ECE342 EMC Test 2 (100 Points, Closed Book, 2 Crib Sheets, 150 Minutes)
October 28, 2005 (KEH)

Name: Solution Copy Box: __________

Note: $\epsilon_0 = 8.854 \times 10^{-12}$ F/m and $\mu_0 = 4\pi \times 10^{-7}$ H/m

1) **(24 points)** A uniform plane electromagnetic wave propagating in a lossless gaseous dielectric medium (NOT free space!) characterized by (ϵ_R, $\mu_R = 1$, $\sigma = 0$) has the following magnetic field intensity

$$\vec{H} = \overset{7.9576}{\cancel{2.6544}}\cos(\underbrace{4\pi \times 10^7}_{\omega} t + \underbrace{0.2\pi}_{\beta} z)\vec{i_y} \text{ mA/m}$$

a) Find the direction of propagation of the wave.

$-z$ direction

b) Find the speed of the wave.

$$v_p = \omega/\beta = \frac{4\pi \times 10^7}{0.2\pi} = 20 \times 10^7 \frac{m}{s} = \boxed{200 \, m/\mu s}$$

c) Find the relative permittivity of the medium, ϵ_R.

$$200 \, m/\mu s = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1/\sqrt{\mu_0 \epsilon_0}}{\sqrt{\epsilon_R}} = \frac{300 \, m/\mu s}{\sqrt{\epsilon_R}} \Rightarrow \epsilon_R = \boxed{2.25}$$

d) Find the frequency (in MHz) of the wave as it moves past an observer located at the *fixed* position $z = z_o$.

$$f = \frac{\omega}{2\pi} = \frac{4\pi \times 10^7}{2\pi} = 20 \times 10^6 \text{ Hz} = \boxed{20 \text{ MHz}}$$

e) Find the precise frequency of the wave (in MHz) as it is received by an observer moving in the $+z$ direction with velocity $\overrightarrow{v_{obs}} = 1000\vec{i_z}$ m/s

$$f = \frac{1}{2\pi}\frac{d}{dt}\left[4\pi \times 10^7 t + 0.2\pi(z_0 + 1000t)\right] = \frac{1}{2\pi}\left[4\pi \times 10^7 + 0.2\pi(1000)\right]$$

$$\boxed{f = 20.0001 \text{ MHz}}$$

f) Find the precise frequency of the wave (in MHz) as it ~~is received by an observer moving~~ in the $-z$ direction with velocity $\overrightarrow{v_{obs}} = -1000\vec{i_z}$ m/s

$$f = \frac{1}{2\pi}\frac{d}{dt}\left[4\pi \times 10^7 t + 0.2\pi(z_0 - 1000t)\right] = \frac{1}{2\pi}\left[4\pi \times 10^7 - 0.2\pi(1000)\right]$$

$$\boxed{f = 19.9999 \text{ MHz}}$$

g) Find the precise frequency of the wave (in MHz) as it is received by an observer moving in the $+y$ direction with velocity $\overrightarrow{v_{obs}} = 1000\vec{i_y}$ m/s

$\boxed{f = 20 \text{ MHz}}$ (z coordinate remains constant $\Rightarrow$ phase is NOT affected by motion in y-dir.)

h) Find the wavelength of the wave (λ), which might be defined as the distance between wave peaks at a fixed time $t = t_o$.

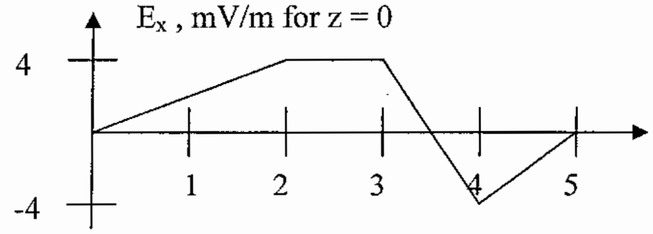
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.2\pi} = \boxed{10 \text{ m}}$$

i) Write an expression for the associated electric field intensity vector, $\vec{E}$.

$$\eta = \sqrt{\mu/\epsilon} = \left(\sqrt{\frac{\mu_0}{\epsilon_0}}\right)\cdot\frac{1}{\sqrt{\epsilon_R}} = 377\Omega\left(\frac{1}{\sqrt{2.25}}\right) = \frac{377}{1.5} = 251.33\,\Omega$$

(−) moving wave $\Rightarrow$ scale by $-\eta$ $\boxed{\vec{E} = -2\cos(4\pi \times 10^7 t + 0.2\pi z)\,\vec{i_x} \text{ V/m}}$

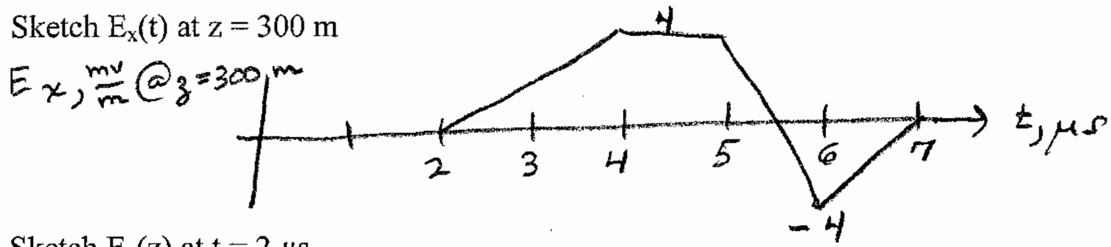
2) (10 points) A uniform plane electromagnetic wave with the electric field oriented in the x direction propagates in the +z direction in a lossless dielectric medium with $\epsilon_R = 4$, $\mu_R = 1$, $\sigma = 0$. (Note that this medium is NOT free space!) The following electric field is observed at z = 0.



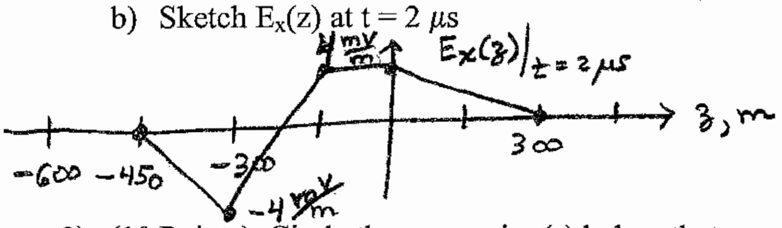
$v = \frac{300 \text{ m}/\mu s}{\sqrt{4}}$

$= 150 \text{ m}/\mu s$

a) Sketch $E_x(t)$ at z = 300 m



b) Sketch $E_x(z)$ at t = 2 µs



3) (10 Points) Circle the expression(s) below that correspond to traveling waves, and for those that are traveling waves, find the wave velocity (be sure to indicate both magnitude and direction.)

$(100t_1 - 25z_1) = (100t_2 - 25z_2) \Rightarrow \frac{z_2 - z_1}{t_2 - t_1} = \frac{\Delta z}{\Delta t} = \frac{100}{25}$

a) sin(100t – 25z) $\quad v_p = \frac{100}{25}\hat{i}_z = \boxed{4\hat{i}_z \text{ m/s}}$

b) sin(100t)

$25x_1 + 100t_1 + 2 = 25x_2 + 100t_2 + 2 \Rightarrow \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{-100}{25}$

c) cos(25x + 100t + 2) $\quad v_p = -\frac{100}{25}\hat{i}_x = \boxed{-4\hat{i}_x \text{ m/s}}$

d) $e^{-x}\cos(2t - 3)$

e) 4u(x – t) $\quad v_p = \frac{+1}{1}\hat{i}_x = \boxed{+\hat{i}_x \text{ m/s}}$

f) $e^{-|0.01y + 3t|}$ $\quad v_p = \frac{-3}{.01}\hat{i}_y = \boxed{-300\hat{i}_y \text{ m/s}}$

g) $e^{-|100y|}$

4) **(12 points)** Consider a 10 kHz VLF (very low frequency) sinusoidal electromagnetic plane wave as it travels through wet marshy soil that is characterized by **σ = 0.01 S/m, μ_R = 1, and ϵ_R = 15.**

a) Calculate the propagation constant (γ), expressed in rectangular form ($\gamma = \alpha + j\beta$). From this information, determine the wavelength of this wave (λ) and the skin depth of this medium at this frequency (δ).

$$\bar{\gamma} = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j2\pi\times10^4(4\pi\times10^{-7})(0.01 + j2\pi10^4 \cdot 15(8.854\times10^{-12}))}$$

$$= \underbrace{\boxed{0.01986}}_{\alpha} + j\,\underbrace{\boxed{0.01988}}_{\beta}$$

$$\boxed{\beta = 0.01988\ \frac{\text{rad}}{\text{m}}}$$
$$\boxed{\alpha = 0.01986\ \frac{\text{Nepers}}{\text{m}}}$$

$$\lambda = 2\pi/\beta = 2\pi/0.01988 = \boxed{316.1\text{ m}}$$

$$\delta = 1/\alpha = \boxed{50.33\text{ m}}$$

Determine how far this wave must travel for its amplitude to be reduced in magnitude by 40 dB.

$$20\log\left[e^{-\alpha z}\right] = -40\text{ dB}$$

$$\Rightarrow z = \boxed{231.9\text{ m}}$$

b) Determine the velocity of this 10 kHz electromagnetic wave in wet marshy soil. (*Remember that the velocity of a wave in a lossy medium is* ***NOT*** *given by 1/sqrt(µϵ)!)*

$$v_p = \omega/\beta = \frac{2\pi\times10\times10^3}{0.01988} = 3.16\text{ m}/\mu s$$

5) **(10 points)** Determine the velocity of propagation (v_p) and the characteristic impedance (Zo) for the following transmission lines:

a) $\underline{L} = 0.50\ \mu H/m$ and $\underline{C} = 50\ pF/m$.

$$v_p = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} = \frac{1}{\sqrt{(0.5\times10^{-6})(50\times10^{-12})}} = \boxed{200\ m/\mu s}$$

$$Z_o = \sqrt{\mathcal{L}/\mathcal{C}} = \sqrt{\frac{0.5\times10^{-6}}{50\times10^{-12}}} = \boxed{100\ \Omega}$$

b) $L = 0.25\ \mu H/m$ and a dielectric characterized by $\epsilon_R = 2.25$, $\mu_R = 1$, $\sigma = 0$.

$$v_p = \frac{300\ m/\mu s}{\sqrt{2.25}} = \frac{300}{1.5} = \boxed{200\ m/\mu s}$$

$$\frac{1}{\sqrt{0.25\times10^{-6}\cdot\mathcal{C}}} = 200\ m/\mu s \Rightarrow \mathcal{C} = 100\ pF/m$$

$$Z_o = \sqrt{\mathcal{L}/\mathcal{C}} = \sqrt{\frac{0.25\times10^{-6}}{100\times10^{-12}}} = \boxed{50\ \Omega}$$

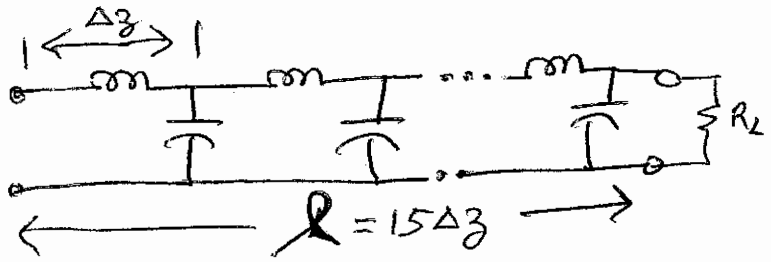
6) **(10 points)** An "LC" analog delay line similar to the one demonstrated in the laboratory is constructed using ***fifteen*** **1 μF** capacitors and fifteen **4.9 mH** inductors.

a) Determine the (approximate) delay time of this analog delay line.

$$\mathcal{L} = \frac{4.9\,mH}{\Delta z} \qquad C = \frac{1\,\mu F}{\Delta z} \qquad \ell = 15\Delta z$$

$$v_p = \frac{1}{\sqrt{\mathcal{L}C}} = \frac{\Delta z}{\sqrt{(4.9\,mH)(1\,\mu F)}} = \Delta z(14285.7)\,m/s$$

$$T = \ell / v_p = \frac{15\cancel{\Delta z}}{14285.7\cancel{\Delta z}} = \boxed{1.05\,ms}$$

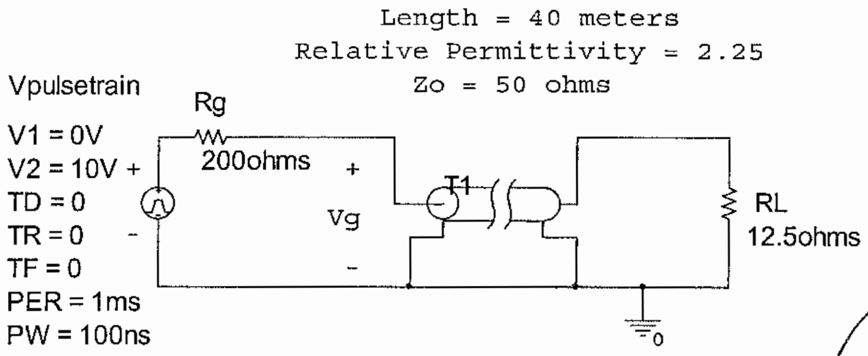


b) Determine the (approximate) load resistance that this line should be terminated in at the receiving end so that there will be *no reflections* received back at the sending end.

$R_L = Z_o$ for no reflections!

$$Z_o = \sqrt{\mathcal{L}/C} = \sqrt{\frac{4.9\,mH}{1\,\mu F}} = \boxed{70\,\Omega}$$

7) **(12 points)** A 40 m length of lossless coaxial cable with Zo = 50 Ω, and polyethylene foam dielectric (with $\epsilon_R = 2.25$) is connected to a pulse generator with an output resistance Rg = 200 Ω. Assume that the generator was first set to deliver a 0 to 10 V pulse train with a 1 ms period and a 100 ns pulse width across its open-circuited output terminals *before* it was connected to the transmission line. If the leading edge of a generated pulse occurs at time t = 0, then sketch the voltage at the sending end (Vg) for $0 < t < 1.5\ \mu s$, assuming that the line is terminated at the receiving end in a resistance of RL = 12.5 Ω.

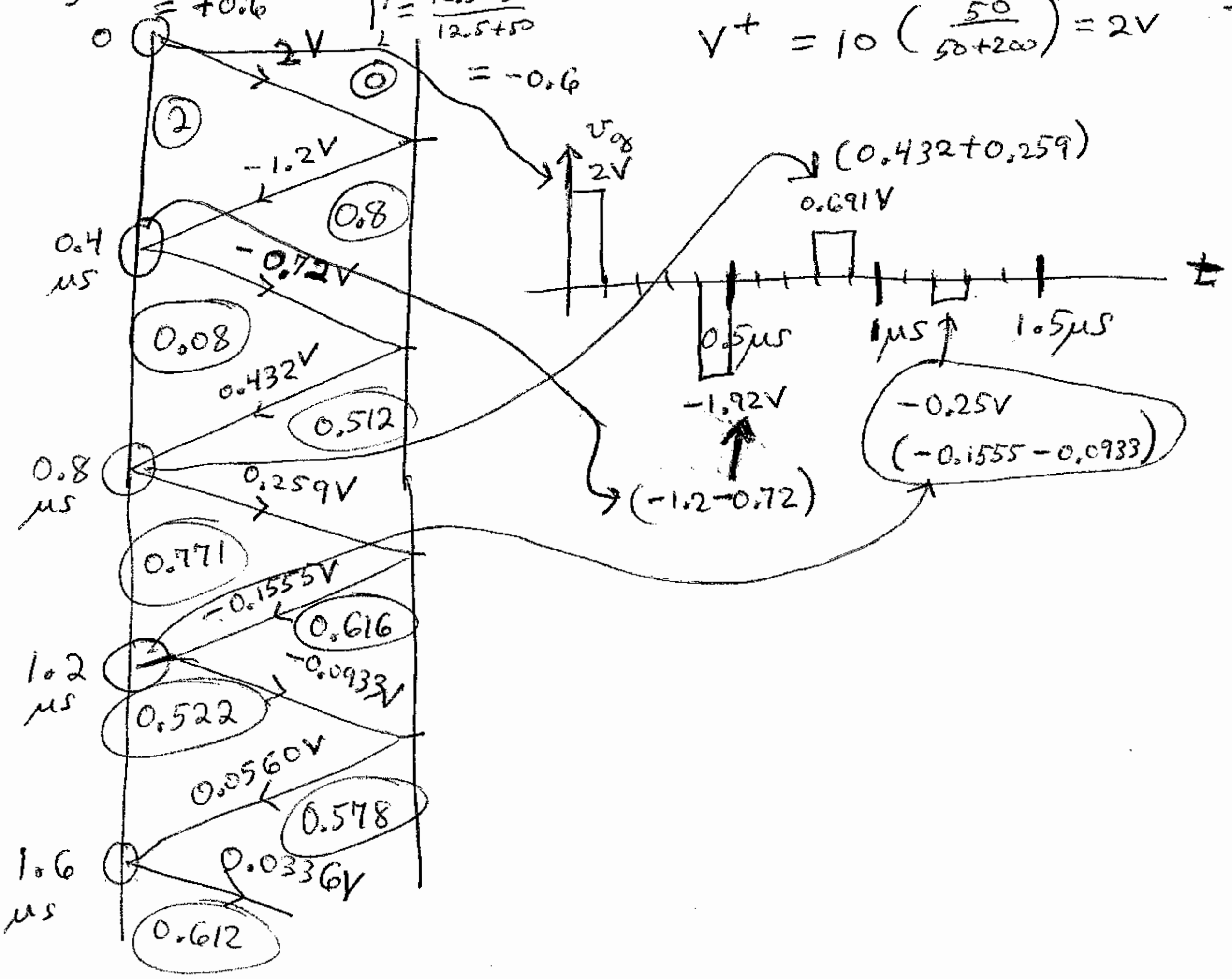


$$T = \frac{l}{v_p} = \frac{40\ \text{m}}{\left(\frac{300}{\sqrt{2.25}}\right)\ \text{m}/\mu s} = 0.2\ \mu s$$

Note: PW < 2T
100ns < 2(200ns)
⇒ Short Pulse problem!

$$\Gamma_g = \frac{200-50}{200+50} = +0.6 \qquad \Gamma_L = \frac{12.5-50}{12.5+50} = -0.6$$

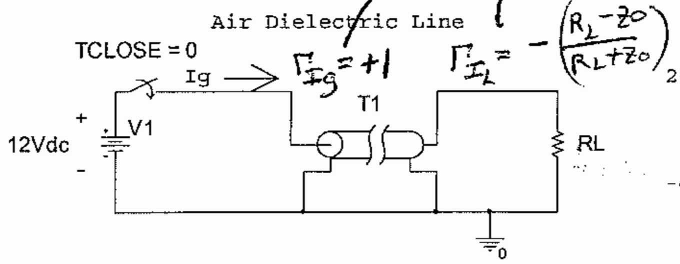
$$V^+ = 10\left(\frac{50}{50+200}\right) = 2V$$

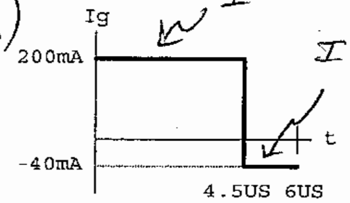


8) **(12 points)** At time t= 0, a 12 V battery with a source resistance of 0 ohms is connected to an unknown length of lossless *air-dielectric* transmission line that is terminated in an unknown resistance, RL. The current flowing into that line "Ig" for $0 < t < 6\ \mu s$ is found to be 200 mA from $0 < t < 4.5\ \mu s$ and then Ig changes to -40 mA from 4.5 ~~$\mu s < t <$~~ 6 μs. *The current that flows after t = 6 μs is not specified.*

Remember: $\Gamma_I = -\Gamma_V$

$\Gamma_{Ig} = +1 \qquad \Gamma_{IL} = -\left(\dfrac{R_L - Z_0}{R_L + Z_0}\right)$





$I^+ + I^- + I^{-+}$

a) Find the length of this line

$$T = \frac{4.5\mu s}{2} = 2.25\mu s = \frac{l}{v_p} \leftarrow 300\times10^6\ m/s \quad \Rightarrow \boxed{l = 675m}$$

b) Find the characteristic impedance of this line, Zo

$$I^+ = 200mA = \frac{12V}{Z_o} \Rightarrow Z_o = \frac{12V}{200mA} = \boxed{60\Omega}$$

$$I^+ + I^- + I^{-+} = -40mA = I^+\left(1 + \Gamma_{I_L} + \Gamma_{I_L}\cdot\Gamma_{I_g}\right)$$

$$\Gamma_{I_L} = -\left(\frac{R_L - 60}{R_L + 60}\right), \quad \Gamma_{I_g} = +1$$

c) Find the unknown load resistance, RL.

$$I^+ + I^- + I^{-+} = 200mA\left(1 - 2\left(\frac{R_L - 60}{R_L + 60}\right)\right) = -40mA$$

$$\Rightarrow \boxed{R_L = 240\Omega}$$

d) What value of current will flow into the line after the battery has been connected to the line for a ***long, long*** time (as t approaches infinity)?

$$I_{SS} = \frac{12V}{240\Omega} = \boxed{50mA}$$

12V I_{SS} $R_L = 60\Omega$

In the dc steady state, Ignore Transmission Line effects!