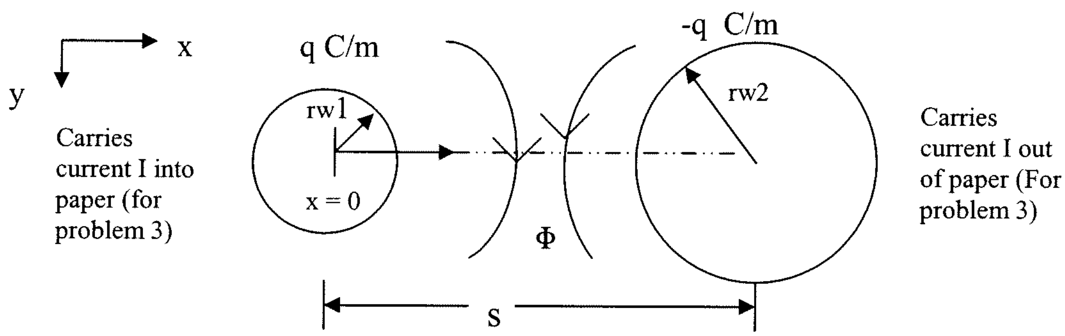
HG-1
Fall, 2006

ECE342 EMC Homework 6

1. From the class notes find the capacitance per unit length "C" of a coaxial transmission line by dividing our result by the line length. Likewise, in Homework # 4, we found the inductance of a length of coaxial cable. Once again, by dividing by the length of the cable, you can find the inductance per unit length, "L". Show that the vp of the V and I waves, given by vp = 1/sqrt(L C) is equal to the vp of the EM wave traveling between the conductors in the dielectric, which we have shown found to be given by vp = 1/sqrt($\mu\epsilon$). In other words, show that, at least for the case of the coaxial cable transmission line,

   $\underline{L}\,\underline{C} = \mu\,\epsilon$ (*"boundary conditions" require this result to be true for ANY transmission line.)*

2. A parallel-wire transmission line with wire radii rw1 and rw2 and center-to-center separation "s" is drawn below: You are to find the capacitance per unit length of this transmission line.



(a) Assume that the left conductor has been charged to "q" coulombs per unit length, and the right conductor has been charged to "-q" coulombs per unit length by putting a dc voltage source of "V" volts across the two conductors. Consider a section of line that is "Length_line" long. First find the **E** field set up by this section of line along a line that joins the wire centers for rw1 < x < s-rw2. (Hint: use symmetry considerations, apply Gauss Law for the Electric Field to each line *acting alone*. Note that with each line in isolation, the charges are uniformly distributed across the cross-section of each conductor. In reality, the charges are NOT distributed uniformly when the conductors are closely spaced, due to the interaction between the two charged wires, but we will ignore this effect, which is called the "proximity effect". Thus our result will hold precisely only for a two-wire line with relatively narrow wire radii rw1 and rw2, that is, our result will yield accurate results whenever s >> rw1, rw2.)

Answer: $\mathbf{E}(x) = q[1/(2\pi\epsilon x) + 1/(2\pi\epsilon(s - x))]\ \mathbf{i}_x$

(b) Now calculate the voltage between the two conductors by evaluating $V = \int_{rw1}^{s-rw2} E(x)dx$

Answer: $V = \dfrac{q}{2\pi\varepsilon}\ln\left(\dfrac{(s - r_{rw1})(s - r_{w2})}{r_{rw1}r_{w2}}\right)$

(c) Now solve for the capacitance C = Q/V = q*Length_line/V, and divide by the length of the line to find the capacitance per unit length, C.

Answer: $\underline{C} = \dfrac{2\pi\varepsilon}{\ln\left(\dfrac{(s - r_{rw1})(s - r_{w2})}{r_{rw1}r_{w2}}\right)}$

3. In this problem we shall find the expression for the inductance per unit length of the parallel wire line of Problem 2. Imagine that the left wire carries current I into the paper, and the right wire carries current I out of the paper. Because the magnetic flux lines are in opposite directions in regions that are not in between the two conductors, and anti-symmetric flux distributions are created on either exterior side, we need only consider the flux that circulates *in between the conductors* in calculating the inductance of the parallel wire line.

(a). By superimposing the results for an individual current-carrying wire in isolation, find the magnetic flux density flowing downward through the rectangular strip between the two conductors that is due to the current "I" flowing through these two lines. (Once again this is an approximation that holds best for very thin wires.)

$$\text{Answer: } B = \left(\frac{\mu I}{2\pi x} + \frac{\mu I}{2\pi(s-x)}\right) i_y$$

(b). Find an expression for the net magnetic flux cutting the rectangular strip (dotted line in the drawing) between the two conductors. Note that this strip has length "Length_line" and width "s –(rw1+rw2)". Hint: If integrating using MAPLE, use the "*AllSolutions*" option. For example, if you were integrating 1/x, you would type: *int(1/x, x = a..b, AllSolutions);*

$$\text{Answer: } \Phi = \frac{\mu I(Length\_line)}{2\pi} \ln\left(\frac{(s-r_{w1})(s-r_{w2})}{r_{w1} r_{w2}}\right)$$

(c). Find the inductance of this section of line by dividing the flux created by the current "I" that creates it, and then divide by the line length to find the inductance per unit length, L.

$$\text{Answer: } \underline{L} = \frac{\mu}{2\pi} \ln\left(\frac{(s-r_{w1})(s-r_{w2})}{r_{w1} r_{w2}}\right)$$

(d). Show for the thin-conductor, parallel-wire line, just as for the coaxial line, $\underline{L}\,\underline{C} = \mu\epsilon$

4. A parallel-wire transmission line is made from two bare wires suspended in air, and each have radius 6.3 mil, and their centers are separated by 100 mil. Find the inductance per meter, the capacitance per meter, the velocity of propagation, and the characteristic impedance Zo. (Ans: 1.08 μH/m, 10.29 pF/m, vp = 300 m/μs, Zo = 323.9 Ω) (Use the approximate formulas in Problems 2 and 3. (1 mil = 0.001 inch = 2.54E-5 meter)
(Assume that $\mu o = 4\pi \times 10^{-7}$ H/m, and $\epsilon o = 10^{-9}/(36\pi)$ F/m)

5. Text Problems 4.2.3 and 4.2.9 (Ans: 53.8 pF/m, 0.299 uH/m, vp/c = 0.831, Zo = 74.6 Ω, vp = 249 m/μs)

6. Text Problem 4.3.1 and 4.3.8 (Answers in textbook.)
(Submit the PSPICE verification; include both the schematic and PROBE plot. Use "T" component for transmission line, select device (it becomes pink), then right click inside pink box and select "Edit Properties" to set its parameters, set its delay time Td = 1 nS and set Zo = 50 ohms. Use "VPULSE" component for 10 V switched on source. Set it to generate a pulse whose duration is longer than 10 ns, so it can model the "switched on" 10 V source. For this problem, suitable parameters for VPULSE are: V1 = 0, V2 = 10V, TD = 0, TR = 0, TF = 0; PER = 200NS, PW=20NS.)

7. Text Problems 4.3.6 and 4.3.13 (Answers: 621.06 m, 75 Ω)
(Submit the PSPICE verification – both schematic and PROBE plot. Simulate the circuit with the calculated line length converted into its corresponding line propagation delay, and the terminating resistance set to 75 ohms, and show that the Vin(t) waveform given in the problem results)

8. Text Problem 4.3.7 (Answers: 80 Ω, 262.9 Ω.) (No need to verify in PSPICE, unless you want to!)

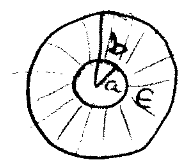
ECE 342 HW6 Solution

1. From the class notes find the capacitance per unit length "$\underline{C}$" of a coaxial transmission line by dividing our result by the line length. Likewise, from Homework #4, Problem 5, we found the inductance of a length of coaxial cable. Once again, by dividing by the length of the cable, you can find the inductance per unit length, "$\underline{L}$". Show that the vp of the V and I waves, given by vp = 1/sqrt($\underline{L}$ $\underline{C}$) is equal to the vp of the EM wave traveling between the conductors in the dielectric, which we have shown is given be vp = 1/sqrt($\mu\epsilon$). In other words, show that, at least for the case of the coaxial cable transmission line,

$$\underline{L}\,\underline{C} = \mu\epsilon$$ *This result is true in general for any transmission line!*

From the course notes (p. 13, Lecture 7)

$$\mathcal{C} = \frac{C}{L} = \frac{2\pi\epsilon}{\ln(b/a)}$$



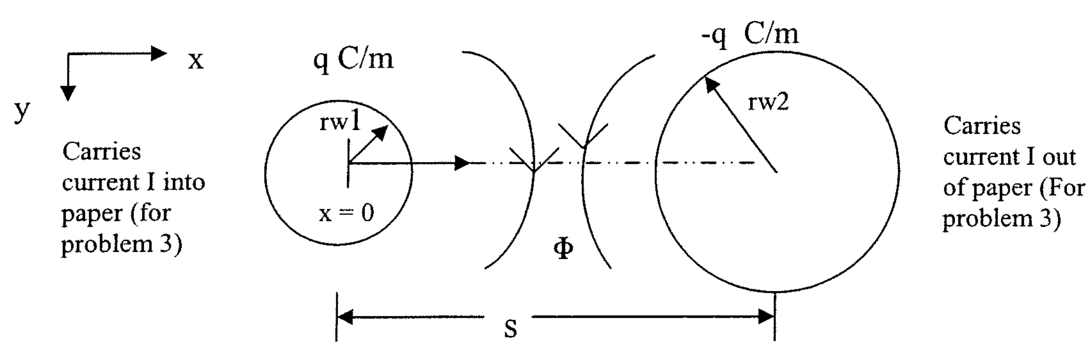
From HW #4, Problem 5

$$\mathcal{L} = \frac{L}{L_{coax}} = \frac{\mu \ln(b/a)}{2\pi}$$

Note that

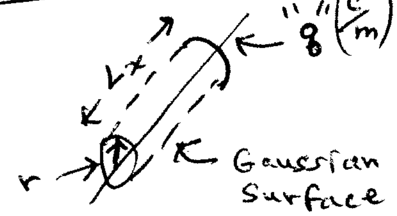
$$\mathcal{L}\,\mathcal{C} = \frac{\mu \ln(b/a)}{2\pi} \cdot \frac{2\pi\epsilon}{\ln(b/a)} = \mu\epsilon$$

2. A parallel-wire transmission line with wire radii rw1 and rw2 and center-to-center separation "s" is drawn below: You are to find the capacitance per unit length of this transmission line.



(a) Assume that the left conductor has been charged to "q" coulombs per unit length, and the right conductor has been charged to "-q" coulombs per unit length by putting a dc voltage source of "V" volts across the two conductors. Consider a section of line that is "Length_line" long. First find the **E** field set up by this section of line along a line that joins the wire centers for rw1 < x < s-rw2. (Hint: use symmetry considerations, apply Gauss Law for the Electric Field to each line *acting alone*. Note that with each line in isolation, the charges are uniformly distributed across the cross-section of each conductor. In reality, the charges are NOT distributed uniformly when the conductors are closely spaced, due to the interaction between the two charged wires, but we will ignore this effect, which is called the "proximity effect". Thus our result will hold precisely only for a two-wire line with infinitesimally narrow wire radii rw1 and rw2. But our result will yield good results whenever s > rw1, rw2.)

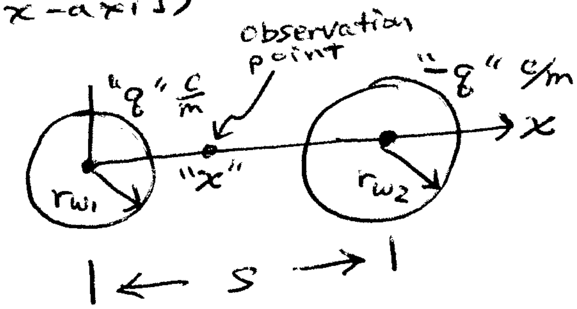
Solution — Applying Gauss Law to single charged filament or wire



$$D_r = \frac{\cancel{Lx} \cdot q}{2\pi r \cancel{Lx}} = \frac{q}{2\pi r}$$

$$\boxed{\vec{D} = \frac{q}{2\pi r}\,\hat{i}_r}$$

(A) Referring to figure of parallel-wire line and using superposition, the $\vec{E}$ field along the line between the centers of the wires (x-axis)



$$E_x(x) = \frac{D(x)}{\epsilon}$$

$$\boxed{E_x(x) = \frac{q}{2\pi\epsilon}\left[\frac{1}{x} + \frac{1}{s-x}\right]}$$

(B) Finding the voltage between the 2 wires

$$V = \int_{x=rw_1}^{s-rw_2} \underbrace{\frac{q}{2\pi\epsilon}\left[\frac{1}{x} + \frac{1}{s-x}\right]}_{=E_x} dx = \frac{q}{2\pi\epsilon}\int_{x=r_{w_1}}^{s-rw_2}\left[\frac{1}{x}+\frac{1}{s-x}\right]dx$$

$$V = \frac{q}{2\pi\epsilon}\ln\left[\frac{(s-r_{w_1})(s-r_{w_2})}{r_{w_1}\cdot r_{w_2}}\right]$$

Note: If integrating in Maple, use "AllSolutions" option as in "int((1/x + 1/(s-x)), x=rw1..(s-r2), AllSolutions);

(C)

$$C = \frac{Q}{V} = \frac{Lx \cdot q}{V} = \frac{\cancel{q}\,Lx}{\frac{\cancel{q}}{2\pi\epsilon}\left(\ln\left[\frac{(s-r_{w_1})(s-r_{w_2})}{r_{w_1}\, r_{w_2}}\right]\right)}$$

$$\mathcal{C} = \frac{C}{Lx} = \boxed{\frac{2\pi\epsilon}{\ln\left[\frac{(s-r_{w_1})(s-r_{w_2})}{r_{w_1}\, r_{w_2}}\right]}}$$

HW6 (3)

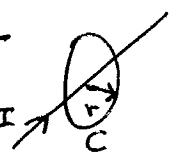
3. In this problem we shall find the expression for the inductance per unit length of the parallel wire line of Problem 2. Imagine that the left wire carries current I into the paper, and the right wire carries current I out of the paper. Because the magnetic flux lines are in opposite directions in regions that are not in between the two conductors, and anti-symmetric flux distributions are created to either exterior side, we need only consider the flux that is developed between the conductors in calculating the inductance of the parallel wire line.

A. By superimposing the results for an individual current-carrying wire in isolation, find the magnetic flux density flowing downward through the rectangular strip between the two conductors that is due to the current "I" flowing through these two lines. (Once again this is an approximation that holds best for very thin wires.)

Answer: $B = (\frac{\mu I}{2\pi x} + \frac{\mu I}{2\pi(s-x)})i_y$

(A) Solution

Single Wire Carrying Current "I"



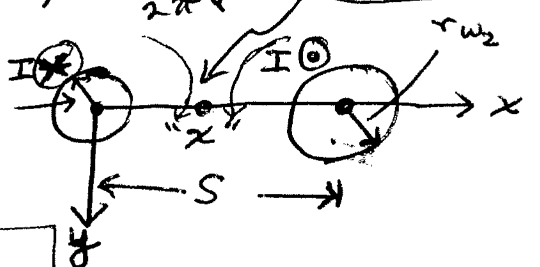
$$\oint_C H \cdot dl = I$$

$$2\pi r \cdot H_\phi = I$$

$$H_\phi = \frac{I}{2\pi r}$$

$$\vec{B} = \mu \vec{H}$$

For two wires



$$B_y(x) = \frac{\mu I}{2\pi}\left[\frac{1}{x} + \frac{1}{s-x}\right]$$

$$\vec{B} = \frac{\mu I}{2\pi}\left[\frac{1}{x} + \frac{1}{s-x}\right]\hat{i}_y$$

(B) Find net magnetic flux cutting strip between centers of wires. (Strip has length "Lx" into paper.)

$$\Phi = \frac{\mu I}{2\pi}\int_0^{Lx}\int_{r_{w_1}}^{s-r_{w_2}}\left[\frac{1}{x} + \frac{1}{s-x}\right]dx = \frac{\mu I\, Lx}{2\pi}\ln\left[\frac{(s-r_{w_1})(s-r_{w_2})}{r_{w_1} r_{w_2}}\right]$$

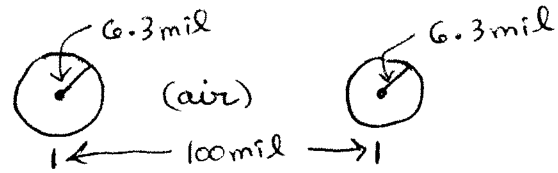
(C)

$$\mathcal{L} = \left(\frac{\Phi}{I}\right)\Big/ Lx = \frac{\mu}{2\pi}\ln\left[\frac{(s-r_{w_1})(s-r_{w_2})}{r_{w_1} r_{w_2}}\right]$$

(D)

$$\mathcal{L}C = \left[\frac{\mu}{2\pi}\ln\left(\frac{(s-r_{w_1})(s-r_{w_2})}{r_{w_1} r_{w_2}}\right)\right]\left[\frac{2\pi\epsilon}{\ln\left(\frac{(s-r_{w_1})(s-r_{w_2})}{r_{w_1} r_{w_2}}\right)}\right] = \mu\epsilon$$

HG (4)



4. A parallel-wire transmission line is made from two bare wires suspended in air, and each have radius 6.3 mil, and their centers are separated by 100 mil. Find the inductance per meter, the capacitance per meter, the velocity of propagation, and the characteristic impedance Zo. (Ans: 1.08 μH/m, 10.30 pF/m, vp = 300 m/μs, Zo = 323.7 Ω)

$$\varepsilon o := \frac{1}{36\cdot\pi}\cdot 10^{-9}\ \frac{F}{m} \qquad \mu o := 4\cdot\pi\cdot 10^{-7}\ \frac{H}{m}$$

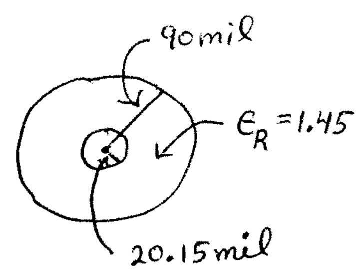
$$L\_per\_meter := \frac{\mu o}{2\cdot\pi}\cdot \ln\left[\frac{(100-6.3)^2}{(6.3)^2}\right] \qquad \boxed{L\_per\_meter = 1.07982\times 10^{-6}\ \frac{H}{m}}$$

$$C\_per\_meter := \frac{2\cdot\pi\cdot\varepsilon o}{\ln\left[\frac{(100-6.3)^2}{(6.3)^2}\right]} \qquad \boxed{C\_per\_meter = 10.28979\times 10^{-12}\ \frac{F}{m}}$$

$$vp := \frac{1}{\sqrt{L\_per\_meter\cdot C\_per\_meter}} \qquad \boxed{vp = 300\times 10^{6}\ \frac{m}{s}}$$

$$Zo := \sqrt{\frac{L\_per\_meter}{C\_per\_meter}} \qquad \boxed{Zo = 323.94583\times 10^{0}\ \Omega}$$

5. Text 4.2.3 and 4.2.9. Given RG6/U coaxial cable with



$$mil := 2.54\cdot 10^{-5}\cdot m \qquad \varepsilon_R := 1.45$$

$$rs := 90mil \qquad rw := 20.15\cdot mil$$

$$C\_per\_meter := \frac{2\cdot\pi\cdot\varepsilon o\cdot\varepsilon_R}{\ln\left(\frac{rs}{rw}\right)} \qquad \boxed{C\_per\_meter = 53.82552\times 10^{-12}\ \frac{F}{m}}$$

$$L\_per\_meter := \frac{\mu o\cdot\varepsilon o\cdot\varepsilon_R}{C\_per\_meter} \qquad \boxed{L\_per\_meter = 299.32108\times 10^{-9}\ \frac{H}{m}}$$

$$vp = \frac{1}{\sqrt{\mu o\cdot\varepsilon o\cdot\varepsilon_R}} = \frac{speed\_in\_free\_space}{\sqrt{\varepsilon_R}} = 0.8305\cdot speed\_in\_free\_space$$

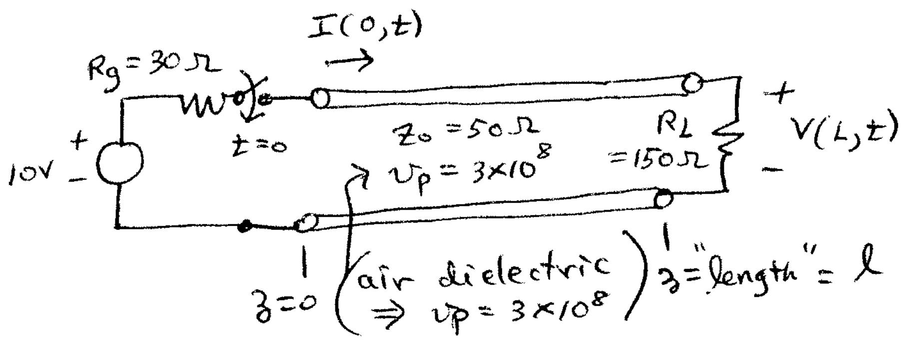
$$Zo := \sqrt{\frac{L\_per\_meter}{C\_per\_meter}} \qquad \boxed{Zo = 74.57179\times 10^{0}\ \Omega}$$

$$vp := \frac{1}{\sqrt{C\_per\_meter \cdot L\_per\_meter}}$$

$$vp = 249.13644 \times 10^{6} \ \frac{m}{s}$$

6. Text 4.3.1. Given a transmission line of length L = 30 cm, vp = 300 m/μs, Zo = 50 Ω, and RL = 150 Ω, driven by a source that consists of a 10 V battery (switched on at t = 0) in series with 30Ω. Find V(L,t) and I(0,t) for 0 < t < 10 ns. Also find the steady state values for V and I on the line as t -> infinity.

$Rg := 30.0 \quad RL := 150.0 \quad Zo := 50.0 \quad vp := 3 \cdot 10^{8} \quad Length := 30 \cdot 10^{-2}$



$$Td := \frac{Length}{vp} \qquad Td = 1 \times 10^{-9}$$

$$Vp1 := 10 \cdot \frac{Zo}{Zo + Rg} \qquad Vp1 = 6.25 \times 10^{0}$$

$$gammaL := \frac{(RL - Zo)}{RL + Zo} \qquad gammaL = 500 \times 10^{-3}$$

$$gammaG := \frac{(Rg - Zo)}{Rg + Zo} \qquad gammaG = -250 \times 10^{-3}$$

$Vm1 := Vp1 \cdot gammaL \qquad Vm1 = 3.125 \times 10^{0}$

$Vp2 := Vm1 \cdot gammaG \qquad Vp2 = -781.25 \times 10^{-3}$

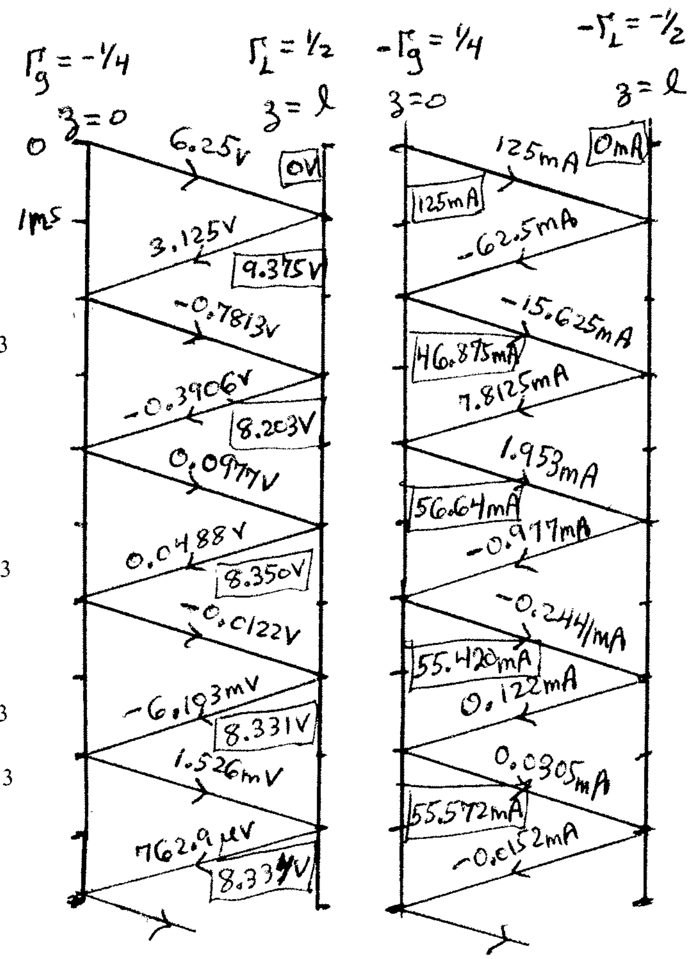
$Vm2 := Vp2 \cdot gammaL \qquad Vm2 = -390.625 \times 10^{-3}$

$Vp3 := Vm2 \cdot gammaG \qquad Vp3 = 97.65625 \times 10^{-3}$

$Vm3 := Vp3 \cdot gammaL \qquad Vm3 = 48.82813 \times 10^{-3}$

$Vp4 := Vm3 \cdot gammaG \qquad Vp4 = -12.20703 \times 10^{-3}$

$$I^{+} = \frac{10V}{30\Omega + 50\Omega} = 125mA$$



$Vm4 := Vp4 \cdot gammaL \qquad Vm4 = -6.10352 \times 10^{-3}$

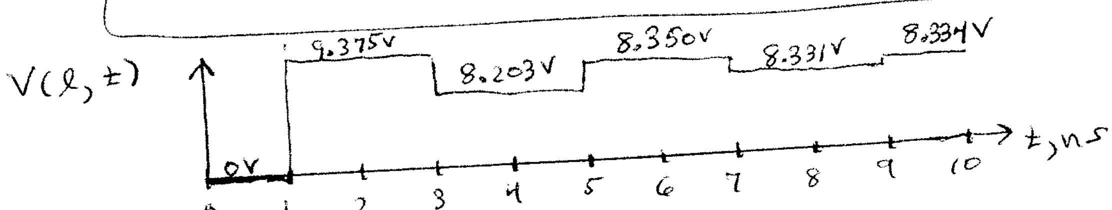
$Vp5 := Vm4 \cdot gammaG \qquad Vp5 = 1.52588 \times 10^{-3}$

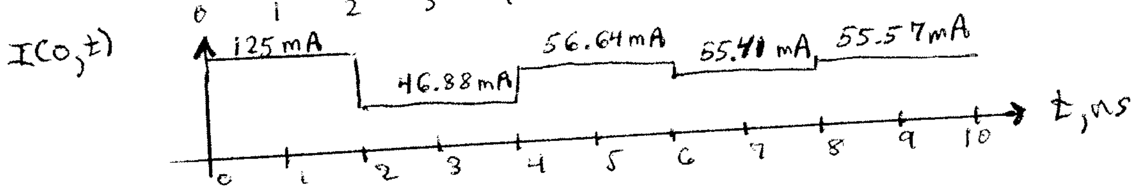
$Vm5 := Vp5 \cdot gammaL \qquad Vm5 = 762.93945 \times 10^{-6}$

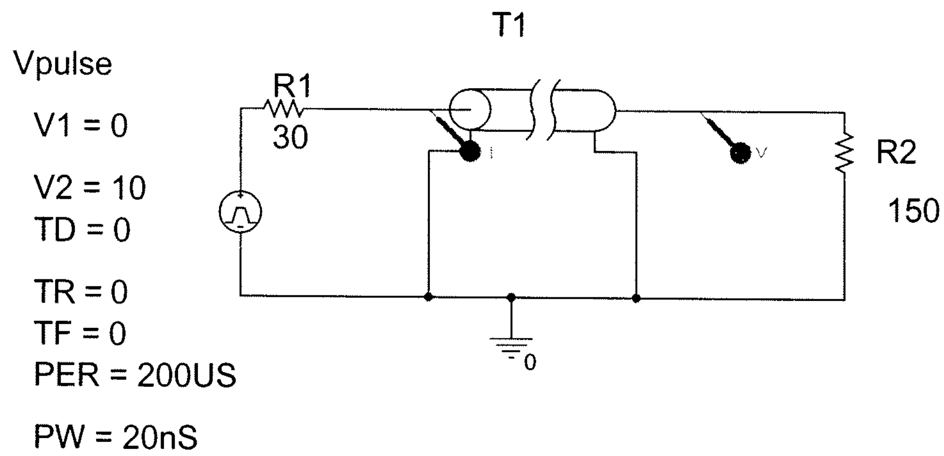
Between 0 and 1 ns	$V_L := 0$	
Between 1 and 3 ns	$V_L := Vp1 + Vm1$	$V_L = 9.375 \times 10^{0}$ V
Between 3 and 5 ns	$V_L := V_L + Vp2 + Vm2$	$V_L = 8.20313 \times 10^{0}$ V
Between 5 and 7 ns	$V_L := V_L + Vp3 + Vm3$	$V_L = 8.34961 \times 10^{0}$ V
Between 7 and 9 ns	$V_L := V_L + Vp4 + Vm4$	$V_L = 8.3313 \times 10^{0}$ V
Between 9 and 11 ns	$V_L := V_L + Vp5 + Vm5$	$V_L = 8.33359 \times 10^{0}$ V

Steady State: $V_{Lss} := 10 \cdot V \cdot \dfrac{RL}{RL + Rg}$ $\qquad V_{Lss} = 8.33333 \times 10^{0}\,V$

Between 0 and 2 ns	$I_o := \dfrac{Vp1}{Zo}$	$I_o = 125 \times 10^{-3}$ A
Between 2 and 4 ns	$I_o := I_o + \dfrac{Vm1}{-Zo} + \dfrac{Vp2}{Zo}$	$I_o = 46.875 \times 10^{-3}$ A
Between 4 and 6 ns	$I_o := I_o + \dfrac{Vm2}{-Zo} + \dfrac{Vp3}{Zo}$	$I_o = 56.64063 \times 10^{-3}$ A
Between 6 and 8 ns	$I_o := I_o + \dfrac{Vm3}{-Zo} + \dfrac{Vp4}{Zo}$	$I_o = 55.41992 \times 10^{-3}$ A
Between 8 and 10 ns	$I_o := I_o + \dfrac{Vm4}{-Zo} + \dfrac{Vp5}{Zo}$	$I_o = 55.57251 \times 10^{-3}$ A
Steady State:	$I_{oss} := \dfrac{10}{Rg + RL}$	$I_{oss} = 55.55556 \times 10^{-3}$ A







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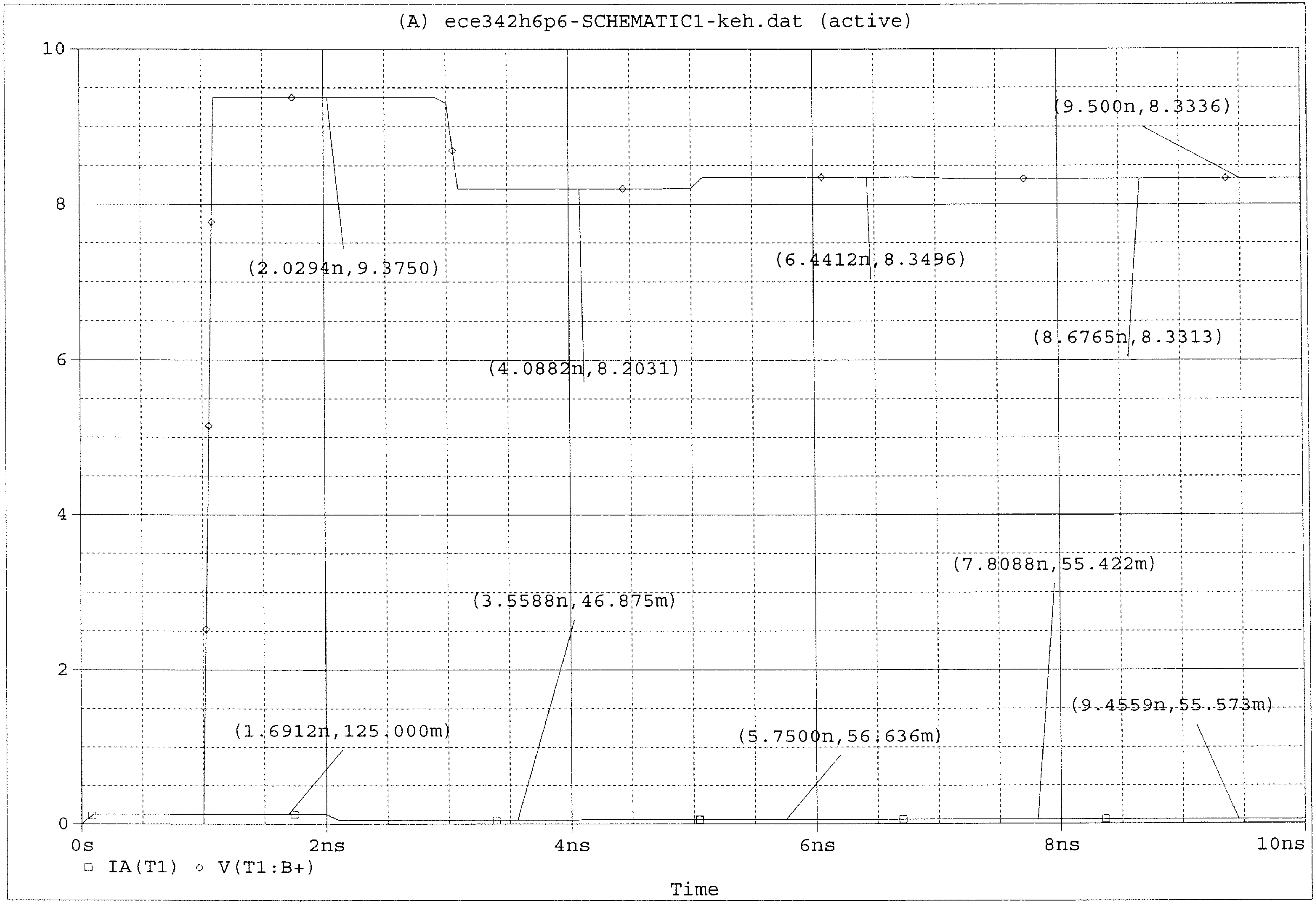

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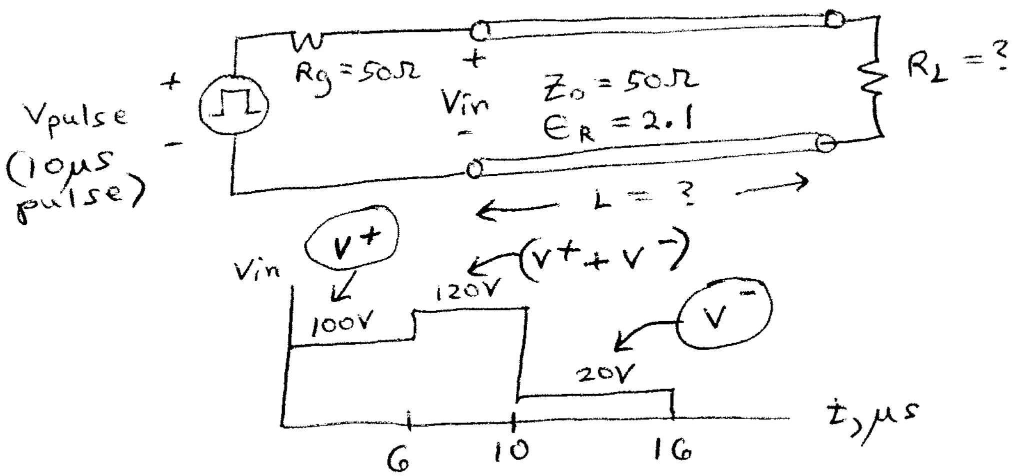
7. Text Problem 4.3.6 and 4.3.13. Given TDR waveform shown below. Assume the TDR has a source impedance of 50 ohms and the coaxial cable it is connected to has Zo = 50 ohms., the dielectric of the cable is Teflon, with $\varepsilon_R := 2.1$



$$vp := \frac{1}{\sqrt{\varepsilon o \cdot \varepsilon_R \cdot \mu o}} \qquad vp = 207.01967 \times 10^{6} \ \frac{m}{s}$$

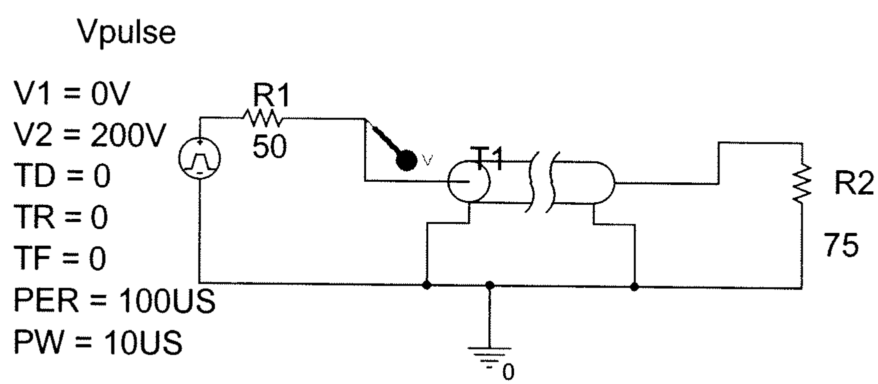
$$Td := \frac{6 \cdot 10^{-9}}{2} \qquad Td = 3 \times 10^{-9} \ s$$

$$Line\_length := vp \cdot Td \qquad \boxed{Line\_length = 621.059 \times 10^{-3} \ m}$$

$$Vplus := 100 \cdot V \qquad Vminus + Vplus = 120 \cdot V \Rightarrow \qquad Vminus := 20 \cdot V$$

$$Vminus = Vplus \cdot \left(\frac{RL - Zo}{RL + Zo} \right)$$

$$RL := -Zo \cdot \frac{(Vminus + Vplus)}{(Vminus - Vplus)} \qquad \boxed{RL = 75 \times 10^{0} \ \Omega}$$



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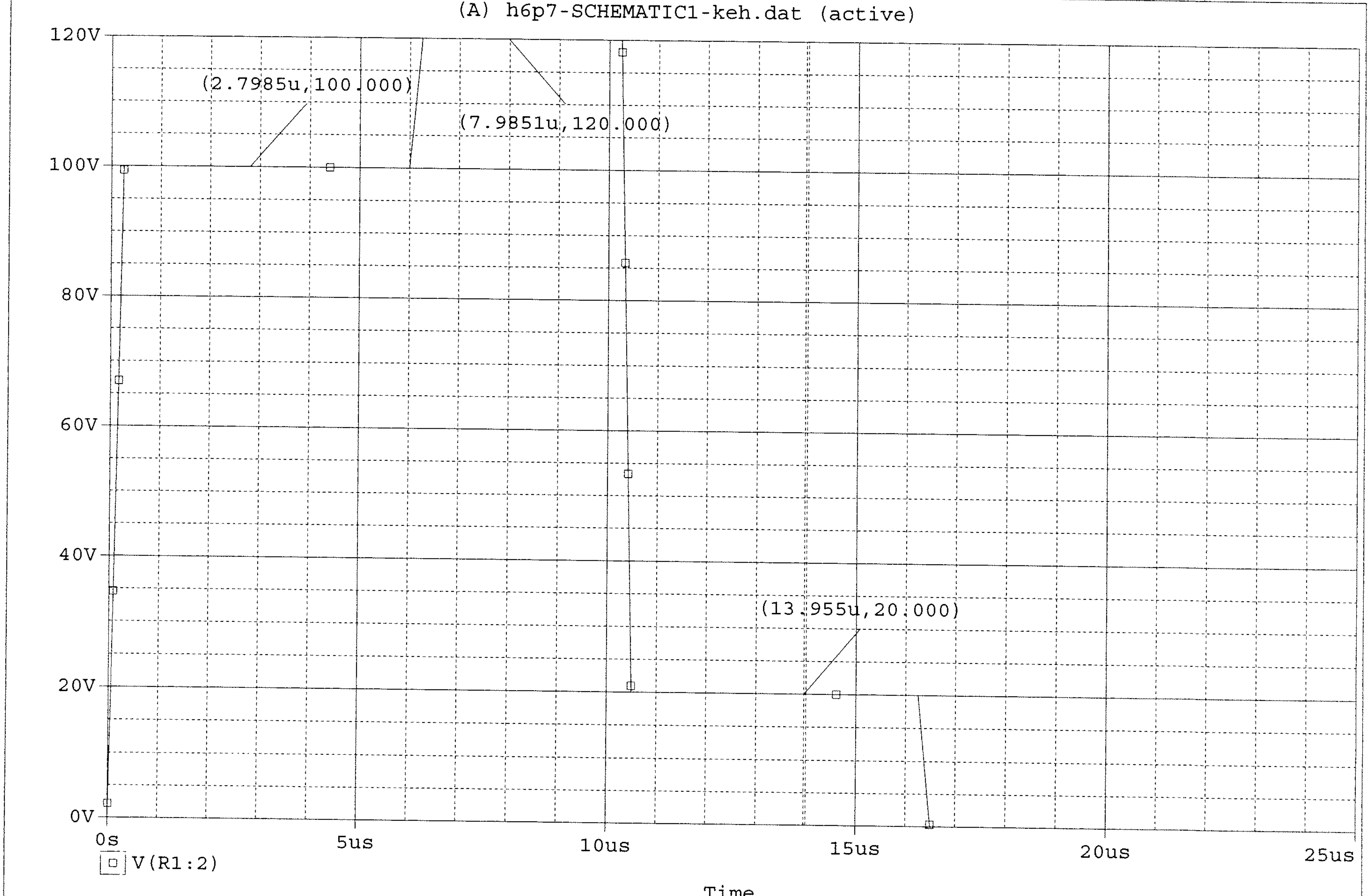

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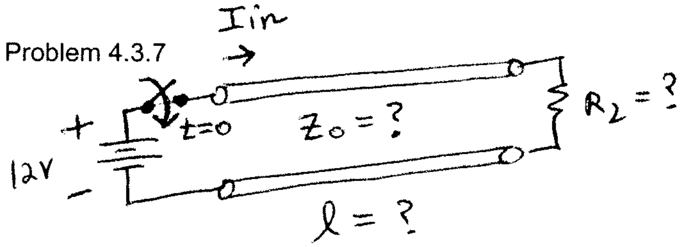
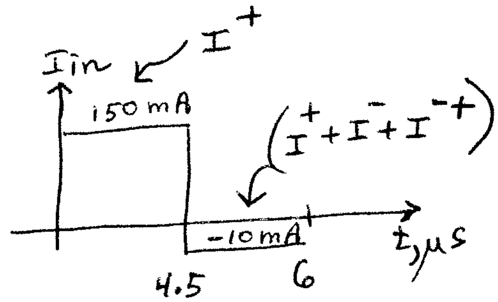
8. Text Problem 4.3.7





$Iplus1 := 150 \cdot mA \qquad Iplus1 + Iminus1 + Iplus2 = -10 \cdot mA$

$$current\_gammaL := -\left(\frac{RL - Zo}{RL + Zo}\right)$$

$Rs := 0 \quad \Rightarrow \quad current\_gammaG := 1$

$$Iminus = Iplus1 \cdot \left[-\left(\frac{RL - Zo}{RL + Zo}\right)\right]$$

$$Iplus2 = Iplus1 \cdot current\_gammaL \cdot current\_gammaG = Iplus1 \cdot \left[-\left(\frac{RL - Zo}{RL + Zo}\right)\right] \cdot 1$$

$Iplus1 \cdot Zo = 12 \cdot V$

$$Zo := 12 \cdot \frac{V}{Iplus1} \qquad \boxed{Zo = 80 \times 10^{0}\,\Omega}$$

$Iplus1 + Iminus + Iplus2 = -10 \cdot mA$

$$Iplus1 + Iplus1 \cdot \left[-\left(\frac{RL - Zo}{RL + Zo}\right)\right] + Iplus1 \cdot \left[-\left(\frac{RL - Zo}{RL + Zo}\right)\right] \cdot 1 = -10 \cdot mA$$

$$RL := Zo \cdot \frac{(3 \cdot Iplus1 + 10 \cdot mA)}{(Iplus1 - 10 \cdot mA)} \qquad \boxed{RL = 262.85714 \times 10^{0}\,\Omega}$$