in the rate of rolloff near the corner frequency ω_0, but in the stop band far from ω_0, the amplitude response for all filters drops $6N$ dB per octave, or $20N$ dB per decade, where N is the order of the filter.

As an example, consider a Butterworth low-pass four-pole filter with f_0 = 10 kHz (ω_0 = 62.83 krad/s). From Table 2.3, the first stage has $k_1 = 1.082$ and $k_2 = 0.924$. Thus, $RC_1 = k_1/\omega_0 = 1.722 \times 10^{-5}$ and $RC_2 = k_2/\omega_0 = 1.471 \times 10^{-5}$. Choosing $R = 10\ \text{k}\Omega$, we have $C_1 = 1722$ pF and $C_2 = 1471$ pF. Similarly, the second stage has $k_1 = 2.613$ and $k_2 = 0.383$. Choosing $R = 10\ \text{k}\Omega$, we have $C_1 = 4159$ pF and $C_2 = 610$ pF.

EXAMPLE

Derive the voltage-response function for the low-pass filter in Figure 2.23.

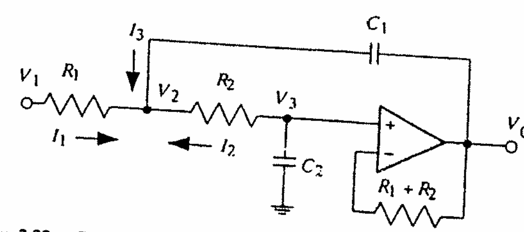
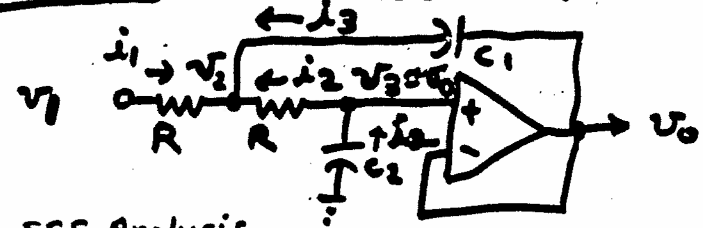


Figure 2.23 Diagram for analysis of unity-gain Sallen-Key, two-pole low-pass Filter.

Example 2^{nd}-order Active Filter (LPF) Anal.



For ($\omega_c = 1$) Butterworth LPF, let

$$\begin{pmatrix} R = 1\,\Omega \\ C_1 = 1.414\,F \\ C_2 = 0.707\,F \end{pmatrix}$$

Use SSS Analysis

Rule #1 $\Rightarrow$ $v_3 \simeq v_0$ (rule #2)

$$\therefore\ i_1 = \frac{v_1 - v_2}{R}\ ;\ i_2 = \frac{v_0 - v_2}{R} = -v_0 j\omega C_2$$

$$i_3 = (v_0 - v_2)\, j\omega C_1$$

KCL $\Rightarrow$ $i_1 + i_2 + i_3 = 0$

$$\frac{v_1 - v_2}{R} - v_0 j\omega C_2 + (v_0 - v_2) j\omega C_1 = 0$$

Solving for v_1

$$v_1/R = v_2/R + j\omega v_0 C_2 + j\omega v_2 C_1 - j\omega v_0 C_1$$

But: $v_2 = v_0(1 + j\omega R C_2)$

$$\frac{v_1}{R} = \frac{v_0}{R}(1 + j\omega R C_2) + j\omega v_0 C_2 + v_0 j\omega C_1 (1 + j\omega R C_2) - j\omega v_0 C_1$$

$$\boxed{H(\omega) = \frac{v_0}{v_1} = \frac{1}{(j\omega)^2 R^2 C_1 C_2 + (j\omega) 2 R C_2 + 1}}$$

For Butterworth Active Filter

$R = 1\,\Omega$, $C_1 = \sqrt{2}\,F$, $C_2 = 1/\sqrt{2}\,F$

$$H(\omega) = \frac{1}{[(j\omega)^2 \cdot 1^2 \cdot \sqrt{2} \cdot 1/\sqrt{2}] + j\omega \cdot 2 \cdot 1 \cdot 1/\sqrt{2} + 1}$$

$$= \frac{1}{-\omega^2 + j\sqrt{2}\,\omega + 1}$$

Finding Magnitude

$$|H(\omega)| = \frac{1}{\sqrt{\underbrace{(1-\omega^2)^2}_{1 - 2\omega^2 + \omega^4} + \underbrace{(\sqrt{2}\,\omega)^2}_{2\omega^2}}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{\omega}{1}\right)^{2\cdot 2}}}$$

This is, indeed, the desired 2^{nd}-order Butterworth transfer curve!!

From "Principles of Electronic Instrumentation", Diefenderfer, WB Saunders, 1972.
(pp. 464-467)

ORIGINS OF NOISE

For reasons of clarity, the word *interference* is preferred to environmental noise. Interference properly denotes that something is hindering a measurement. Furthermore, the source of interference can often be identified and eliminated. This is not true of fundamental noise, which is stochastic in nature.

There are two factors of nature which force a lower limit on us regarding noise. Often we disregard them until they are forced on us in a dramatic manner. All matter that is not at absolute zero temperature has thermal fluctuations associated with it. Secondly, a number of phenomena with which we work are quantized. That is, such phenomena as charge, energy and light do not change smoothly but change in steps. The net result is that there is always (unless at absolute zero temperature) agitation associated with these processes which cannot be eliminated. This agitation is the fundamental source of noise.

There are three types of noise generally associated with transistors, other solid state devices, and the components involved in circuitry. These are thermal noise, shot noise, and flicker noise.

Thermal noise arises from the random movement of electrons and other free carriers. Other names have been given to this source of noise: Johnson noise, after the discoverer of the phenomenon, and Nyquist noise, after the man who showed that this noise was a consequence of the Second Law of Thermodynamics. If the measurement of thermal noise is made over a sufficiently long time to virtually eliminate random fluctuations, the noise will, of course, be zero. At any instant of time, however, there is a net movement in a given direction. This net current produces a voltage which is observed and designated thermal noise, v_N. The thermal noise voltage (*rms*) is related to the resistance of the material, the temperature, and the bandwidth:

$$v_N^2 = 4kTRB \tag{12–1}$$

where k is Boltzmann's constant (1.38×10^{-23} joule/°K). The temperature, T, is in °K, and R is in ohms. The bandwidth, B (in Hertz), is the frequency band of the measuring instrument ($f_{max} - f_{min}$). The meaning of the above equation is clear. Thermal noise increases with the temperature, the bandwidth and the resistance. Since there is no frequency term in the expression, thermal noise is frequency independent; e.g., it appears at *all* frequencies. As a result, it is often referred to as white noise. The magnitude of thermal noise is not insignificant. For example, a 100 KΩ resistor at room temperature has an *rms* noise value of 4 μV when measured by an instrument with a 10 KHz bandwidth. This magnitude in itself is hardly large. However, when that resistor forms the input to an amplifier whose gain is 10^5, the noise at he output of the amplifier is nearly half a volt!

The noise power, P_N, can be obtained from the above equation by lividing both sides by R:

$$P_N = 4kTB \tag{12–2}$$

The noise power depends only upon the temperature and the bandwidth of the measuring instrument. This quantity is useful in defining a commonly used figure of merit for electronic devices, the noise figure, NF; the definition is

$$NF = \frac{\left(\dfrac{\text{Signal Power}}{\text{Noise Power}}\right)_{\text{input}}}{\left(\dfrac{\text{Signal Power}}{\text{Noise Power}}\right)_{\text{output}}} \tag{12–3}$$

This expression can be rewritten, making use of the definition of power gain, as

$$NF = \frac{P_{N\,\text{output}}}{P_{N\,\text{input}}} \cdot \frac{1}{A_p} \tag{12–4}$$

It is clear that $NF = 1$ for an ideal noise-free amplifier, whereas $NF > 1$ for a real amplifier.

The total noise power of an amplifier, regardless of the sources of noise, can be expressed in terms of the contributions from each of the n stages of the amplifier,

$$P_{N_{(\text{total})}} = P_{N_1}(A_{P_1} \cdots A_{P_N}) + P_{N_2}(A_{P_2} \cdots A_{P_N}) + \cdots + P_{N_n}A_{P_N} \tag{12–5}$$

This expression can be simplified by using the definition of NF. This yields

$$NF_{\text{total}} = NF_1 + \frac{NF_2}{A_{P_1}} + \frac{NF_3}{A_{P_1}A_{P_2}} + \cdots \tag{12–6}$$

This result has immediate significance for any designer or user of electronic equipment. Recall that the power gain is usually quite large. This means that equation 12–6 may be approximated as

$$NF_{\text{total}} \approx NF_1 \tag{12–7}$$

Thus, in most practical circuits, the major contribution to the system noise figure comes from that of the first stage. This result has a clear warning for the user when considering a signal pickup and subsequent amplification. Since the total noise figure is essentially that of the first stage, the first stage of amplification (often the pre-amplifier) should be as free from noise as possible. Since connections from the signal source to the amplifier assembly often introduce a significant amount of noise, the pre-amplifier should be placed as close to the signal source as possible. The output of the pre-amplifier then can be carried to the main amplifier.

Thermal noise, in transistors, comes from the resistance of the device, principally from the base area as the charge carrier is depleted.

Shot noise is the result of the statistical fluctuations of charge carriers across a junction. The expression, derived by Schottky for the vacuum diode, is applicable to all junctions or interfaces:

$$i_N^2 = 2qIB \qquad (12\text{–}8)$$

where i_N is the *rms* noise current in amperes, q is the charge of an electron (1.59×10^{-19} coulombs), I is the d.c. current in amperes, and B has its earlier definition. Since shot noise is associated with random movement of charge carriers, this means that vacuum tubes (including phototubes and photomultipliers), diodes, transistors and the like *all* evidence the effect. In transistors, the currents through the emitter-base and collector-base diodes are the sources of shot noise. If necessary, the noise power for shot noise can be expressed in terms of the square of the shot current times the resistance:

$$P_N = i_N^2 R = 2qIRB \qquad (12\text{–}9)$$

where R is the equivalent resistance of the junction.

As with thermal noise, shot noise is independent of frequency. This means that it appears at all frequencies. The difference between thermal noise and shot noise is that the latter is related to the d.c. current through the junction. Immediately, this suggests that reduction in shot noise is accomplished by limiting the current through the device.

The third source of fundamental noise, flicker noise, increases with decreasing frequency. Because of this, it is often called $1/f$ noise. The phenomenon giving rise to flicker noise is not well understood. Partly because of this, the third name euphemistically given this noise is *excess noise.* Several observations characterize the noise, including the fact that $1/f$ noise is significantly greater in solid state devices than in vacuum devices, and its effect is seldom important above 1 KHz. One important conclusion may be drawn from $1/f$ noise behavior: when sensitive measurements are to be made, avoid d.c. !

Note!

These three fundamental sources of noise are seen to be operative in a graph of NF versus frequency for a transistor, as in Figure 12.1. The frequency region below 1 KHz is due, almost completely, to flicker noise. The magnitude of NF in the intermediate region is clearly affected by thermal noise, while the steep increase in NF at high frequencies is a combination of both shot and thermal noise effects.

As often as not, fundamental noise sources are not at the root of measurement problems. Rather, they are of environmental origin. The most common source of interference in the United States is 60 Hz power distribution. Most people are cognizant of this, but ignore the higher harmonics at 120 Hz, 180 Hz, and even 240 Hz. Another troublesome source of interference is the AM radio frequency band. Because a conductor acts as an antenna for this band, this is often a problem. Also, noise from the brushes in nearby electric motors will invariably be noticed on sensitive equipment. Even X-ray equipment can contribute greatly to environmental noise. Figure 12.2 summarizes these various sources of noise and, more importantly, shows the frequency regions that are relatively free of noise.

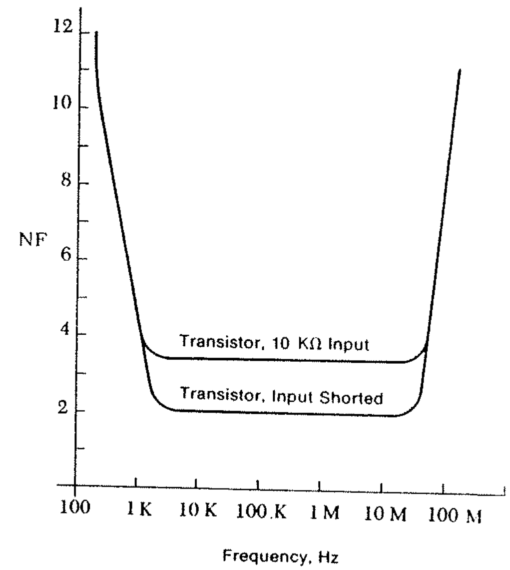


Figure 12.1 The noise figure (NF) of a transistor *versus* frequency.

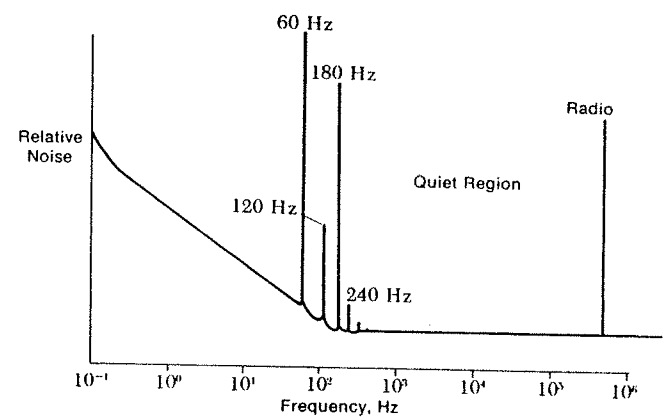


Figure 12.2 The noise spectrum, showing regions of quiet and interference.

referred input

Figure 3.16 shows how variations in bias current contribute to overall noise. The noise currents flow through the external equivalent resistances so that the total rms noise voltage is

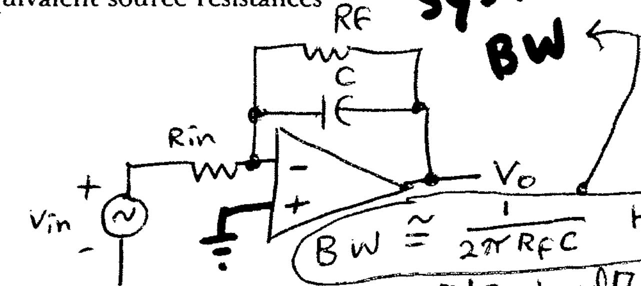
$$v_t \cong \{[v_n^2 + (i_n R_1)^2 + (i_n R_2)^2 + 4\kappa T R_1 + 4\kappa T R_2] \mathrm{BW}\}^{1/2} \tag{3.22}$$

v_t referred to input of AMP.

System BW

where

R_1 and R_2 = equivalent source resistances



$BW \cong \frac{1}{2\pi R_F C}$ Hz

Noise voltage @output $v_{OT} = v_t \cdot \left[\underbrace{\text{In-band Gain}}_{\left(= \frac{R_F}{R_i}\right)}\right]$

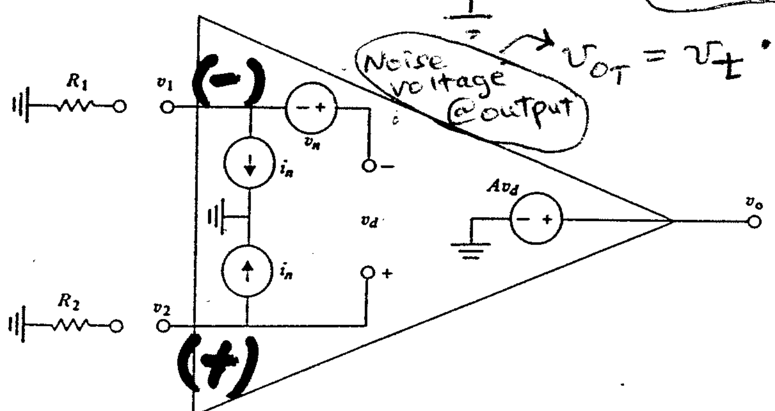


Figure 3.14 Noise sources in an op amp. The noise-voltage source v_n is in series with the input and cannot be reduced. The noise added by the noise-current sources i_n can be minimized by using small external resistances.

v_n = mean value of the rms noise voltage, in $\mathrm{V \cdot Hz^{-1/2}}$, across the frequency range of interest
i_n = mean value of the rms noise current, in $\mathrm{A \cdot Hz^{-1/2}}$, across the frequency range of interest

v_n, i_n found in OP AMP spec. sheets

κ = Boltzmann's constant (Appendix)
T = temperature, K
BW = noise bandwidth, in Hz

$K = 1.38 \times 10^{-23}$ J/K

Derivation of "Characteristic Noise Resistance" for OP AMP.

1. Use "Linear Superposition" to find component of V_T due to R_2 alone: (Set $R_1 = 0$)

(3.22) becomes

$$V_t = \{[v_n^2 + (i_n R_2)^2 + 4kTR_2]BW\}^{1/2}$$

Noise Power of noise voltage in R_2 is

$$P_t\Big|_{R_2} = \frac{V_t^2}{R_2} = \left[\frac{v_n^2}{R_2} + i_n^2 R_2 + 4kT\right] BW$$

To find value of R_2 that yields minimum noise power in R_2

$$\frac{d\,P_t|_{R_2}}{dR_2} = \left[-\frac{v_n^2}{R_2^2} + i_n^2 + 0\right] BW = 0$$

$$\Rightarrow \boxed{R_2 = \frac{v_n}{i_n}} \text{ to minimize } P_t\Big|_{R_2}$$

$\therefore \frac{v_n}{i_n} \triangleq$ Characteristic Noise Resistance

Likewise, we can show

$$\boxed{R_1 = \frac{v_n}{i_n}} \text{ to minimize } P_t\Big|_{R_1}$$

v_n = mean value of the rms noise voltage, in V·Hz$^{-1/2}$, across the frequency range of interest
i_n = mean value of the rms noise current, in A·Hz$^{-1/2}$, across the frequency range of interest
κ = Boltzmann's constant (Appendix)
T = temperature, K
BW = noise bandwidth, Hz

NOISE

$R_1, R_2 \Rightarrow$ "Source Resistances"

Figure 3.16 shows how variations in bias current contribute to overall noise. The noise currents flow through the external equivalent resistances so that the total rms noise voltage is

$$v_t \cong \{[v_n^2 + (i_nR_1)^2 + (i_nR_2)^2 + 4\kappa TR_1 + 4\kappa TR_2]\text{BW}\}^{1/2} \quad (3.22)$$

internal Johnson Noise
Shot Noise
Johnson Noise gen. externally in source resistances

where R_1 and R_2 = equivalent source resistances

Referred to input side

EXAMPLE 3.3 Because the electromagnetic flowmeter (Section 8.3) produces a signal in the microvolt range, it requires a low-noise amplifier. Typical source resistance is 300 Ω and bandwidth is 100 Hz. For the circuit shown in Figure E3.3, calculate the transformer turns ratio and also the noise voltage without and with the transformer.

$R_1, R_2 < 10k \Rightarrow$ Bipolar OP AMP Lowest Noise
$> 10k \Rightarrow$ FET OP AMP

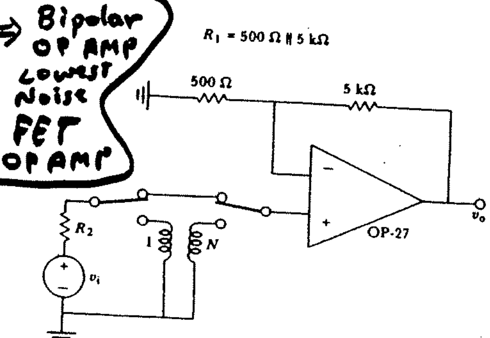
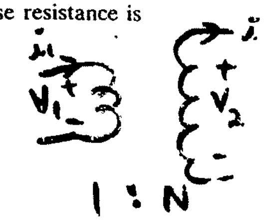


Figure E3.3 The OP-27 is a low-noise op amp. For ac inputs, the noise obtained using a direct input (switches up) can be reduced by using an input transformer (switches down).

ANSWER For 400 Hz, a typical electromagnetic flowmeter frequency, the specification sheet for the OP-27 gives $v_n = 3 \times 10^{-9}$ V·Hz$^{-1/2}$ and $i_n = 0.4 \times 10^{-12}$ A·Hz$^{-1/2}$. The characteristic noise resistance is

$$R_n = \frac{v_n}{i_n} = (3 \times 10^{-9})/(0.4 \times 10^{-12})$$
$$= 7.5\ \text{k}\Omega$$



The transformer turns ratio is

$$N = \left(\frac{R_n}{R_2}\right)^{1/2} = \left(\frac{7.5\ \text{k}\Omega}{300\ \Omega}\right)^{1/2} = 5$$

$\frac{V_2}{V_1} = N, \quad \frac{I_2}{I_1} = \frac{1}{N}$

For Figure E3.3, the R_1 of Figure 3.16 is much smaller than R_2 seen through the transformer, so we neglect noise originating in R_1. For the case without the transformer, (3.22) reduces to

500Ω > 300Ω$(5)^2$

$\frac{P_{sec}}{P_{prim}} = \frac{V_2 I_2}{V_1 I_1} = \frac{N}{N} = 1$

$$v_t = \{[v_n^2 + (i_nR_2)^2 + 4\kappa TR_2]\text{BW}\}^{1/2}$$
$$= \{[9 \times 10^{-18}) + (0.16 \times 10^{-24})(9 \times 10^4)$$
$$+ 4(1.38 \times 10^{-23})(300)(300)]100\}^{1/2}$$
$$= \{[(900 + 1.4 + 497)10^{-20}]100\}^{1/2} = \underline{37\ \text{nV}}$$

$\frac{Z_2}{Z_1} = \frac{V_2/I_2}{V_1/I_1} = N^2$

Note that v_n dominates the other noise sources. When the transformer is added, v_i is multiplied by 5 prior to mixing with v_n, and the total noise referred to the input becomes

Referring Noise to input (posn. of R_2)

$$v_t' = \left\{\left[\left(\frac{v_n}{5}\right)^2 + (i_nR5)^2 + 4\kappa TR_2\right]\text{BW}\right\}^{1/2}$$
$$= \left\{\left[\frac{9 \times 10^{-18}}{25} + (0.16 \times 10^{-24})(9 \times 10^4)(25)\right.\right.$$
$$\left.\left. + 4(1.38 \times 10^{-23})(300)(300)\right]100\right\}^{1/2}$$
$$= \{[(36 + 36 + 497)10^{-20}]100\}^{1/2} = \underline{24\ \text{nV}}$$

Note that proper transformer matching of the source to the amplifier has made v_n negligible and reduced the total noise voltage by 35%. The proper turns ratio can be incorporated into an existing transformer, as shown in Figure 8.6, thus eliminating the need for a separate transformer.