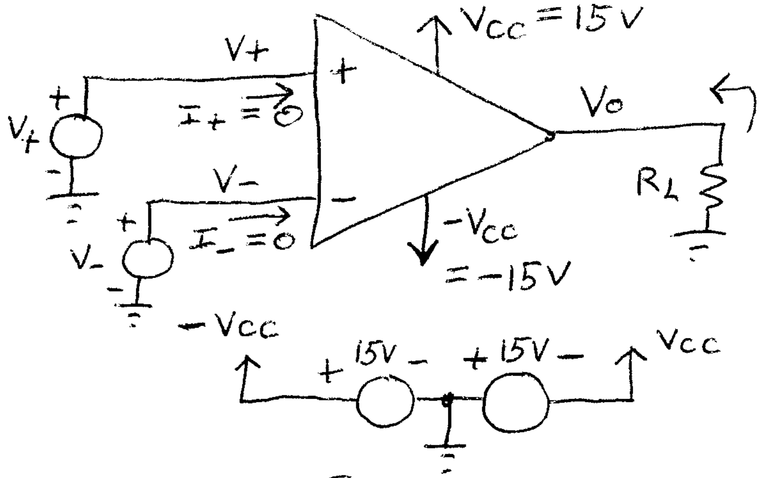
Operational Amplifier (OPAMP)

⇒ "High Gain Differential Voltage Amplifier"



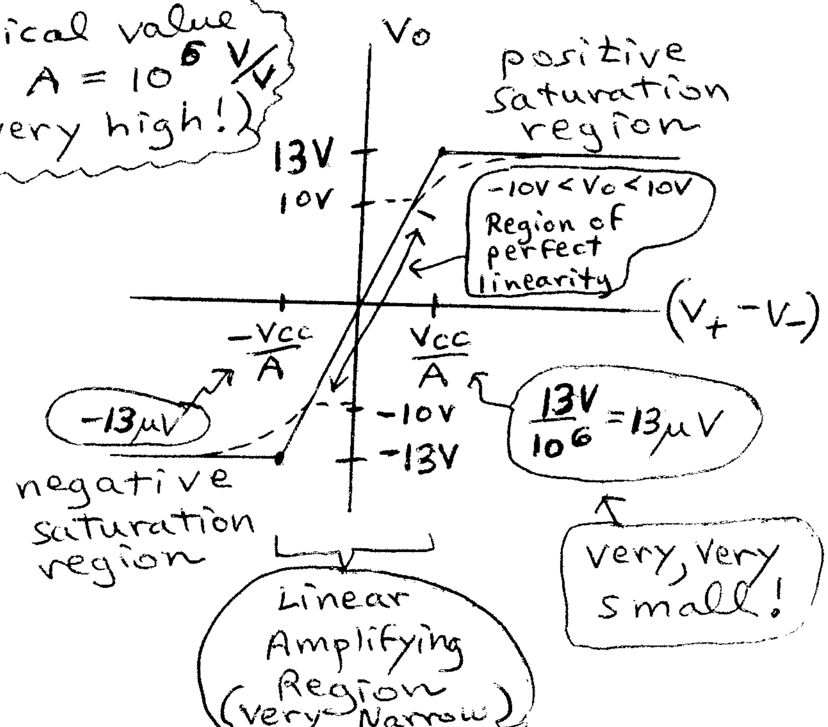
"Typical" Op Amps have protective current limiting circuitry that limits I_o to:

$-25mA < I_o < +25mA$

$$V_o = \begin{cases} -V_{cc} & \text{for } A(V_+ - V_-) \le -V_{cc} \\ A(V_+ - V_-) & \text{for } -V_{cc} < A(V_+ - V_-) < +V_{cc} \\ +V_{cc} & \text{for } A(V_+ - V_-) \ge +V_{cc} \end{cases}$$

Typical value of $A = 10^6$ V/V (very high!)

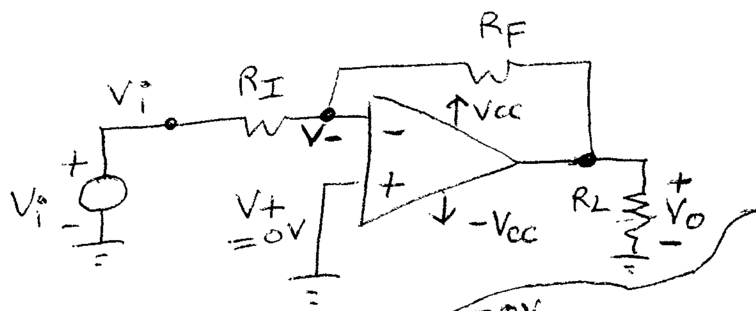
As long as I_o is within $(-25mA, +25mA)$ current range, V_o is independent of I_o, and hence V_o is independent of the load resistance R_L



Rule #1 of OPAMP: ("Virtual Voltage Rule") When OPAMP is operating in its linear amplifying range, $V_+ \approx V_-$, since $(V_+ - V_-)$ must lie between $+15\mu V$ and $-15\mu V$.

Rule #2 of OPAMP: No current flows into the (+) input or the (−) input of the OPAMP ($I_+ = 0$, $I_- = 0$)

Inverting Amplifier Configuration



A is called the "open Loop" voltage gain of OP AMP

$$V_o = A(\underbrace{V_+}_{=0V} - V_-) = -A(V_-)$$

Since no current flows into (−) input of OP AMP, by linear superposition we see

$$V_- = \underbrace{V_i \frac{R_F}{R_F + R_I}}_{\substack{\text{Contribution} \\ \text{to } V_- \text{ due to} \\ V_i \text{ acting alone}}} + \underbrace{V_o \frac{R_I}{R_F + R_I}}_{\substack{\text{Contribution} \\ \text{to } V_- \text{ due to} \\ V_o \text{ acting alone}}}$$

$$V_o = -A\, V_i \left(\frac{R_F}{R_F + R_I}\right) - A\, V_o \left(\frac{R_I}{R_F + R_I}\right)$$

$$V_o \left[1 + A\left(\frac{R_I}{R_F + R_I}\right)\right] = V_i \left(-A\left(\frac{R_F}{R_F + R_I}\right)\right)$$

"Closed Loop" Voltage Gain

$$A_V \triangleq \frac{V_o}{V_I} = \frac{\left(\frac{-A R_F}{R_F + R_I}\right)}{1 + A\left(\frac{R_I}{R_F + R_I}\right)} = \frac{\left(\frac{-A R_F}{\cancel{R_F + R_I}}\right)}{\left(\frac{R_F + R_I + A R_I}{\cancel{R_F + R_I}}\right)}$$

$$A_V = \frac{-R_F}{\left(\frac{R_F + R_I}{A}\right) + R_I}$$

Assuming $\frac{R_F + R_I}{A} \ll R_I$

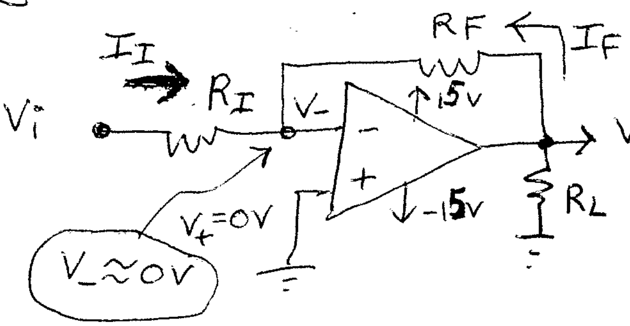
(This assumption is usually true, since $A \approx 10^6$!)

OR Assuming $\frac{R_F + R_I}{R_I} \ll A$,

$$\Rightarrow \boxed{A_V = \frac{-R_F}{R_I}}$$

Note that the closed-loop voltage gain is independent of "A" as long as A is LARGE

The "Inverting Amplifier Configuration" closed-loop Voltage Gain is more easily calculated using the two "rules of the OP AMP":



By Rule #1 (Virtual Voltage Rule) V_- is "virtually at ground" since V_+ is at ground.

Thus $I_I = \dfrac{V_i - 0}{R_I} = \dfrac{V_i}{R_I}$

But because Rule #2 says that no current can flow into the (+) terminal of the OP AMP,

$$KCL \Rightarrow I_F = -I_I = -\frac{V_i}{R_I}$$

$$\text{Thus} \quad V_o = I_F R_F + \underbrace{V_-}_{\approx 0V} = I_F R_F = \frac{-V_i}{R_I} R_F$$

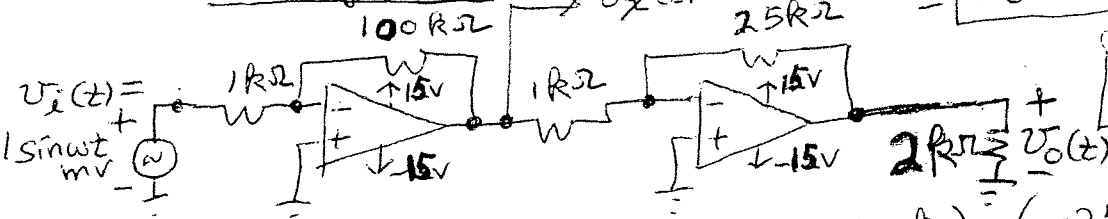
$$\therefore \boxed{A_V \triangleq \frac{V_o}{V_i} = -\frac{R_F}{R_I}}$$

Valid while OP AMP is in its linear amplifying region while $-13V < V_o < 13V$

$-V_i\left(\dfrac{R_F}{R_I}\right)$

Example #1

$v_x(t) = (1 \sin\omega t \text{ mV})\left(\dfrac{-100k}{1k}\right) = \boxed{-0.1 \sin\omega t \text{ Volts}}$



Find $v_o(t)$, $v_x(t)$

$$v_o(t) = \underbrace{(1 \sin\omega t \text{ mV})\left(\frac{-100k}{1k}\right)}_{v_x(t)} \cdot \left(\frac{-25k}{1k}\right) = \boxed{2.5 \sin\omega t \text{ Volts}}$$

Example #2

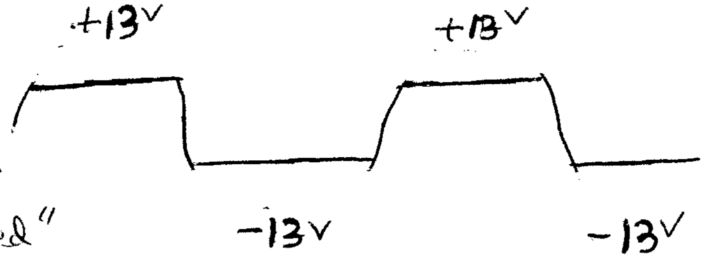
Now what would happen if $v_i(t)$ amplitude in EX#1 were increased to

$$v_i(t) = 10 \sin \omega t \text{ mV} \quad ?$$

$$v_x(t) = \left(\underset{\text{mV}}{10 \sin \omega t}\right)\left(\frac{-100k}{1k}\right) = -1 \sin \omega t \text{ Volts}$$

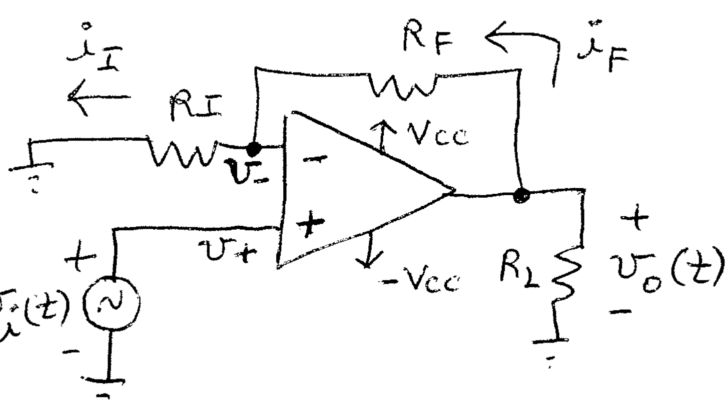
$$v_y(t) = v_x(t)\left(\frac{-25k}{1k}\right) = \begin{cases} 13\text{V}, & \text{when } 2500\,v_i(t) > 13\text{V} \\ +25 \sin \omega t \text{ Volts}, & |2500\,v_i(t)| < 13\text{V} \\ -13\text{V}, & \text{when } 2500\,v_i(t) < 13\text{V} \end{cases}$$

$v_y(t)$ →



Note:
$v_y(t)$ Output becomes distorted, or "clipped" because v_i amplitude has been raised to a level that requires $v_y(t)$ to rise above the +13V limit = $V_{cc} - 2V$ voltage and below the -13V limit = $-V_{cc} + 2V$

Non-Inverting OP AMP Configuration



Find the closed-loop voltage gain $A_V \triangleq \frac{v_o}{v_i}$

$$v_+ = v_i \Rightarrow v_- \approx v_i \quad \text{(Rule \#1 OP AMP)}$$

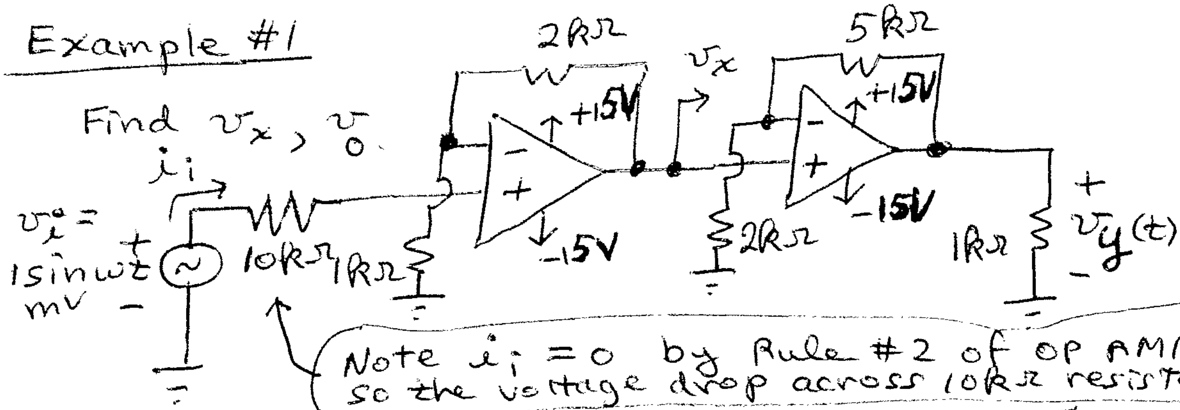
$$\therefore i_I = \frac{v_-}{R_I} = \frac{v_i}{R_I}$$

$$i_F = i_I \quad \text{(Rule \#2 OP AMP)} \Rightarrow v_o = i_F R_F + v_-$$

$$\therefore v_o = \frac{v_i}{R_I} R_F + v_i \Rightarrow \boxed{A_V \triangleq \frac{v_o}{v_i} = \frac{R_F}{R_I} + 1}$$

Example #1

Find v_x, v_o.



Note $i_i = 0$ by Rule #2 of OP AMPs, so the voltage drop across 10kΩ resistor = 0!

$$v_x = (1\sin\omega t \text{ mV})\left(1 + \frac{2k}{1k}\right) = \boxed{3\sin\omega t \text{ mV}}$$

$$v_y = v_x\left(1 + \frac{5k\Omega}{2k\Omega}\right) = \boxed{10.5\sin\omega t \text{ mV}}$$

Example #2

Repeat Ex #1 if v_i were changed to $v_i = 3\text{ V}$ (dc)

$$v_x = (3\text{ V})\left(1 + \frac{2k}{1k}\right) = \boxed{9\text{ V}}$$

$$v_y = \underbrace{(3\text{ V})\left(1 + \frac{2k}{1k}\right)}_{v_x = 9\text{V}}\left(1 + \frac{5k}{2k}\right) = 31.5\text{V} > 13\text{V}$$

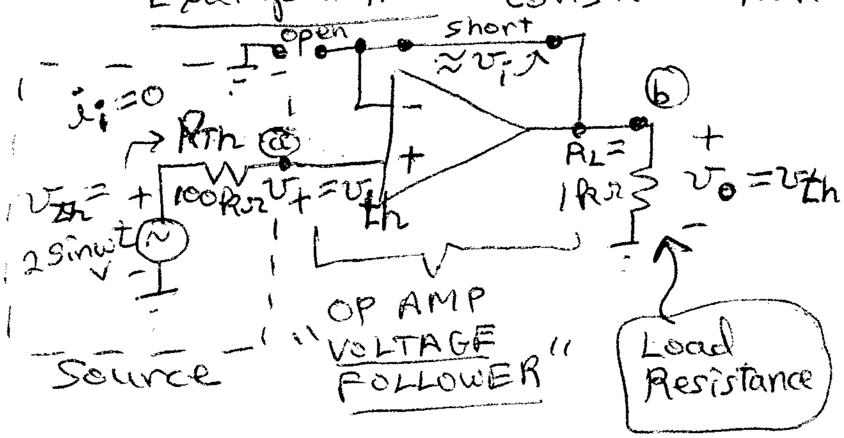
$$\Rightarrow v_y = \boxed{+13\text{V}}$$

positive saturation

Example #3

Consider Non-Inverting OP AMP with

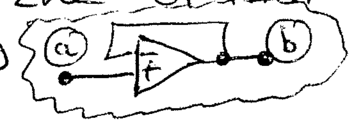
$$\boxed{\begin{array}{ll} R_F = 0\,\Omega & \text{(short circuit)} \\ R_I = \infty\,\Omega & \text{(open circuit)} \end{array}}$$

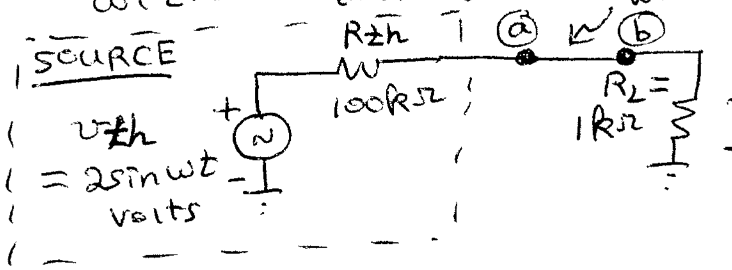


$$\Rightarrow A_v \triangleq \frac{v_o}{v_i} = \left(1 + \frac{0}{\infty}\right) = 1.0$$

This is called a "voltage follower" configuration, where $v_o = v_{Th} = 2\sin\omega t$ V

To appreciate the usefulness of the "op amp follower" circuit with a voltage gain of 1.0 over a "wire", let us replace the OP AMP with a wire:



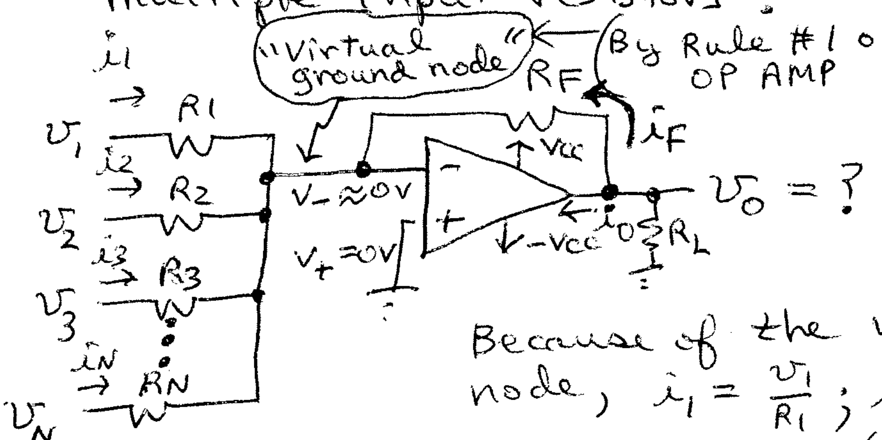


$$+\ v_o\ - = v_{th}\frac{1k}{(1k+100k)} = 0.0198 \sin \omega t \text{ volts}$$

With the op amp voltage follower removed, far less signal gets delivered to the load due to the presence of the 100kΩ resistor. We say that the op amp follower circuit eliminates the effects of "Loading" a source with a high Thevenin resistance when it is connected across a relatively low load resistance "R_L".

Example #4 Inverting, Weighted Summing Circuit

Consider an inverting OP AMP configuration with multiple input resistors:



Find v_o

Because of the virtual ground node, $i_1 = \frac{v_1}{R_1}$; $i_2 = \frac{v_2}{R_2}$; $i_3 = \frac{v_3}{R_3}$; $i_N = \frac{v_N}{R_N}$

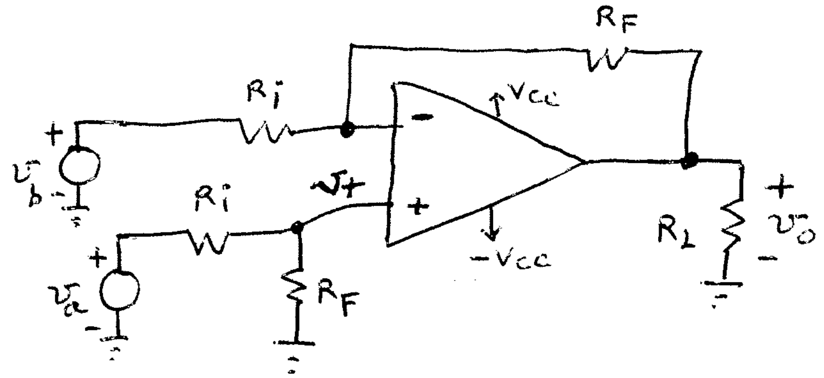
∴ By Rule #2 of OP AMP ($i_- = 0$)

$$KCL \Rightarrow i_F = -(i_1 + i_2 + i_3 + \dots + i_N)$$

$$\therefore v_o = i_F R_F = -\frac{R_F}{R_1}v_1 - \frac{R_F}{R_2}v_2 - \frac{R_F}{R_3}v_3 - \dots \frac{R_F}{R_N}v_N$$

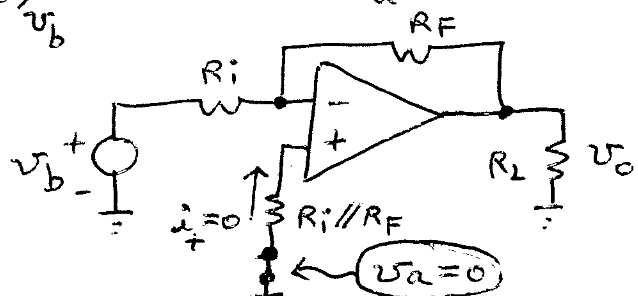
(valid as long as $|v_o| < V_{cc} - 2$, and $|i_o| < 25mA$.)

Difference Amplifier Circuit



Use Linear Superposition to find v_o

[a] $(v_o)_{v_b} = ?$ Set $v_a = 0\,V$ (replace it by a short circuit)

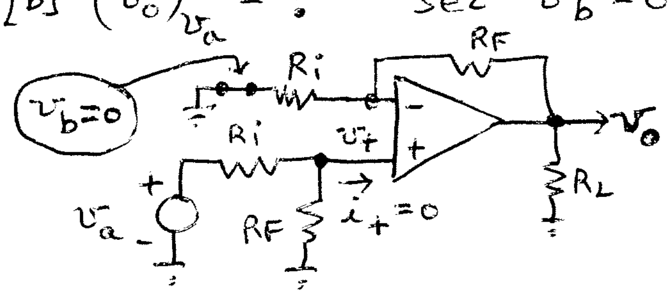


The $R_i/\!/R_F$ resistance may be replaced by a short circuit, since there is no voltage dropped across $R_i/\!/R_F$, since $i_+ = 0$ (Rule #2 of OP AMP).

Thus we have the "inverting OP AMP configuration"

and
$$(v_o)_{v_b} = v_b\left(\frac{-R_F}{R_i}\right)$$

[b] $(v_o)_{v_a} = ?$ Set $v_b = 0\,V$ (replace by a short circuit)



Because $i_+ = 0$, we may use a voltage divider eqn to find v_+, then from v_+ to v_o, we have a "non-inverting OP AMP config."

$$v_o = \underbrace{v_a\left(\frac{R_F}{R_F+R_i}\right)}_{v_+}\cdot\left(1+\frac{R_F}{R_i}\right) = v_a\left(\frac{R_F}{R_i}\right)$$

Thus the complete output voltage is given by

$$\boxed{v_o = (v_o)_{v_a} + (v_o)_{v_b} = \frac{R_F}{R_I}(v_a - v_b)}$$

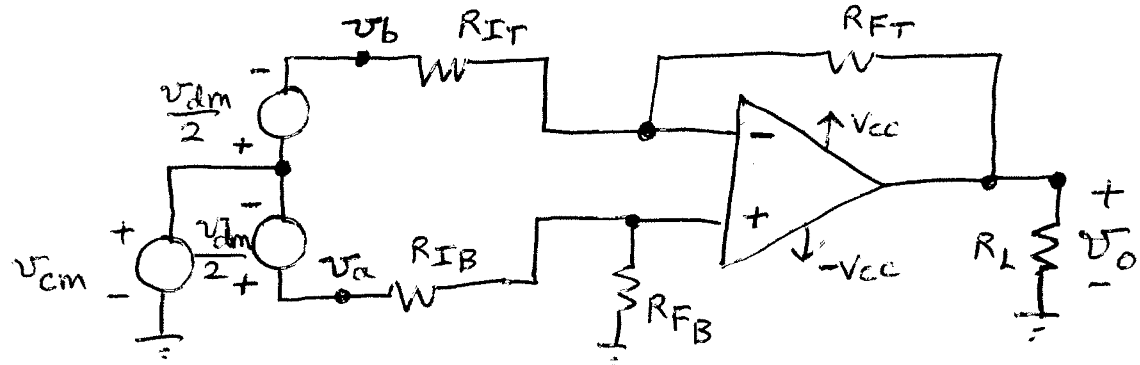
Let us define the <u>differential voltage</u> and the <u>common-mode voltage</u> as

$$\boxed{\begin{array}{lll} v_{dm} \triangleq v_a - v_b & & \text{(differential voltage)} \\ v_{cm} \triangleq \dfrac{v_a + v_b}{2} & & \text{(common-mode voltage)} \end{array}}$$

Solving for v_a and v_b $\Rightarrow$

$$\boxed{\begin{array}{l} v_a = v_{cm} + \frac{1}{2} v_{dm} \\ v_b = v_{cm} - \frac{1}{2} v_{dm} \end{array}}$$

Our difference amplifier may be drawn in terms of v_{dm} and v_{cM} sources instead of v_a and v_b:



Also note that the R_F & R_I resistors have been differentiated using the subscripts T (top) and B (bottom).

$$v_o = \left(v_{cm} - \frac{v_{dm}}{2}\right)\left(\frac{-R_{F_T}}{R_{I_T}}\right) + \left(v_{cm} + \frac{v_{dm}}{2}\right)\left(\frac{R_{F_B}}{R_{F_B}+R_{I_B}}\right)\left(\frac{R_{F_T}+R_{I_T}}{R_{I_T}}\right)$$

$$v_o = v_{cm}\underbrace{\left[\left(\frac{R_{F_B}}{R_{F_B}+R_{I_B}}\right)\left(\frac{R_{F_T}+R_{I_T}}{R_{I_T}}\right) - \frac{R_{F_T}}{R_{I_T}}\right]}_{=A_{v_{CM}}} + v_{dm}\underbrace{\left[\left(\frac{R_{F_B}}{R_{F_B}+R_{I_B}}\right)\left(\frac{R_{F_T}+R_{I_T}}{2R_{I_T}}\right) + \frac{R_{F_T}}{2R_{I_T}}\right]}_{A_{vdm}}$$

(we desire this to be as close to 0 as possible!)

(Typ. >>1)

If the R_I's and R_F's are perfectly matched

$$R_{I_T} = R_{I_B} = R_I; \quad R_{F_T} = R_{F_B} = R_F$$

then

$$v_o = \underbrace{v_{cm}}_{=\frac{1}{2}(v_a+v_b)}\underbrace{(0)}_{A_{vcm}=0} + \underbrace{v_{dm}}_{=(v_a-v_b)}\underbrace{\left(\frac{R_F}{R_I}\right)}_{A_{vdm}=R_F/R_I}$$

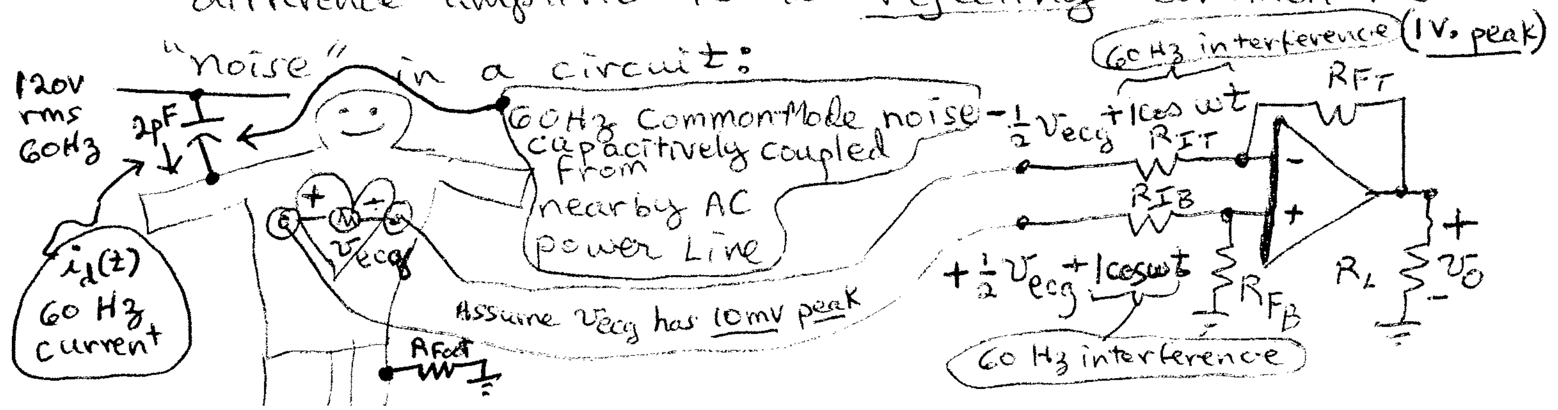
But if the R_I's and R_F's are not perfectly matched (which is always the case in the real world)

$A_{v_{cm}} << 1$, A_{vdm} is usually $>> 1$

A "Figure of merit" for a difference amplifier

$$CMRR \triangleq \left|\frac{A_{vdm}}{A_{vcm}}\right| \quad \text{(Should be very large!)}$$

The higher the CMRR, the better the difference amplifier is at rejecting common-mode "noise" in a circuit:



Ex #1 (Ideal Case)

Let $R_{F_T} = R_{F_B} = 100\,k\Omega$ and $R_{I_T} = R_{I_B} = 1\,k\Omega$

Then $A_{vcm} = 0$, $A_{vdm} = \frac{100k}{1k} = 100 \Rightarrow CMRR = \frac{100}{0}$ $\boxed{CMRR = \infty}$

$$\therefore v_o = \tfrac{1}{2}\left(\left[\frac{v_{ecg}}{2} + \cos\omega t\right] + \left[-\frac{v_{ecg}}{2} + \cos\omega t\right]\right) A_{vcm} +$$

$$\left\{\left[\frac{v_{ecg}}{2} + \cos\omega t\right] - \left[\frac{-v_{ecg}}{2} + \cos\omega t\right]\right\} A_{vdm}$$

$$v_o = \cos\omega t \cdot A_{vcm} + v_{ecg} \cdot A_{vdm}$$

$$v_o = \cos\omega t \cdot (0) + v_{ecg} \cdot (100) = \boxed{100\, v_{ecg}}$$

$\therefore$ all of the interfering 60 Hz common-mode signal is rejected by OP AMP difference amplifier, while the ECG waveform is amplified by a factor of 100!

Ex #2 (Real Case)

Assume R_F's and R_I's are not perfectly matched

Let $R_{F_T} = 101\,k\Omega$, $R_{F_B} = 99\,k\Omega$

$R_{I_T} = 1.1\,k\Omega$, $R_{I_B} = 0.9\,k\Omega$

Find $v_o(t)$

$$A_{vcm} = \left[\left(\frac{R_{FB}}{R_{FB} + R_{IB}}\right)\left(\frac{R_{FT} + R_{IT}}{R_{IT}}\right) - \frac{R_{FT}}{R_{IT}}\right] = \boxed{0.163}$$

$$A_{vdm} = \frac{1}{2}\left[\left(\frac{R_{FB}}{R_{FB} + R_{IB}}\right)\left(\frac{R_{FT} + R_{IT}}{R_{IT}}\right) + \frac{R_{FT}}{R_{IT}}\right] = \boxed{91.9}$$

$$\Rightarrow \boxed{CMRR = \frac{91.9}{0.163} = 563.8}$$

$$v_o = \underbrace{\cos\omega t\,(0.163)}_{\text{Noise}} + \underbrace{91.9\,(v_{ecg})}_{\text{Signal}}$$

(If v_{ecg} has 10 mV peak, it is well above noise @ output!)