ECE341 EMC Homework #5

1) For each of the following traveling wave functions, find the velocity of propagation both in magnitude and direction.

   a) $(t-0.1y)^2$ (b) $e^{-|0.02z+t|}$ (c) $[\sin(5x-1000t)]^2$

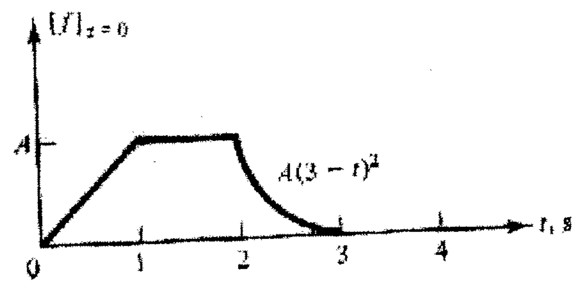
   Answers: ($10\mathbf{i}_y$ m/s, $-50\mathbf{i}_z$ m/s, $200\mathbf{i}_x$ m/s)

2) The time variation for z = 0 of a traveling wave function f(z,t) representing a traveling wave propagating in the +z direction with a velocity of 200 m/s is shown below. Find the value of the function for each of the following cases:

   a) z = 300 m, t = 1.9 s b) z = 400 m, t = 1.9 s c) z = 300 m, t = 2.9 s
   d) z = -400 m, t = 0.90 s e) z = -600 m, t = -0.9 s

   Hint: Consider, for example, Problem (2a), You are given the function vs. t for z = 0, so plot the wave function vs. t for z = 300 m (Do this by shifting the entire plot to the right by 1.5 s), then read off the value corresponding to t = 1.9 s.

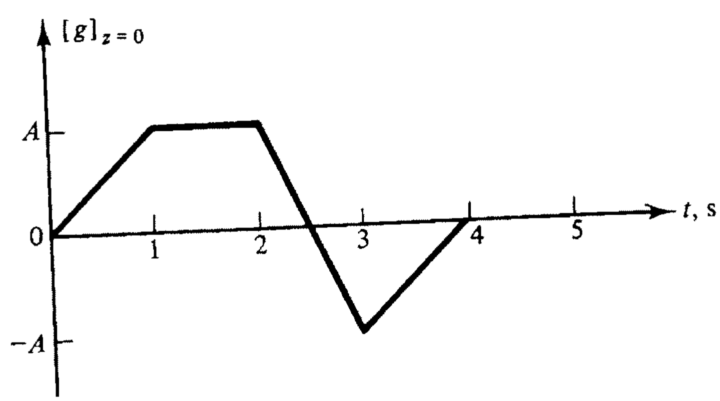
   Answers: (0.4A, 0, A, 0.01A, 0.81A)



3) The time variation for z = 0 of a function g(z,t) representing a traveling wave propagating in the –z direction with velocity magnitude of 300 m/s is shown in the figure below. Find the value of the function for each of the following cases:

   (a) z = 150 m, t = 0.3 s (b) z = 150 m, t = 2.3 s (c) z = 150 m, t = -1.3 s
   (d) z = -300 m, t = 2.3 s (e) z = -600 m, t = 5.3 s

   Answers: (0.8A, -0.6A, 0, A, -0.7A)



4) (a) Plot the "g" wave from Problem 3 as a function of distance at the fixed time t = 2 s. Remember this wave is traveling in the –z direction with a velocity magnitude of 300 m/s.
(Partial answer: your plot should show that g(2s,300m) = -A)
(b) Now assume the "g" wave is traveling in the +z direction with the same velocity magnitude. Once again plot the wave as a function of distance at the fixed time t = 2 s.
(Partial answer: your plot should show that g(2s,-300m) = -A.

5) Fill in the blanks in the Radio Frequency Classification Table below, assuming radio wave propagation in air (free space):

Designation	Frequency Range	Wavelength Range
ELF (Extremely Low Frequency)	30 – 30000 Hz	10,000 – 10~~0~~ km
VLF (Very Low Frequency)	3 – 30 kHz	________
LF (Low Frequency)	30 – 300 kHz	________
MF (Medium Frequency)	________	1000 – 100 m
HF (High Frequency)	________	100 – 10 m
VHF (Very High Frequency)	30 – 300 MHz	________
UHF (Ultra-High Frequency)	300 – 3000 MHz	________
Microwaves	________	30 – 1 cm
Millimeter Waves	________	10 – 1 mm

6) A uniform plane electromagnetic wave propagating in free space has the following electric field intensity (note that the axes have been rotated from the orientation used to derive the plane wave in class.)

$$\mathbf{E} = 37.7\cos(9\pi \times 10^7 t + 0.3\pi y)\mathbf{i}_x \text{ V/m}$$

Find: (a) the frequency in MHz of the E field observed by a fixed observer in space
(b) the wavelength (c) the direction of propagation of the wave
(d) the associated magnetic field intensity **H**
Answers: (45 MHz, 6.667 m, $-\mathbf{i}_y$ direction, $0.1\cos(9\pi \times 10^7 t + 0.3\pi y)\mathbf{i}_z$ A/m

7) Consider an EM wave traveling in the +z direction with $\mathbf{E} = A\cos(\omega t - \beta z)\mathbf{i}_x$. Show that the frequency observed by an observer moving at speed "v_{obs}" along the z axis in the +z direction is given by $\omega_{observed} = \omega(1 - v_{obs}/v_p)$, where $v_p = \omega/\beta = 1/\sqrt{\mu\varepsilon}$ = wave velocity in the medium. The difference between the source frequency ω and the frequency observed by a moving observer is called the Doppler-Shift, $\omega(v_{obs}/v_p)$. *(Hint: Recall that frequency is the time rate of change of the argument (phase) of the cosine function in the given E field expression. At a fixed point in space, z is a constant, and so the observed angular frequency is simply ω. But in this case, the observer is moving, so z is no longer a constant, and we must replace z by "$z_0 + v_{obs}t$", and differentiate the phase of the cosine function with respect to time.)*

8) For a police radar gun operating at a frequency of 30.0000000 GHz, find the (Doppler-shifted) frequency received at a speeding car that is traveling *directly toward* the police radar gun at 30 m/s. Repeat if the speeding car is traveling *directly away from* the police radar gun at 30 m/s. (Answers:30.000003 GHz, 29.999997 GHz)

9) Text Problem 3.24 (p. 118)
10) Text Problem 3.26 (p. 118)
11) Text Problem 3.31 (p. 119) Consider ONLY the following two cases: 1 MHz (AM broadcast frequency) and 100 MHz (FM broadcast frequency). *Hint: Since the wave attenuates as $e^{-\alpha z}$, the most accurate formula for skin depth δ is $1/\alpha$, where α is the real part of the propagation constant γ that is calculated directly from its definition, without any approximation being made.*

ECE 342 HW 5 Solution

1.(a) $(t - 0.1y)^2$

to follow a point on this wave, we require that

$(t_1 - 0.1y_1) = (t_2 - 0.1y_2)$

$\Rightarrow |\vec{v}_p| = \frac{\Delta y}{\Delta t} = \frac{y_2 - y_1}{t_2 - t_1} = 10\ m/s$

$\Rightarrow \boxed{\vec{v}_p = 10\,\hat{i}_y\ m/s}$

(b) $e^{-|0.02z + t|}$

to follow a point on this wave, we require that

$0.02z_1 + t_1 = 0.02z_2 + t_2$

$\Rightarrow |\vec{v}_p| = \frac{\Delta z}{\Delta t} = \frac{z_2 - z_1}{t_2 - t_1} = -50\ m/s$

$\Rightarrow \vec{v}_p = \boxed{-50\,\hat{i}_z\ m/s}$

(c) $[\sin(5x - 1000t)]^2$

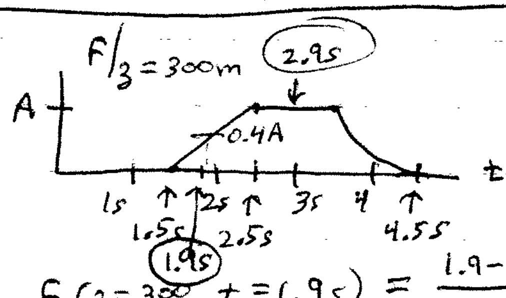
to follow a point on this wave, we require that

$5x_1 - 1000t_1 = 5x_2 - 1000t_2$

$|\vec{v}_p| = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{1000}{5} = 200\ m/s$

$\Rightarrow \vec{v}_p = \boxed{200\,\hat{i}_x\ m/s}$

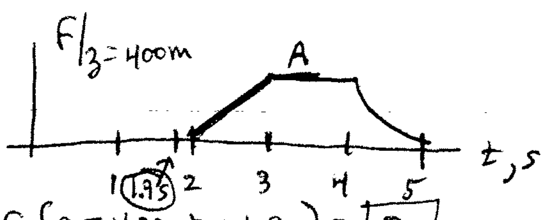
2.(a)



$F|_{z=0m}$ wave is delayed (shifted to right) by $\frac{300\ m}{200\ m/s} = 1.5\ s$

$F(z = 300\ m, t = 1.9s) = \frac{1.9 - 1.5}{1} A = \boxed{0.4A}$

2 (b)



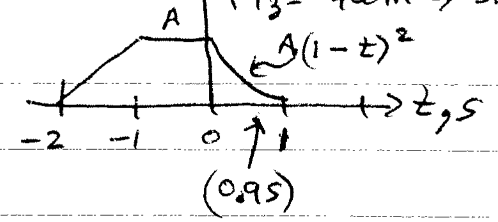
$F|_{z=0}$ wave is shifted to right by $\frac{400m}{200\ m/s} = 2s$

$F(z = 400\ m, t = 1.9s) = \boxed{0}$

HW5 (2)

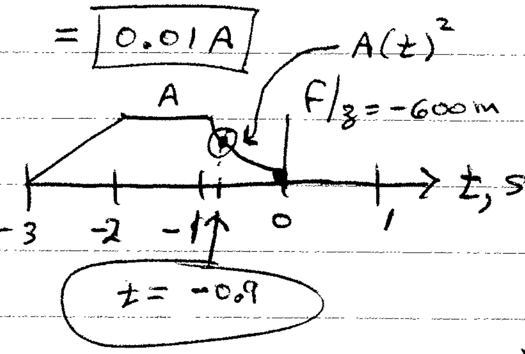
2(c) Same plot as 2(a) $f(z=300\text{m}, t=2.9\text{s}) = \boxed{A}$

2(d) $f|_{z=-400\text{m}} \Rightarrow$ shift $f|_{z=0}$ plot to left by $\frac{400}{200} = 2\text{s}$



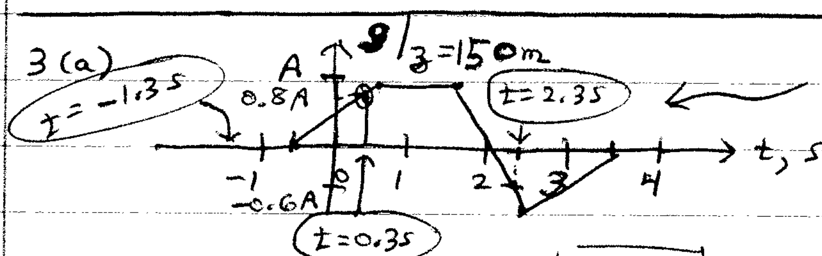
$$f(z=-400\text{m}, t=0.9\text{s}) = A(1-t)^2\Big|_{t=0.9} = A(1-0.9)^2$$
$$= \boxed{0.01A}$$

2(e)



$$f(z=-600\text{m}, t=-0.9\text{s}) = A(-0.9)^2 = \boxed{0.81A}$$

3(a)



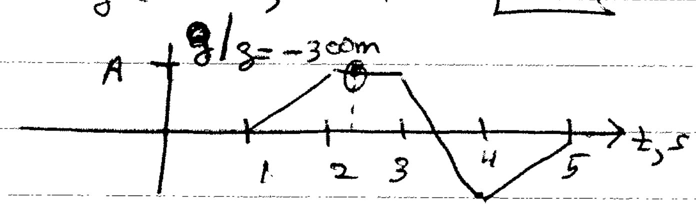
$g|_{z=0}$ wave is shifted to left by $\frac{150\text{m}}{300\text{m/s}} = \frac{1}{2}\text{s}$

$g(150\text{m}, 0.3\text{s}) = \boxed{0.8A}$

3(b) $g(150\text{m}, 2.3\text{s}) = \boxed{-0.6A}$

3(c) $g(150\text{m}, -1.3\text{s}) = \boxed{0}$

3(d)

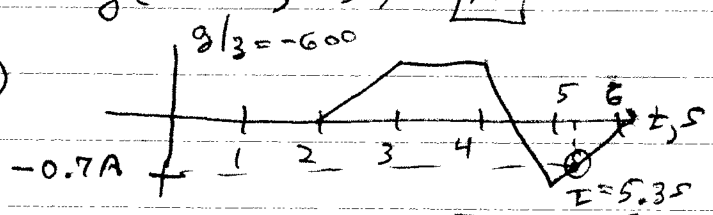


$g|_{z=0}$ is shifted to right by $\frac{300}{300} = 1\text{s}$

$g(-300\text{m}, 2.3\text{s}) = \boxed{A}$

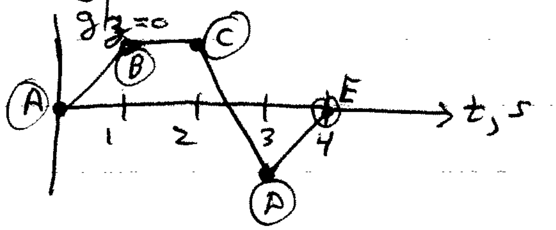
$g|_{z=0}$ is shifted to right by $\frac{600}{300} = 2\text{s}$

3(e)



$g(-600\text{m}, 5.3\text{s}) = \boxed{-0.7A}$

4. Referring to the $g|_{z=0}$ plot that is given



(Note g is traveling in $-z$ direction @ 300 m/s)

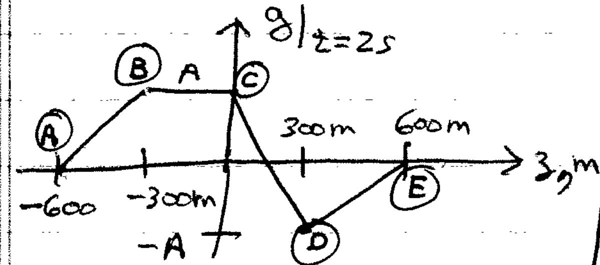
Point (A) travels to $z=(2-0)(-300) = -600$ m at $t=2$ s

(B) travels to $z=(2-1)(-300) = -300$ m at $t=2$ s

(C) is at $z=(2-2)(-300) = 0$ m at $t=2$ s

(D) is at $z=(2-3)(-300) = 300$ m at $t=2$ s

(E) is at $z=(2-4)(-300) = 600$ m at $t=2$ s



(Now assume g travels in $+z$ direction @ 300 m/s)

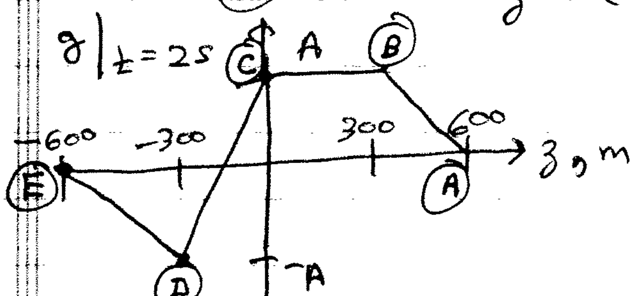
Point (A) Travels to $z=(2-0)(300) = 600$m @ $t=2$s

(B) $z=(2-1)(300) = 300$ m

(C) $z=(2-2)(300) = 0$ m

(D) $z=(2-3)(300) = -300$ m

(E) $z=(2-4)(300) = -600$ m

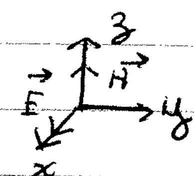


5. Fill in the table: Note $\lambda = v_p / f = \frac{300 \times 10^6 \text{ m/s}}{f}$

Designation	Frequency Range	Wavelength Range
ELF (Extremely Low Frequency)	30 – 30000 Hz	10,000 – 10 km
VLF (Very Low Frequency)	3 – 30 kHz	100 km – 10 km
LF (Low Frequency)	30 – 300 kHz	10 km – 1 km
MF (Medium Frequency)	300 – 3000 kHz	1000 – 100 m
HF (High Frequency)	3 MHz – 30 MHz	100 – 10 m
VHF (Very High Frequency)	30 – 300 MHz	10 m – 1 m
UHF (Ultra-High Frequency)	300 – 3000 MHz	1 m – 0.1 m
Microwaves	3 GHz – 30 GHz	30 – 1 cm
Millimeter Waves	30 GHz – 300 GHz	10 – 1 mm

Given the following $\vec{E}$ wave:

6. $\vec{E} = 37.7 \cos(\underbrace{9\pi \times 10^7}_{\omega} t + \underbrace{0.3\pi}_{\beta} y)\, \hat{i}_x$ V/m



($\vec{E} \times \vec{H}$ along the $-y$ direction)

(a) frequency = ?

$\omega = 9\pi \times 10^7$ r/s

$\Rightarrow f = \frac{\omega}{2\pi} = \frac{9\pi \times 10^7}{2\pi} = 4.5 \times 10^7$ Hz

$= \boxed{45 \text{ MHz}}$

(b) Wavelength = ?

$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.3\pi} = \boxed{6.667 \text{m}}$

(c) Because phase argument is "$\omega t + \beta y$" $\Rightarrow$

Direction of propagation is $\boxed{-\hat{i}_y \text{ direction}}$

$\boxed{\text{or "} -y \text{ direction"}}$

(d) Because this wave moves in "$-y$" direction, we must scale E_x by "-377Ω" to find H_z

$\vec{H} = \frac{E_x}{-377}(-\hat{i}_z) = \boxed{0.1 \cos(9\pi \times 10^7 t + 0.3\pi y)\, \hat{i}_z \frac{A}{m}}$

7. $\vec{E} = A\cos(\omega t - \beta z)$

If observer is in motion with speed "v_{obs}"

then $z = z_0 + v_{obs} t$

$$\vec{E} = A\cos(\omega t \underbrace{- \beta(z_0 + v_{obs} t)}_{\text{phase}})$$

$$\omega_{obs} = \frac{d}{dt}(\text{phase}) = \frac{d}{dt}(\omega t - \beta(z_0 + v_{obs} t))$$

$$= \omega - \beta v_{obs} = \omega\left(1 - \frac{v_{obs}}{\left(\frac{\omega}{\beta}\right)}\right) \quad (= v_p)$$

$$\boxed{\omega_{obs} = \omega\left(1 - \frac{v_{obs}}{v_p}\right)}$$ (Doppler shift formula)

(dividing both sides by 2π)

8. $f_{obs} = f\left(1 - \frac{v_{obs}}{v_p}\right)$

(a) Traveling toward radar gun, v_{obs} is negative

$$v_{obs} = -30 \text{ m/s}$$

$$f_{obs} = 30\,\text{GHz}\left(1 - \frac{(-30)}{(3\times10^8)}\right) = \boxed{30.000003 \text{ GHz}}$$

(b) Traveling away from radar gun, v_{obs} is positive

$$v_{obs} = 30 \text{ m/s}$$

$$f_{obs} = 30\,\text{GHz}\left(1 - \frac{30}{3\times10^8}\right) = \boxed{29.999997 \text{ GHz}}$$

9. (Text 3.24) Given $\vec{E} = 100\cos(\omega t - \beta x)\hat{i}_z$ V/m

Note we worked out case where $(E_x \hat{i}_x) \times (H_y \hat{i}_y)$ is along direction of propagation, $\hat{i}_z$. So in this problem $\vec{E} = E_z \hat{i}_z$, $\vec{E} \times \vec{H}$ along direction of propagation, $+\hat{i}_x$

$\Rightarrow$ $\vec{H}$ must lie along $-\hat{i}_y$, since $\boxed{\hat{i}_z \times (-\hat{i}_y) = \hat{i}_x}$

HW5 6

$$\overset{\text{Given}}{\lambda = 25\,cm} = 0.25\,m = \frac{2\pi}{\beta} \Rightarrow \beta = \frac{2\pi}{0.25} = 8\pi\ r/m$$

$$v_p = \frac{\omega}{\beta} \Rightarrow \omega = v_p \cdot \beta = \left(2\times10^8 \frac{m}{s}\right)\left(8\pi\ r/m\right) = 16\pi\times10^8\ r/s$$

$$f = \frac{\omega}{2\pi} = 8\times10^8\ Hz = \boxed{800\ MHz}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\epsilon_0}}\cdot\frac{1}{\sqrt{\epsilon_R}} = \frac{v_{p\,air}}{\sqrt{\epsilon_R}} = \frac{3\times10^8\ m/s}{\sqrt{\epsilon_R}}$$

Given

$$v_p = 2\times10^8\ m/s = \frac{3\times10^8\ m/s}{\sqrt{\epsilon_R}}$$

$$\epsilon_R = \left(\frac{3\times10^8}{2\times10^8}\right)^2 = \boxed{2.25}$$

Given

$$\boxed{\vec{E} = 100\cos(\underbrace{16\pi\times10^8}_{(=1.6\pi\times10^9)} t - \underbrace{8\pi}_{(=25.13)} x)\,\hat{i}_z\ V/m}$$

$$\eta = \frac{E_z}{-H_y} = \sqrt{\frac{\mu}{\epsilon}} = \frac{\sqrt{\mu_0/\epsilon_0}}{\sqrt{\epsilon_R}} = \frac{377\Omega}{\sqrt{2.25}} = 251.33\Omega$$

$$\boxed{\vec{H} = \underbrace{\frac{-100}{251.33}}_{(=-0.398)} \cos(16\pi\times10^8 t - 8\pi x)\,\hat{i}_y\ A/m}$$

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10. (Text P 3.26) $\sigma = 2\,S/m$, $\epsilon_R = 9$, $\mu_R = 16$, $f = 1\,GHz$

$$\bar{\gamma} = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j2\pi f\mu_R\mu_0(\sigma + j2\pi f\epsilon_R\epsilon_0)}$$

$$\bar{\gamma} = \underbrace{314.06}_{=\alpha} + j\underbrace{402.25}_{=\beta} \Rightarrow \boxed{\begin{array}{l}\alpha = 314.06\ 1/m \\ \beta = 402.25\ r/m\end{array}}$$

$$\bar{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = 195.1 + j152.3\ \Omega = 247.6\angle 38^\circ\ \Omega$$

(or)

HW5 7

11. (Text P. 3.31) (Wet Soil) $\Rightarrow \sigma = 10^{-2}$ S/m, $\epsilon_r = 15$, $\mu_r = 1$

(a) $f = 1\,MHz \Rightarrow \omega = 2\pi \times 10^6$ r/s

$$\bar{\gamma} = \sqrt{j\omega\mu_0\mu_R(\sigma + j\omega\epsilon_0\epsilon_R)} = \underbrace{0.1906}_{\alpha} + j\underbrace{0.2071}_{\beta}$$

$2\pi \times 10^6$

attenuation const

$$\Rightarrow \boxed{\alpha = 0.1906 \text{ 1/m}}, \quad \boxed{\beta = 0.2071 \text{ r/m}}$$ ← phase const.

$$\boxed{v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{0.2071} = 3.03 \times 10^7 \text{ m/s}}$$ ← phase velocity

$$\boxed{\delta = \frac{1}{\alpha} = \frac{1}{0.1906} = 5.25\,m}$$ ← skin depth

$$\boxed{\bar{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = 28.1\angle 42.6^\circ}$$ ← intrinsic impedance

@ $f = 100\,MHz \Rightarrow \omega = 2\pi \times 10^8$ r/s

$$\bar{\gamma} = 0.485 + j8.132 \Rightarrow \boxed{\begin{array}{l}\alpha = 0.485 \text{ 1/m} \\ \beta = 8.132 \text{ r/m}\end{array}}$$

$$\boxed{v_p = \frac{\omega}{\beta} = 7.72 \times 10^7 \text{ m/s}}$$

$$\boxed{\delta = \frac{1}{\alpha} = \frac{1}{0.485} = 2.06\,m}$$

$$\boxed{\bar{\eta} = 96.93\angle 3.42^\circ}$$

$$20\log\left[e^{-\alpha z}\right] = -20\,dB \Rightarrow \begin{cases} z = 12.08\,m \text{ @ } 1\,MHz \\ z = 4.74\,m \text{ @ } 100\,MHz \end{cases}$$

(z = distance wave travels until its amplitude is reduced by 20dB)