Why Bother Studying Transmission Line Theory?

We shall find that any application involving the conduction of an alternating current (freq = f) over a distance "ℓ" where

(1-1) $$\ell \gtrsim .1\,[\text{free-space wavelength}] = .1\left[\frac{300\,m/\mu s}{f}\right]$$

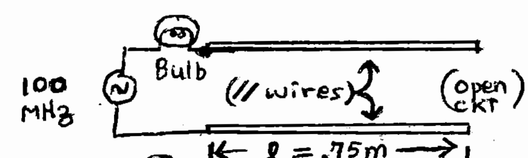
requires that the conduction path be treated as a "transmission line." Such situations occur frequently in the work of the ...... (a) RF Communications Engineer (Especially @ UHF/Microwaves)

(b) High-Speed Digital Computer Designer ($f = 1/T_{RISE}$)

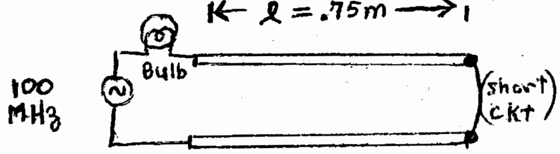
(c) Digital/Analog Communications System Designer (Large "ℓ")

> When equation (1-1) holds, conventional circuit theory **WILL NOT** suffice! Instead, techniques from ~~this~~ course must be used to understand the system behavior.

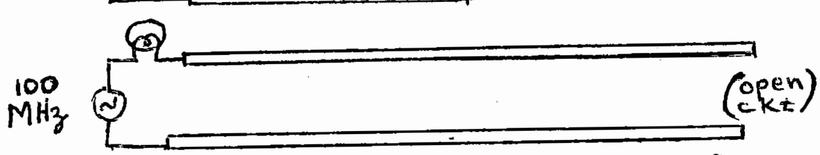
Example Try using AC CKT Theory to answer the following questions....



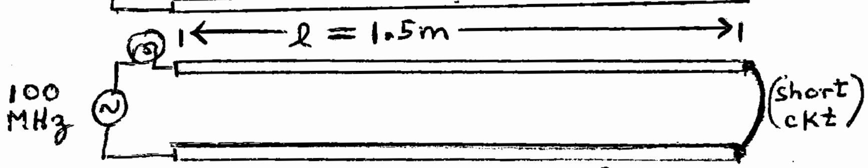
Will Bulb Light? (Ans: Yes!)



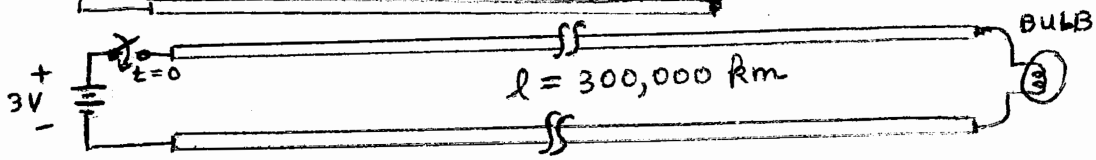
Will Bulb Light? (Ans: No!)



Will Bulb Light? (Ans: No)



Will Bulb Light? (Ans: Yes)



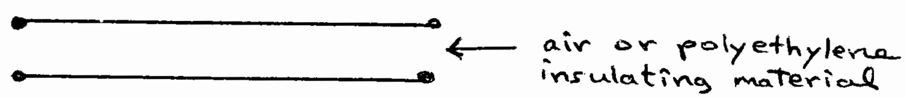
WHEN DOES BULB Light? (Ans: 1 sec)

I. Transmission Line Wave Equations

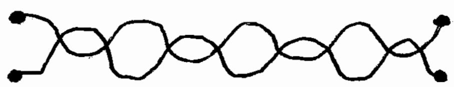
A transmission line is merely any 2-conductor system used for transferring electrical energy from point A to point B.

Familiar Examples of Transmission Lines:

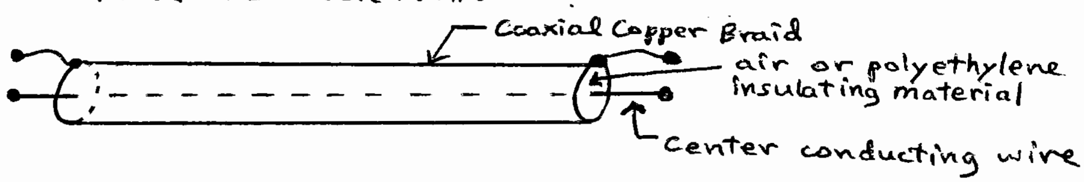
1. Parallel — Wire Line "TV Twin lead"



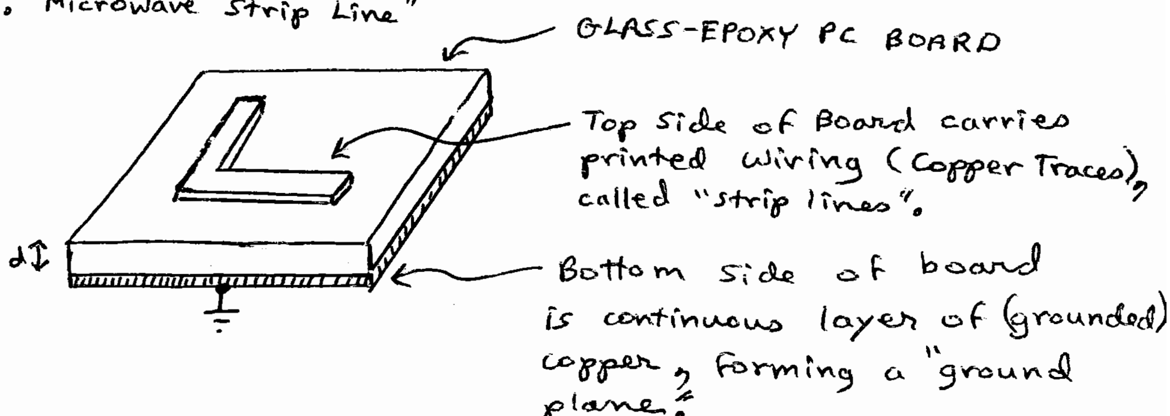
2. Twisted Pair Line "Telephone Cable"



3. Coaxial Line "Cable TV Line"



4. Microwave "Strip Line"



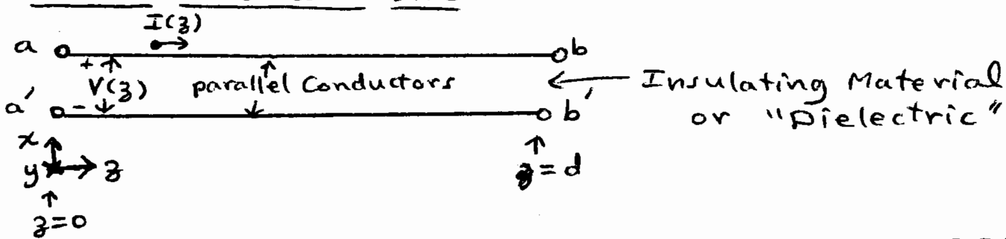
Electrically speaking, the strip line "sees" its reflected image at a distance "d" below the ground plane, thus this strip line technique is a convenient way to create "parallel conductor" transmission Lines on a PC board!

A. Transmission Line Modelling

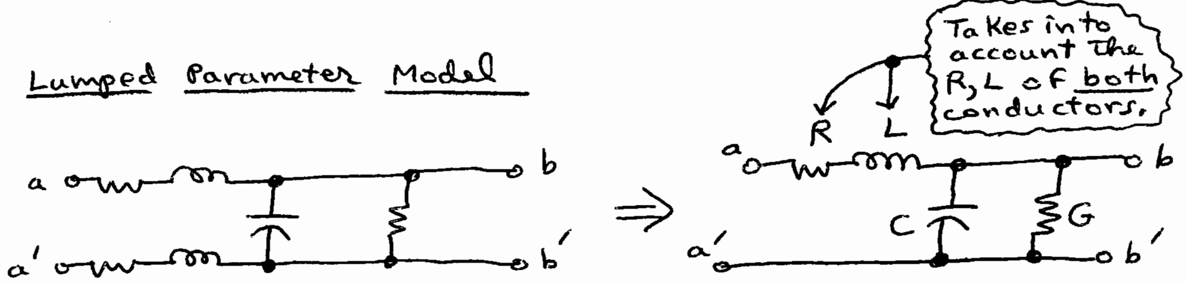
Some sort of mathematical model of a transmission line is needed if we desire to analyze a system that incorporates a transmission line.

One might first be tempted to model a transmission line in terms of its "lumped parameters", as shown below:

Actual Transmission Line



Lumped Parameter Model

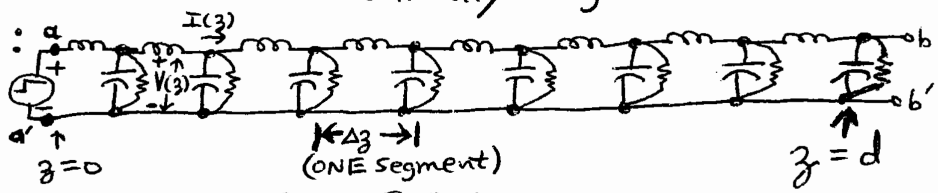


where R = net resistance of both conductors
L = net inductance of both conductors
C = net interconductor capacitance
G = net interconductor conductance (through the insulating material)

This lumped parameter model will be perfectly acceptable in applications involving the transmission of very slowly changing signals over relatively short line lengths (d), where it may be claimed that $V(z)$ and $I(z)$ remain constant over the entire transmission line length $(0 < z < d)$.

The problem with the lumped parameter model is that, in reality, the line inductance, resistance, capacitance, and interline conductance are actually continuously intermingled, or distributed! Thus, a more precise model is needed that does NOT lump these distributed properties into single, discrete R, L, C, and G component values.

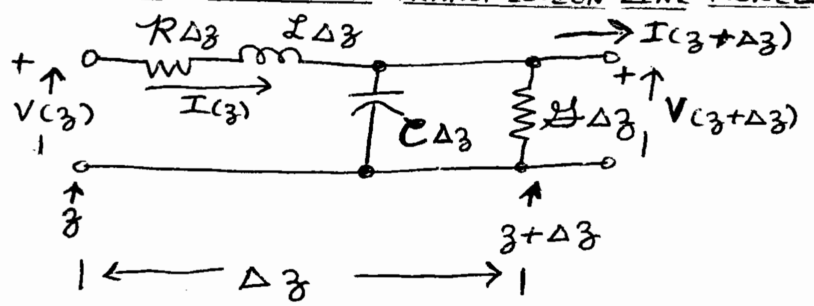
The "distributed parameter" model involves chopping the line up into a series of infinitesimally long



Consider the response of such a distributed parameter model to a step function. It is easy to see that $V(z)$ and $I(z)$ will NOT remain constant at a fixed time (t) for $0 < z < d$. Thus, a lumped model just doesn't suffice!

Each incremental line segment is modelled as shown below:

DISTRIBUTED PARAMETER TRANSMISSION LINE MODEL



where $\mathcal{R}$ = Conductor Resistance/unit length

$\mathcal{L}$ = Conductor Inductance/unit length

$\mathcal{C}$ = Interline Capacitance/unit length

$\mathcal{G}$ = Interline Conductance/unit length

[Note that $\mathcal{R}$, $\mathcal{L}$, $\mathcal{C}$, and $\mathcal{G}$ depend upon line construction, and are constant throughout the length of a parallel-conductor transmission line.]

B. Transmission Line Equations

Referring to the distributed parameter model for an incremental segment of transmission line of length Δz,

$$\text{KVL} \Rightarrow V(z+\Delta z) - V(z) = -\mathcal{R} I(z)\Delta z - \mathcal{L}\Delta z \frac{\partial I(z)}{\partial t}$$

$$\text{KCL} \Rightarrow I(z+\Delta z) - I(z) = -\mathcal{C}\Delta z \frac{\partial V(z+\Delta z)}{\partial t} - \mathcal{G}\Delta z V(z+\Delta z)$$

Dividing both sides by Δz $\Rightarrow$

$$\frac{V(z+\Delta z) - V(z)}{\Delta z} = -\mathcal{R} I(z) - \mathcal{L}\frac{\partial I(z)}{\partial t}$$

$$\frac{I(z+\Delta z) - I(z)}{\Delta z} = -\mathcal{C}\frac{\partial V(z+\Delta z)}{\partial t} - \mathcal{G} V(z+\Delta z)$$

Now letting the incremental line length, $\Delta z \to 0$

$$(1\text{-}2) \quad \frac{\partial V(z)}{\partial z} = -\mathcal{R} I(z) - \mathcal{L}\frac{\partial I(z)}{\partial t}$$

$$(1\text{-}3) \quad \frac{\partial I(z)}{\partial z} = -\mathcal{C}\frac{\partial V(z)}{\partial t} - \mathcal{G} V(z)$$

Lossy Transmission Line Equations

In an ideal (lossless) transmission line, $\mathcal{R} \to 0$ & $\mathcal{G}$ 0. In fact, well constructed transmission lines come very close to this lossless case! The equations for a lossless line become:

$$(1\text{-}4) \quad \frac{\partial V}{\partial z} = -\mathcal{L}\frac{\partial I}{\partial t}$$

$$(1\text{-}5) \quad \frac{\partial I}{\partial z} = -\mathcal{C}\frac{\partial V}{\partial t}$$

Lossless Transmission Line Equations

Differentiating (1-4) wrt "z" and (1-5) wrt "t" yields

$$(1\text{-}6) \quad \frac{\partial^2 V}{\partial z^2} = -\mathcal{L}\frac{\partial}{\partial z}\left[\frac{\partial I}{\partial t}\right] = -\mathcal{L}\frac{\partial}{\partial t}\left[\frac{\partial I}{\partial z}\right]$$

$$(1\text{-}7) \quad \frac{\partial}{\partial t}\left[\frac{\partial I}{\partial z}\right] = -\mathcal{C}\frac{\partial^2 V}{\partial t^2}$$

Substituting (1-7) into (1-6) yields

$$(1\text{-}8) \qquad \boxed{\frac{\partial^2 V}{\partial z^2} = \mathcal{L}\,\mathcal{C}\,\frac{\partial^2 V}{\partial t^2} \quad \text{(Voltage Wave Eqn)}}$$

Likewise, differentiating (1-4) wrt "t" and (1-5) wrt "z" $\Rightarrow$

$$(1\text{-}9) \qquad \boxed{\frac{\partial^2 I}{\partial z^2} = \mathcal{L}\,\mathcal{C}\,\frac{\partial^2 I}{\partial t^2} \quad \text{(Current Wave Eqn)}}$$

C. Solution to the Transmission Line Wave Equations

The solution to (1-8) is given by

$$(1\text{-}10) \qquad V(z,t) = f(t - z\sqrt{\mathcal{L}\mathcal{C}}) + g(t + z\sqrt{\mathcal{L}\mathcal{C}})$$

where f and g are arbitrary functions of $(t \pm z\sqrt{\mathcal{L}\mathcal{C}})$.

Verification: Assume V is of the form (1-10). Then the left side of eqn 1-8 is given by (Let $u = t - z\sqrt{\mathcal{L}\mathcal{C}}$ and $w = t + z\sqrt{\mathcal{L}\mathcal{C}}$)

$$\frac{\partial V}{\partial z} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial z} + \frac{\partial g}{\partial w}\cdot\frac{\partial w}{\partial z} \Rightarrow \frac{\partial^2 V}{\partial z^2} = \frac{\partial^2 f}{\partial u^2}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{\partial f}{\partial u}\left(\frac{\partial^2 u}{\partial z^2}\right) + \frac{\partial^2 g}{\partial w^2}\left(\frac{\partial w}{\partial z}\right)^2 + \frac{\partial g}{\partial w}\left(\frac{\partial^2 w}{\partial z^2}\right)$$

But Note: $\frac{\partial u}{\partial z} = -\sqrt{\mathcal{L}\mathcal{C}}$, $\frac{\partial w}{\partial z} = +\sqrt{\mathcal{L}\mathcal{C}}$, $\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 w}{\partial z^2} = 0$; Hence the above simplifies to

$$\frac{\partial^2 V}{\partial z^2} = \mathcal{L}\mathcal{C}\left[\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 g}{\partial w^2}\right] = \mathcal{L}\mathcal{C}\left[\frac{\partial^2 f}{\partial u^2}\underbrace{\left(\frac{\partial u}{\partial t}\right)^2}_{=1} + \frac{\partial^2 g}{\partial w^2}\underbrace{\left(\frac{\partial w}{\partial t}\right)^2}_{=1}\right]$$

$$\frac{\partial^2 V}{\partial z^2} = \mathcal{L}\mathcal{C}\left[\frac{\partial^2 f}{\partial t^2} + \frac{\partial^2 g}{\partial t^2}\right] = \mathcal{L}\mathcal{C}\,\frac{\partial^2 V}{\partial t^2}$$

$\leftarrow$ NOTE THAT THIS IS SIMPLY THE RIGHT HAND SIDE OF WAVE EQUATION 1-8 !!

The function $f(t - z\sqrt{\mathcal{L}\mathcal{C}})$ represents a travelling wave moving in the positive z direction, and will thus be called a "(+) wave." To illustrate this, consider two pairs of t and z, say (t_1, z_1) and (t_2, z_2). In order to "follow" a certain point on the travelling wave, say this point is at $z = z_1$ @ time t_1, and at $z = z_2$ @ time t_2; then we must demand that

$$t_1 - z_1\sqrt{\mathcal{L}C} = t_2 - z_2\sqrt{\mathcal{L}C}$$

so that the function $f(t - z\sqrt{\mathcal{L}C})$ has the same value for these two pairs of t and z.

$$\Rightarrow (z_2 - z_1)\sqrt{\mathcal{L}C} = t_2 - t_1$$

$$\Rightarrow \frac{z_2 - z_1}{t_2 - t_1} = \frac{1}{\sqrt{\mathcal{L}C}}$$

Thus in a time $t_2 - t_1$, the function is displaced by the distance $(t_2 - t_1)\frac{1}{\sqrt{\mathcal{L}C}}$. The velocity of propagation of the travelling wave is $1/\sqrt{\mathcal{L}C}$. We denote this by v_p.

(1-11) $$\boxed{v_p = \frac{1}{\sqrt{\mathcal{L}C}}}$$

Similarly, $g(t + z\sqrt{\mathcal{L}C})$ represents a travelling wave moving in the negative z direction, and will be called a "(−) wave". To illustrate this, we set

$$t_1 + z_1\sqrt{\mathcal{L}C} = t_2 + z_2\sqrt{\mathcal{L}C}$$

$$\Rightarrow (z_2 - z_1)\sqrt{\mathcal{L}C} = t_1 - t_2$$

$$\Rightarrow \frac{z_2 - z_1}{t_2 - t_1} = -\frac{1}{\sqrt{\mathcal{L}C}}$$

The minus sign indicates propagation in the negative z direction.

We may now write the solution for the voltage waves on a lossless line as:

(1-12) $$V(z, t) = V^+(t - z/v_p) + V^-(t + z/v_p) = V^+ + V^-$$

where the actual functional form of V^+ and V^- depend on the line excitation + boundary conditions.

The corresponding solution for the current waves, $I(z,t)$, may be found by substituting (1-12) into either of the transmission line equations (1-4) or (1-5). using (1-4),

$$\frac{\partial V}{\partial z} = -\mathcal{L}\frac{\partial I}{\partial t}\,,\quad \text{we have (by chain rule)}$$

$$\frac{\partial I}{\partial t} = -\frac{1}{\mathcal{L}}\left[-\frac{1}{v_p}\,\frac{\partial V^+(t-z/v_p)}{\partial(t-z/v_p)} + \frac{1}{v_p}\,\frac{\partial V^-(t+z/v_p)}{\partial(t+z/v_p)}\right]$$

Integrating wrt t, and using $v_p = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}}$ we get

$$I = \frac{1}{\sqrt{\mathcal{L}/\mathcal{C}}}\left[V^+(t-z/v_p) - V^-(t+z/v_p)\right]$$

or

$$I = \underbrace{\frac{V^+}{Z_0}}_{\text{"}I^+\text{"}} + \underbrace{\frac{-V^-}{Z_0}}_{\text{"}I^-\text{"}} = I^+ + I^- \qquad (1-13)$$

Where $Z_0 \triangleq \sqrt{\mathcal{L}/\mathcal{C}} = \frac{V^+}{I^+} = \frac{-V^-}{I^-}$, and is called the

<u>characteristic</u> <u>impedance</u> of the transmission line.

Note that Z_0 is purely real ... but it does <u>NOT</u> <u>dissipate power</u> in the sense of a physical resistor at a certain point on a Line would dissipate power as heat. Rather, the characteristic impedance of a line determines the rate at which energy is <u>propagated</u> down the (lossless) line! Z_0 simply may be thought of as the ratio of the (+) Voltage wave to (+) Current wave on a transmission line.

In Summary, the soln to the wave equations permits the voltages and currents on a transmission line to be given by

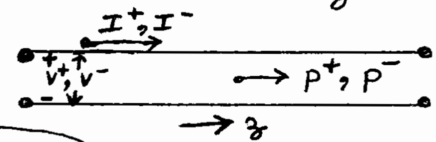
$$V = V^+ + V^- \quad \text{(both a (+) and (-) wave!)}$$

$$I = I^+ + I^-$$

<u>where</u> $I^+ = \frac{V^+}{Z_0}$ and $I^- = -\frac{V^-}{Z_0}$

A note on polarities and reference directions

Note that the polarities for the line voltage and direction of current flow are the same for both (+) and (−) waves. The power flow is then always referenced in the +z direction for both waves. Note that

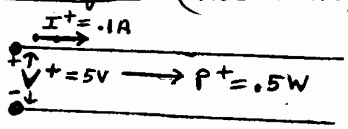


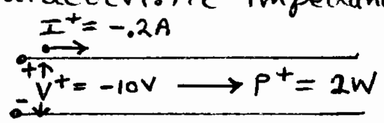
$$P^+_{(z)} = V^+_{(z)} I^+_{(z)} = (V^+)\left(\frac{V^+}{Z_0}\right) = \frac{(V^+_{(z)})^2}{Z_0} \leftarrow \text{(power carried by (+) wave at point } z\text{)}$$

$$P^-_{(z)} = V^-_{(z)} I^-_{(z)} = (V^-)\left(-\frac{V^-}{Z_0}\right) = -\frac{(V^-_{(z)})^2}{Z_0} \leftarrow \text{(power carried by (−) wave at point } z\text{)}$$

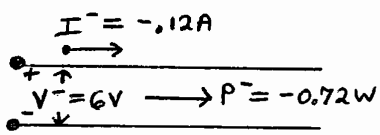
The minus sign in the result for P^- ensures that the actual power flow for the (−) wave is in the negative z direction.

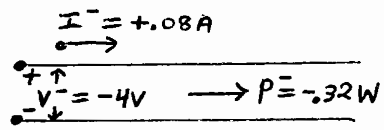
Examples (Assuming the line characteristic impedance, $Z_0 = 50\Omega$)





$\left(I^+ = \frac{V^+}{50\Omega}\right)$



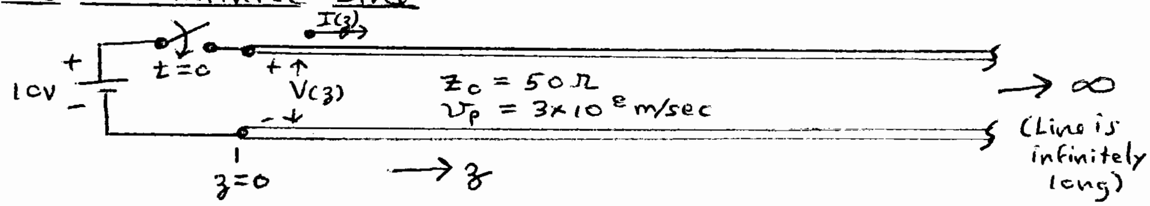


$\left(I^- = \frac{-V^-}{50\Omega}\right)$

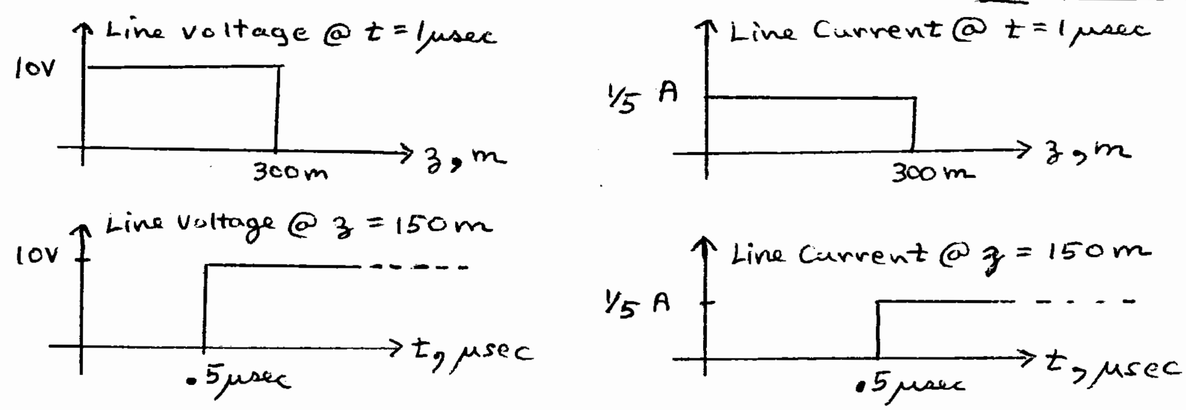
As long as we follow the conventions for the polarities of the voltages and currents, the numerical result will always come out such that P^+ is positive and P^- is negative.

II. Transient Waves on a Transmission Line

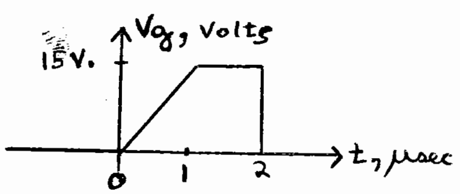
A. The semi-infinite Line

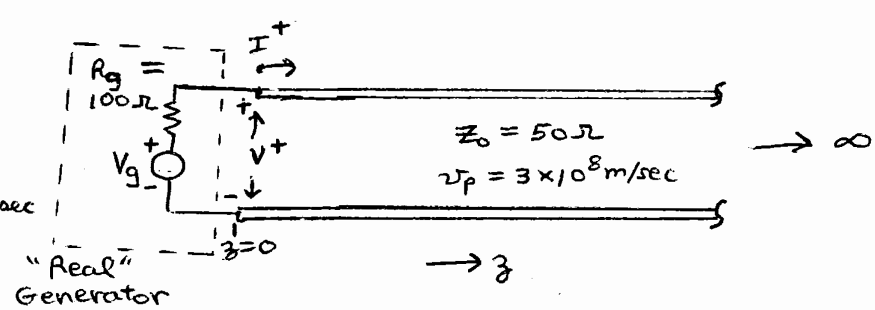


We assume that the switch is closed @ $t=0$. The general solution for the line voltage and current consists of a superposition of (+) and (−) waves. But in this case, a (−) wave will NOT be present, since there would have to be a source, or some kind of termination which would reflect part of the power in the (+) wave located at the far (right) end of the line. An incident (+) wave is set up at $z=0$ by the closing of the switch. Given that for this line $v_p = 3\times10^8$ m/sec, we can draw sketches of line voltage and line current versus either distance (at a fixed time) or versus time (at a fixed distance). For Example:



Another Example:





A (+) wave voltage appears at $z=0$ which is a fraction of the source voltage waveform. To find this fraction, consider the ckt at the input of the line, writing a KVL equation:

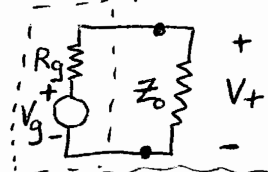
$$KVL \Rightarrow V_g - I^+ R_g - V^+\big|_{z=0} = 0$$

$$\text{or} \quad V_g - \frac{V^+}{Z_0} R_g - V^+\big|_{z=0} = 0$$

$$V^+\big|_{z=0}\left(1 + \frac{R_g}{Z_0}\right) = V_g$$

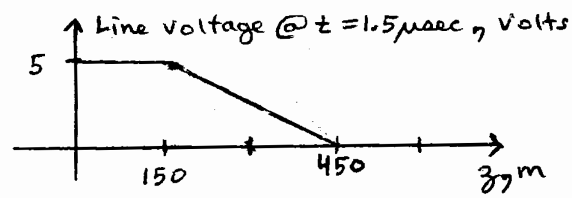
$$\Rightarrow \quad V^+\big|_{z=0} = V_g \left(\frac{Z_0}{R_g + Z_0}\right)$$

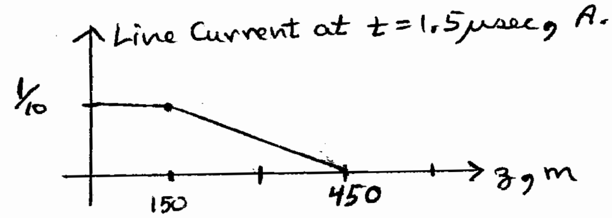
Equiv ckt seen by Generator:

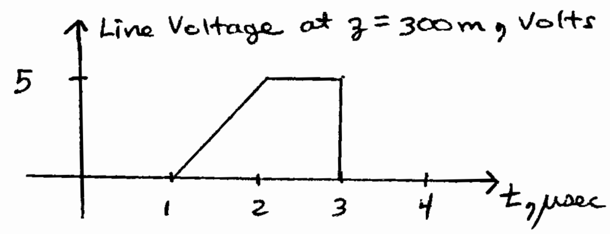


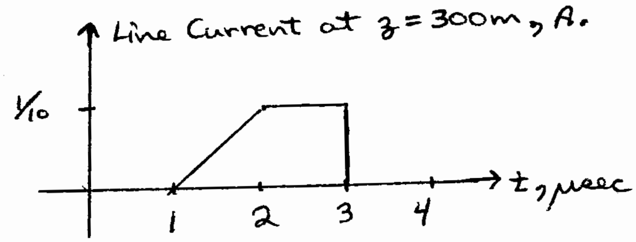
Thus the semi-infinite line behaves like a resistance equal to its characteristic impedance at the line's input terminals!

We can now draw sketches of the line voltage and line current as follows:



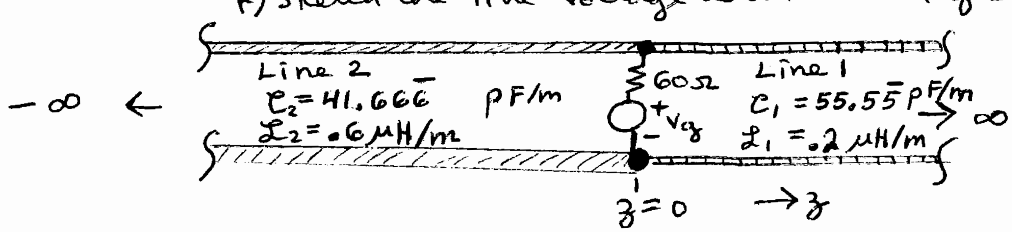


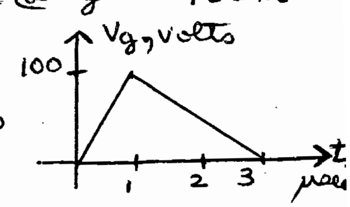




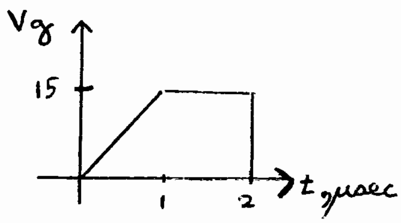
HW #1 In the system shown below, the lines are lossless ($\mathcal{R}, \mathcal{G} = 0$)

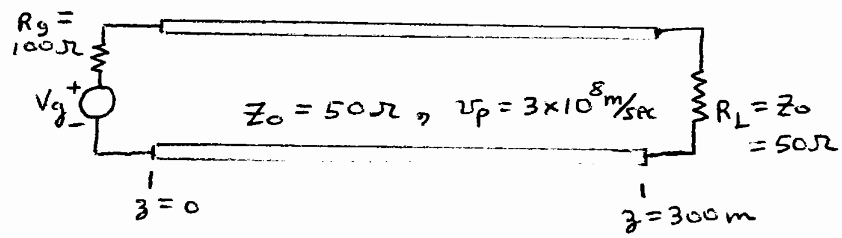
a) The phase velocity, v_p, for line 1 and line 2
b) The characteristic impedance of line 1 and line 2
c) Sketch the line voltage as a function of z @ $t = 2\mu sec$
d) Sketch the line current as a function of z @ $t = 4\mu sec$
e) Sketch the line voltage as a function of t @ $z = 300m$
f) Sketch the line voltage as a function of t @ $z = -400m$





B. Line of Finite Length, terminated in its Characteristic Impedance

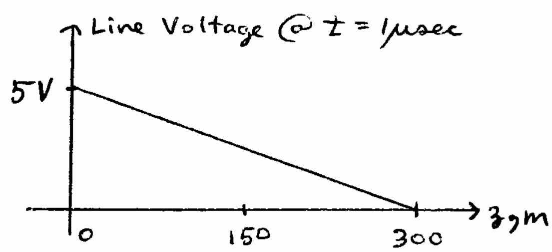


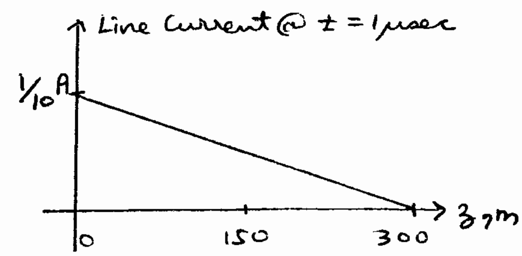


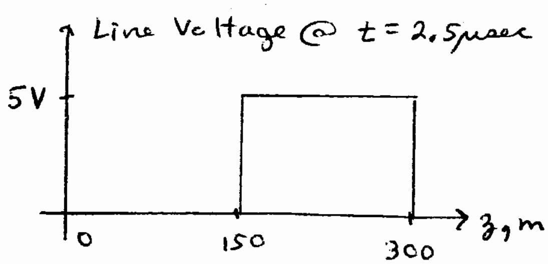
A (+) wave appears @ $z = 0$ $= \left(\frac{Z_0}{R_g + Z_0}\right) V_g$. It travels down the line at a rate of $v_p = 300$ m/μsec. The leading edge of the (+) wave reaches the load (R_L) at $t = 1$ μsec.

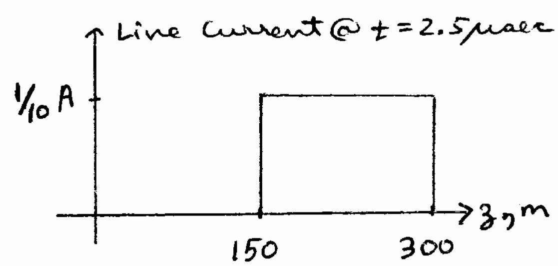
Since the ratio of this (+) voltage wave ($V+$) to (+) current wave ($I+$) $= Z_0$, which is equal to R_L, this (+) wave satisfies the boundary condition imposed by R_L at the load end, hence V^+ & I^+ simply are absorbed by the load, with no reflection back from this load occurring. Thus no (−) waves are present on this line.

We may draw sketches of line voltage and current as follows:



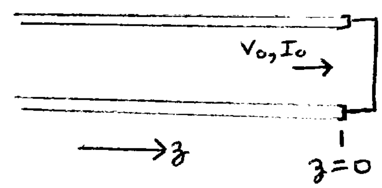






C. Line Terminated in a Short Circuit

Assume that a (+) wave of voltage V_o and current I_o ($= V_o/Z_o$) is incident on the short circuit at $t = 0$.



Since the short circuit requires that the net voltage across it $= 0$, which is not satisfied by the (+) wave alone, a (−) wave will be set up such that

$$V_o + V^- = 0 \quad \Rightarrow \quad V^- = -V_o$$

Thus the (−) wave, or reflected wave, voltage is exactly the negative of the (+) wave, or incident wave, voltage.

The (−) wave current is given by

$$I^- = \frac{-V^-}{Z_o} = -\frac{-V_o}{Z_o} = \frac{V_o}{Z_o} = I_o = \left(\begin{array}{c}\text{Incident}\\ \text{Current Wave}\end{array}\right)$$

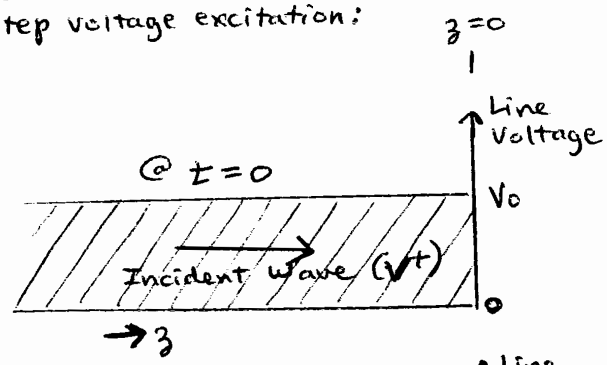
Note that the current wave is reflected <u>without</u> inversion! (The reflected current equals the incident current.)

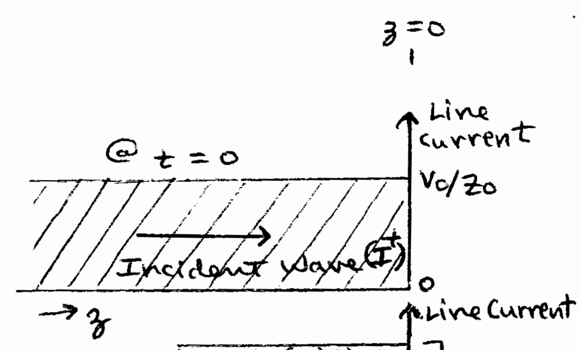
The power in the reflected wave (V^-, I^-) is given by

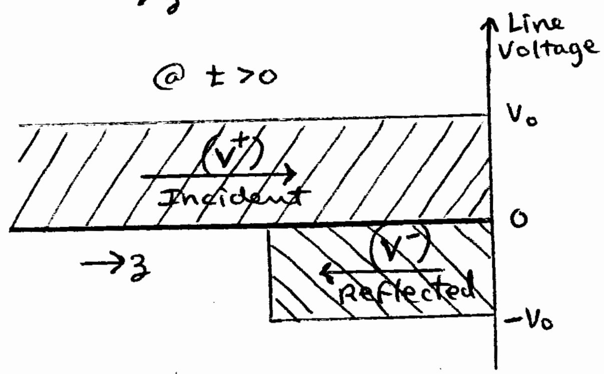
$$P^- = (V^-)(I^-) = (-V_o)(I_o) = -P^+$$

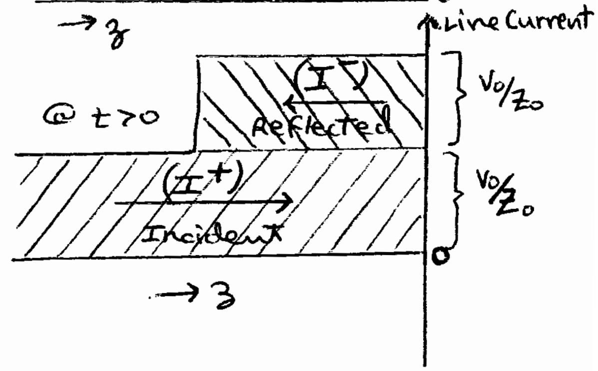
Thus the entire incident power is reflected back toward the source, as might be expected, since a short circuit (a 0-Ω resistor) cannot be expected to absorb (dissipate) any power.

We may draw sketches of line voltage and line current for this short-circuited line as follows, assuming simple step voltage excitation:

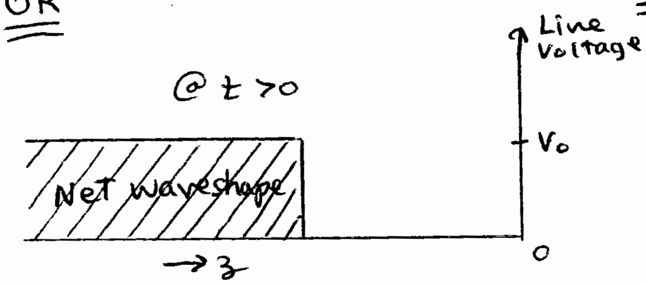




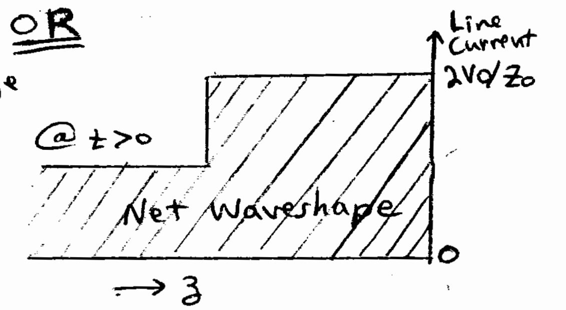




OR

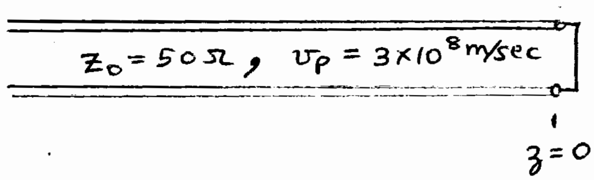


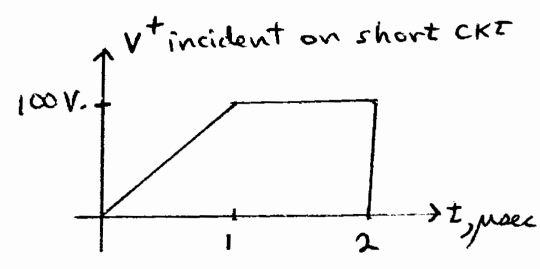
OR



An example involving a more "interesting" waveshape:

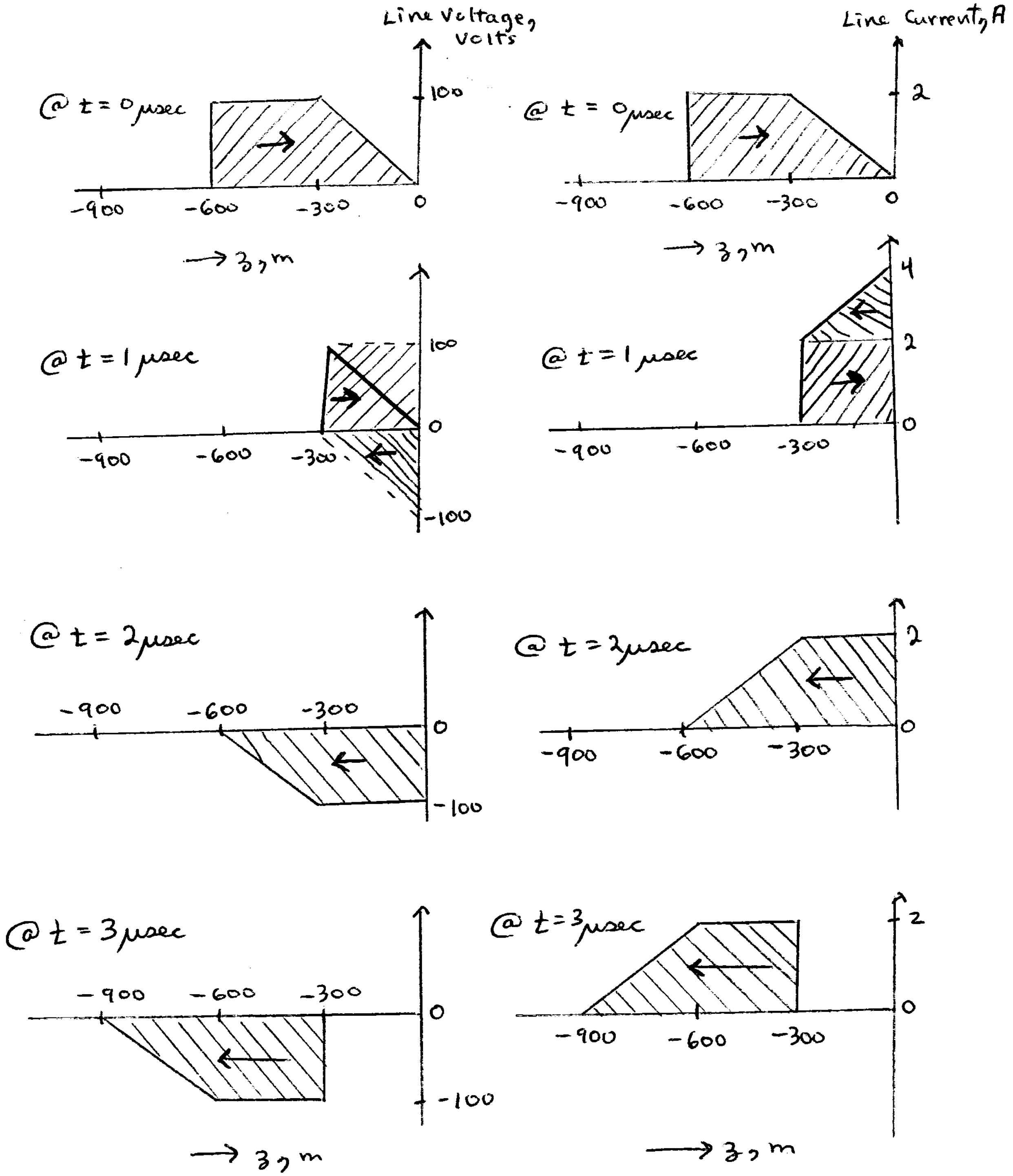
Consider





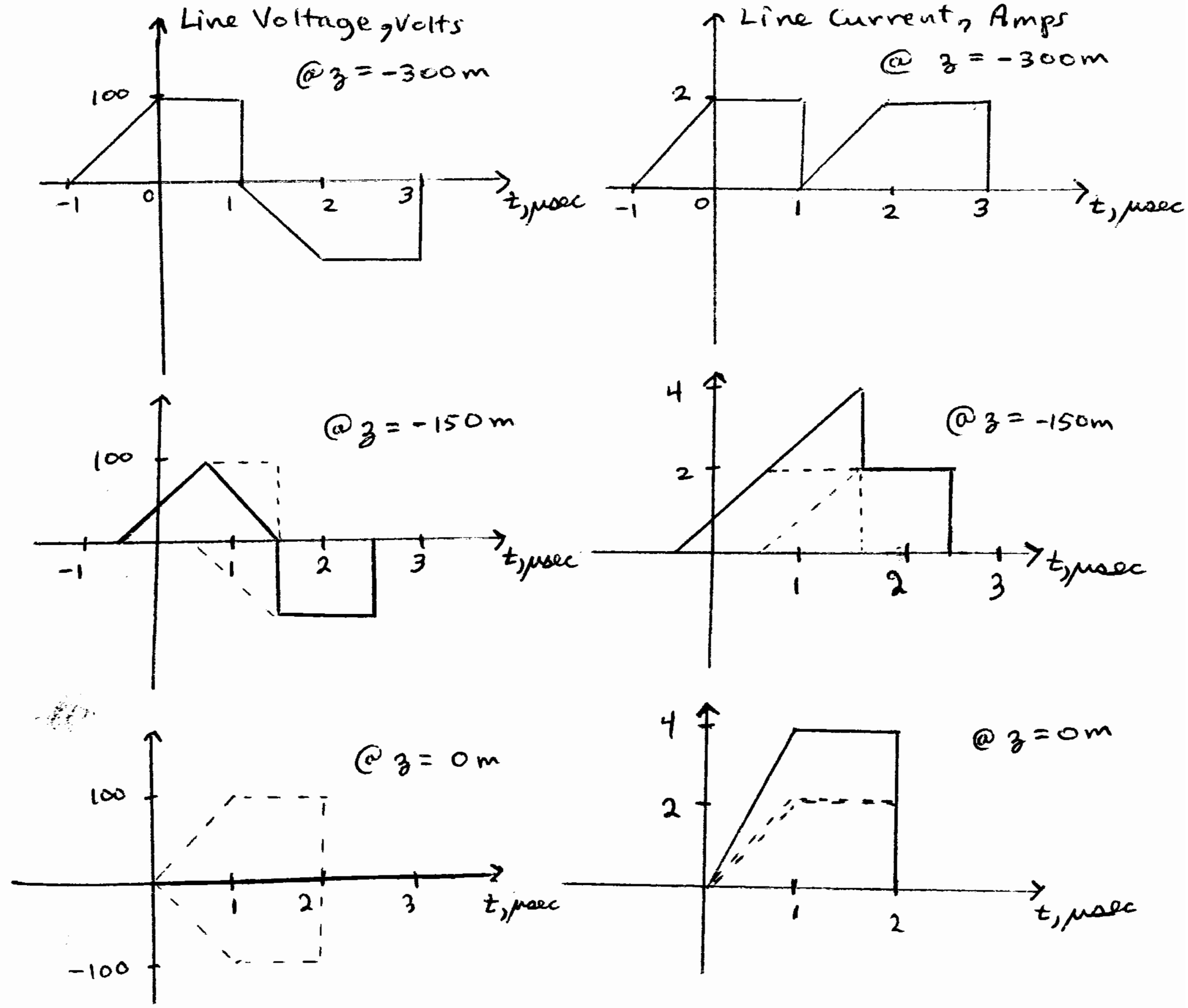
Line Voltage and Current plotted vs Distance

⇒ "SNAPSHOTS at known instant of time"



Line voltage and Current plotted vs time

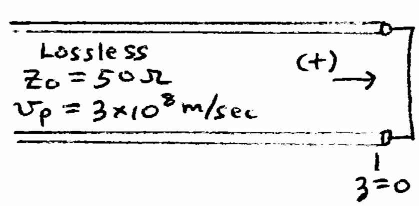
⇒ "Oscilloscope readings at a given distance down line"

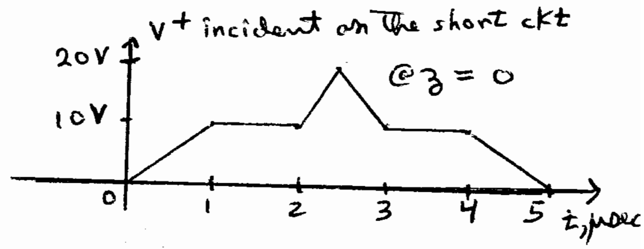


Homework # 2 A (+) wave with the voltage waveform shown below is incident on the short ckt termination with its leading edge at $t = 0$. SKETCH

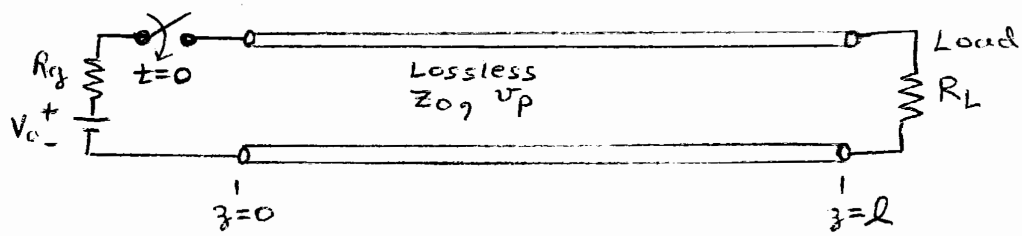
(a) Line voltage & Line current as a function of z @ $t = 3\mu sec$ and $t = 4\mu sec$

(b) Line voltage & Line current as a function of t @ $z = -1200m$ and $z = -300m$

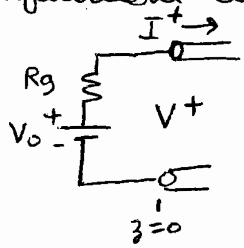




D. More General Case: Line Terminated in Arbitrary Load Resistance



The moment the switch is closed at $t=0$, a (+) wave (V^+, I^+) will be set up and begin travelling down the line. Until a reflection from the load arrives back at the source, the source is not affected by what is connected to the far end of the line. Thus we have the following equivalent circuit at $z=0$, $t=0^+$:



$V_o - I^+R_g - V^+ = 0$ (Boundary Condition (KVL))

$I^+ = V^+/Z_o$ (Transmission Line Requiremen

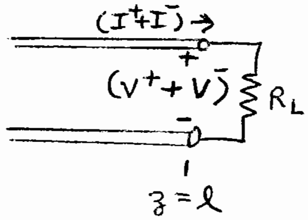
Solving for V^+, we obtain

$$\boxed{V^+ = V_o\left(\frac{Z_o}{R_g+Z_o}\right), \quad I^+ = \frac{V^+}{Z_o} = \frac{V_o}{R_g+Z_o}}$$

Thus the voltage source (initially) views the characteristic impedance of the line across $z=0$.

The (+) wave reaches the termination at time $t = \ell/v_p$. Since the ratio of the voltage to the current in the (+) wave is equal to Z_o, whereas the load resistance requires that the voltage-to-current ratio at the termination be equal to R_L, a (−) wave (reflected wave) will be set up. Let the voltage and current associated with this wave be V^- and I^-.

Thus, we have the following equivalent circuit at $z = \ell$, $t = \ell / v_p$



Voltage across $R_L = V^+ + V^-$

Current through $R_L = I^+ + I^-$

According to the (Ohm's Law) Boundary condition at $z = \ell$

$$V^+ + V^- = R_L (I^+ + I^-)$$

But

$$I^+ = \frac{V^+}{Z_o} \quad \text{and} \quad I^- = -\frac{V^-}{Z_o} \quad \text{(transmission line requirements)}$$

Thus

$$V^+ + V^- = R_L \left(\frac{V^+}{Z_o} - \frac{V^-}{Z_o} \right)$$

or,

$$V^- \left(1 + \frac{R_L}{Z_o}\right) = V^+ \left(\frac{R_L}{Z_o} - 1 \right)$$

or,

$$V^- = V^+ \left(\frac{R_L - Z_o}{R_L + Z_o} \right)$$

Let us now define a quantity known as the "voltage reflection coefficient," denoted by the symbol Γ_V, as the ratio of the reflected voltage (V^-) to the incident voltage (V^+). Thus

$$\boxed{\Gamma_V \triangleq \frac{V^-}{V^+} = \frac{R_L - Z_o}{R_L + Z_o}}$$

Note that the current reflection coefficient, Γ_I, may be defined as

$$\Gamma_I = \frac{I^-}{I^+} = \frac{-V^-/Z_o}{V^+/Z_o} = \frac{-V^-}{V^+} = -\Gamma_V$$

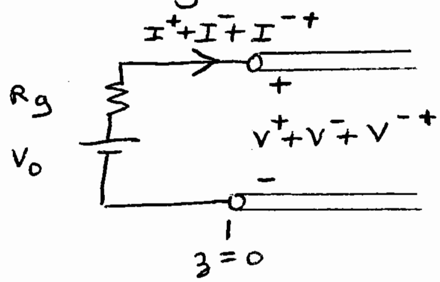
Finally, the power reflection coefficient is given by

$$\Gamma_p = \frac{V^- I^-}{V^+ I^+} = \left(\frac{V^-}{V^+}\right)\left(\frac{I^-}{I^+}\right) = \Gamma_V \Gamma_I = -\Gamma_V^2$$

The minus sign indicates that power is carried in the negative z direction by the reflected wave.

The reflected (−) wave travels back to the source end. It arrives there at time $t = \frac{2\ell}{v_p}$. Since the boundary condition at $z=0$, which was satisfied initially by the (+) wave alone, is now violated by the added presence of this (−) wave, a "reflection of the reflection" will be set up, travelling back toward the load.

We will call this wave the (−+) wave (the (+) wave resulting from the (−) wave). Letting the voltage and current associated with this wave be denoted V^{-+} and I^{-+}. We then have the following situation at $t = 2\ell/v_p$:



Total Line Voltage$\big|_{z=0} = V^+ + V^- + V^{-+}$

Total Line Current$\big|_{z=0} = I^+ + I^- + I^{-+}$

According to the boundary condition (by KVL)

$$V^+ + V^- + V^{-+} = V_0 - R_g\,(I^+ + I^- + I^{-+})$$

But the transmission line properties indicate that

$$I^+ = \frac{V^+}{Z_0}, \quad I^- = \frac{-V^-}{Z_0}, \ \underline{\text{and}} \ I^{-+} = \frac{V^{-+}}{Z_0}$$

<u>Thus</u>

$$V^+ + V^- + V^{-+} = V_0 - \frac{R_g}{Z_0}\left(V^+ - V^- + V^{-+}\right)$$

We know that $V^+ = V_0\left(\frac{Z_0}{Z_0 + R_g}\right)$, thus we have

$$\frac{V_0 Z_0}{R_g + Z_0} + V^- + V^{-+} = V_0 - V_0\left(\frac{R_g}{R_g + Z_0}\right) + \frac{R_g V^-}{Z_0} - \frac{R_g}{Z_0} V^{-+}$$

or, $$V^{-+}\left(1 + \frac{R_g}{Z_0}\right) = V^-\left(\frac{R_g}{Z_0} - 1\right)$$

or
$$\boxed{\frac{V^{-+}}{V^{-}} = \frac{R_g - Z_0}{R_g + Z_0}}$$

Thus the (−) wave views the source as if it were simply the source's resistance alone. The effect of the voltage source is taken into account only when determining the outflow of the original (+) wave. Hence the voltage reflection coefficient formula may be used repeatedly, at either end of the line (where R_g is used in place of R_L at the source end).

Before proceeding further, let's examine this formula for some special cases:

(a) $R_L = Z_0$ (line terminated in its characteristic impedance)

$$\Gamma_V = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0 \quad \Rightarrow \text{No Reflection}$$

(b) $R_L = 0\,\Omega$ (short-circuited line)

$$\Gamma_V = \frac{0 - Z_0}{0 + Z_0} = -1 \quad \Rightarrow \begin{cases} V^- = -V^+ & \text{(all of voltage reflected with inversion.)} \\ I^- = I^+ & \text{(all of current reflected without inversion.)} \end{cases}$$

(c) $R_L = \infty\,\Omega$ (open-circuited line)

$$\Gamma_V = \frac{\infty - Z_0}{\infty + Z_0} = +1 \quad \Rightarrow \begin{cases} V^- = V^+ & \text{(all of voltage reflected without inversion)} \\ I^- = -I^+ & \text{(all of current reflected with inversion)} \end{cases}$$

Now, returning to the (− +) wave, this wave reaches the load end of the line at $t = 3\ell/v_p$, setting up another reflected wave, called the "(− + −) wave". This process of bouncing back and forth goes on indefinitely until the steady state is reached at $t = \infty$. Thus, in the steady state, a superposition of an infinite number of (+) and (−)

waves exists on the line. Let us denote the reflection coefficient at the load as $\Gamma_R = \frac{R_L - Z_0}{R_L + Z_0}$, and the reflection coefficient that exists at the source as $\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$, then the line voltage in the steady state can be expressed as

$$V_{ss} = V^+(1 + \Gamma_R + \Gamma_R\Gamma_g + \Gamma_R^2\Gamma_g + \Gamma_R^2\Gamma_g^2 + \Gamma_R^3\Gamma_g^2 + \cdots)$$

$$= \frac{V_0 Z_0}{R_g + Z_0}\Big[\left(1 + \Gamma_R\Gamma_g + (\Gamma_R\Gamma_g)^2 + (\Gamma_R\Gamma_g)^3 + \cdots\right) + \Gamma_R\left(1 + \Gamma_R\Gamma_g + (\Gamma_R\Gamma_g)^2 + (\Gamma_R\Gamma_g)^3 + \cdots\right)\Big]$$

$$= \frac{V_0 Z_0}{R_g + Z_0}(1 + \Gamma_R)\left(1 + \Gamma_R\Gamma_g + (\Gamma_R\Gamma_g)^2 + (\Gamma_R\Gamma_g)^3 + \cdots\right)$$

$$\doteq \frac{V_0 Z_0}{R_g + Z_0}\left(\frac{1 + \Gamma_R}{1 - \Gamma_R\Gamma_g}\right)$$

→ This step was performed using the summation of a geometric series:

$$1 + a + a^2 + \cdots = \frac{1}{1-a} \qquad (\text{where } a = \Gamma_R\Gamma_g)$$

Proof: $(1 + a + a^2 + \cdots) - a(1 + a + a^2 + \cdots) = 1$

$\Rightarrow (1-a)(1 + a + a^2 + \cdots) = 1$

$\Rightarrow 1 + a + a^2 + \cdots = \frac{1}{1-a}$

$$V_{ss} = \frac{V_0 Z_0}{R_g + Z_0}\;\frac{1 + \left(\frac{R_L - Z_0}{R_L + Z_0}\right)}{1 - \left(\frac{R_L - Z_0}{R_L + Z_0}\right)\left(\frac{R_g - Z_0}{R_g + Z_0}\right)} = V_0\left(\frac{R_L}{R_L + R_g}\right)$$

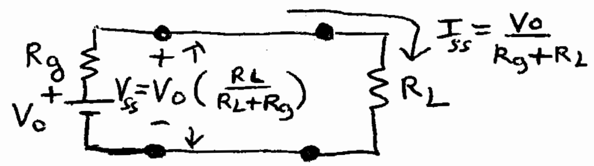
The steady state current is given by

$$I_{ss} = I^+\left[1 + (-\Gamma_R) + (-\Gamma_R)(-\Gamma_g) + (-\Gamma_R)^2(-\Gamma_g) + (-\Gamma_R)^2(-\Gamma_g)^2 + \cdots\right]$$

$$= \frac{V_0}{R_g + Z_0}\left[(1 + \Gamma_R\Gamma_g + \cdots) - \Gamma_R(1 + \Gamma_R\Gamma_g + \cdots)\right]$$

$$I_{ss} = \frac{V_o}{R_g+Z_o}\left(\frac{1-\Gamma_R}{1-\Gamma_R\Gamma_g}\right) = \frac{V_o}{R_g+Z_o}\;\frac{1-\left(\frac{R_L-Z_o}{R_L+Z_o}\right)}{1-\left(\frac{R_L-Z_o}{R_L+Z_o}\right)\left(\frac{R_g-Z_o}{R_g+Z_o}\right)} = \frac{V_o}{R_g+R_L}$$

These results are simply the line voltage and line current which we would expect in the steady state, since the line inductance behaves like a short circuit, and the line capacitance behaves like an open circuit. In the steady state, our transmission line problem becomes a simple "DC circuit analysis" problem !!

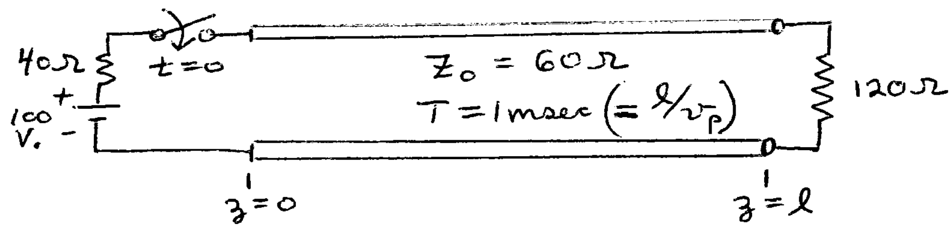


Note that any DC resistive circuit will exhibit transmission line effects (voltage + currents bouncing back and forth, i.e. "ringing") when first turned on, until the steady state (DC) conditions are reached (usually in a very short time, depending on circuit dimensions)!

E. Bounce Diagram Analysis of Transmission Line

From the last section it should be apparent that when $R_g \neq Z_o$ and $R_L \neq Z_o$, transient pulse analysis of a transmission line can become complex. Such analysis is facilitated with the aid of a "bounce diagram."

Let us introduce this technique using a numerical example:



First we note that

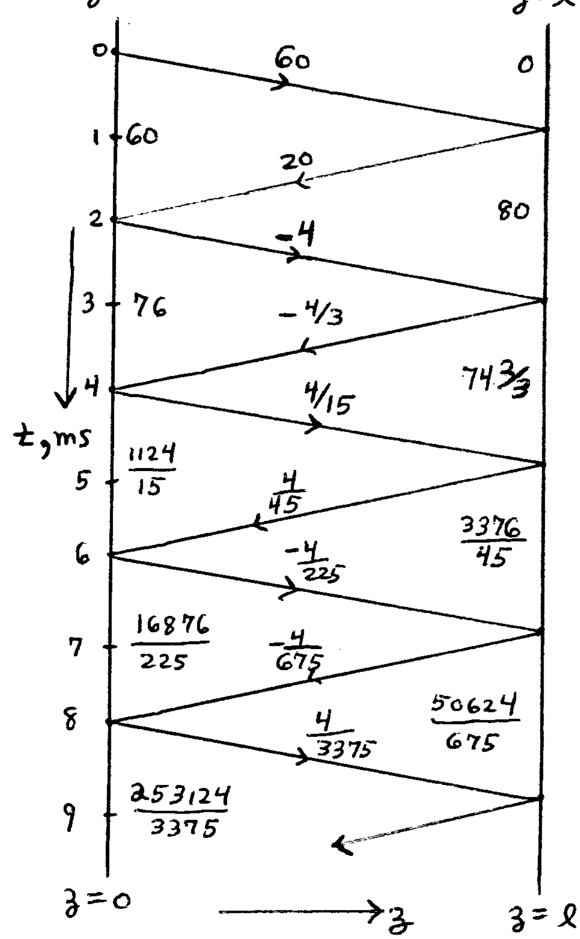
Voltage carried by the initial (+) wave $= \left(\frac{60}{40+60}\right) 100V = 60V.$

current carried by the initial (+) wave $= \frac{60V}{60\Omega} = 1A.$

Voltage reflection coefficient at load, $\Gamma_R = \frac{120-60}{120+60} = \frac{1}{3}$

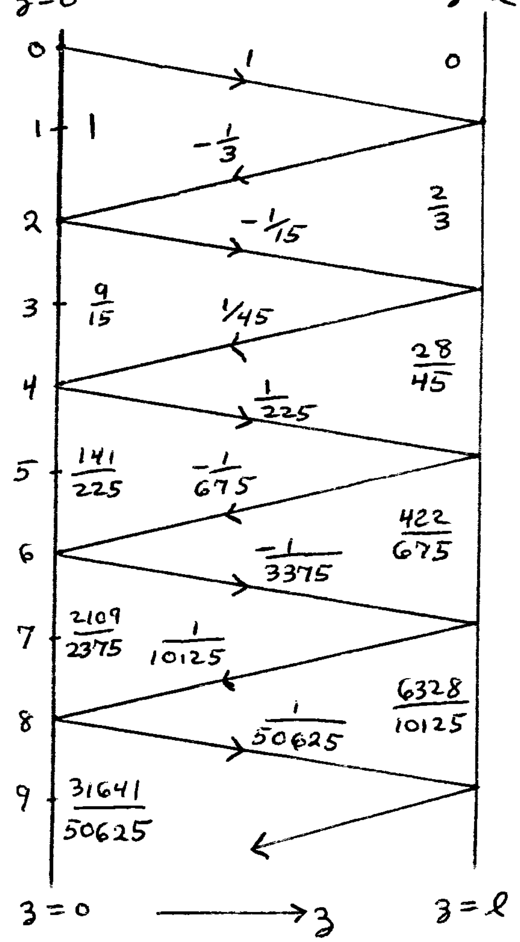
Voltage reflection coefficient at source, $\Gamma_g = \frac{40-60}{40+60} = -\frac{1}{5}$

$\Gamma_g = -\frac{1}{5}$ z=0 $\Gamma_R = 1/3$ z=l



Voltage Bounce Diagm

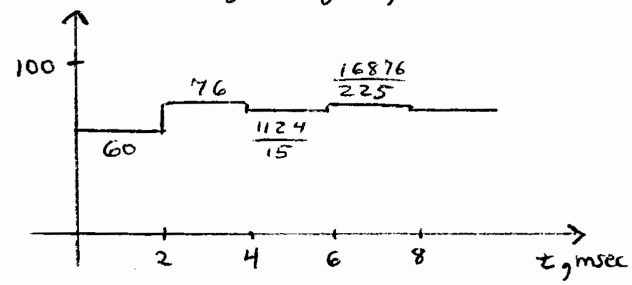
$-\Gamma_g = +\frac{1}{5}$ z=0 $-\Gamma_R = -1/3$ z=l



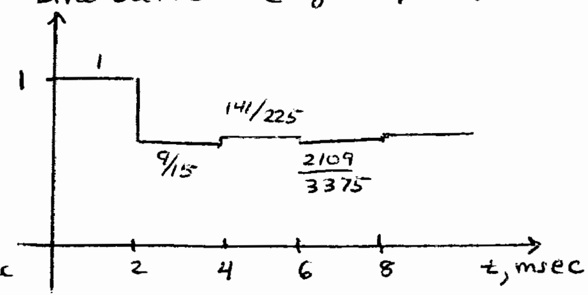
Current Bounce Diagm

The following voltage and current waveforms may be drawn from the above bounce diagrams:

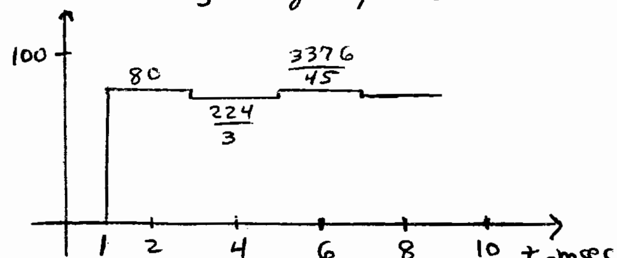
Line Voltage @ $z = 0$, Volts



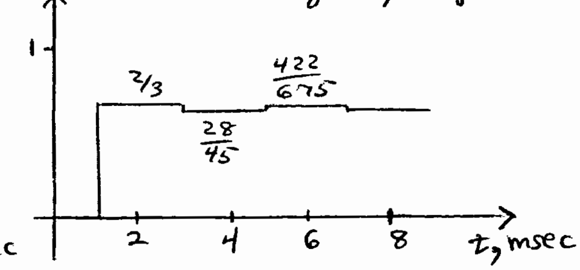
Line Current @ $z = 0$, Amps



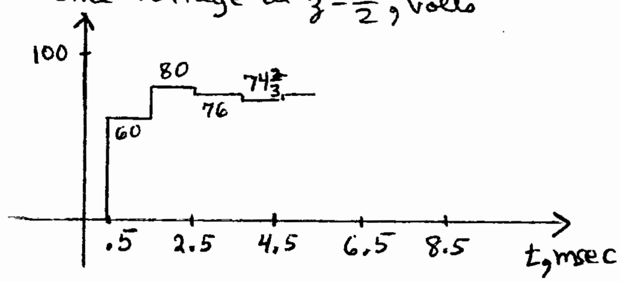
Line Voltage at $z = \ell$, volts



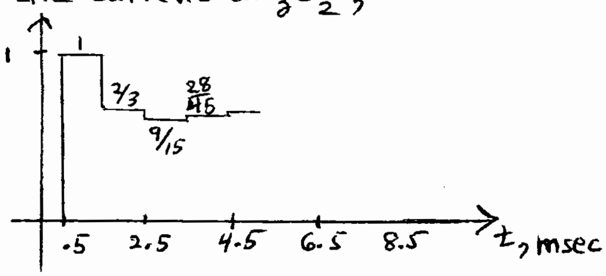
Line Current at $z = \ell$, Amps



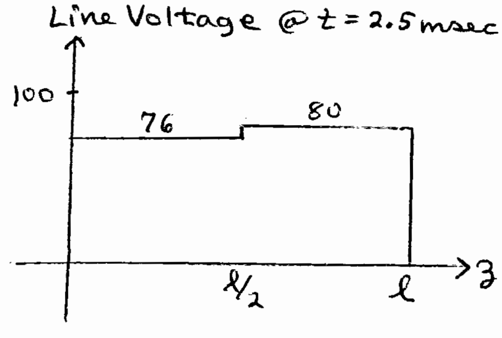
Line Voltage at $z = \frac{\ell}{2}$, volts



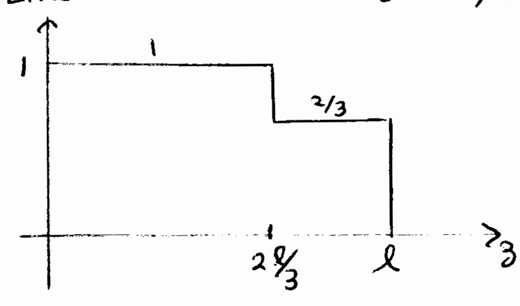
Line Current at $z = \frac{\ell}{2}$,



Line Voltage @ $t = 2.5$ msec, Volts



Line Current at $t = 1\frac{1}{3}$ msec, Amps



In the steady state,

$$(+) \text{ wave voltage} = 60 - 4 + \frac{4}{15} - \frac{4}{225} + \cdots$$
$$= 60\left(1 - \frac{1}{15} + \frac{1}{(15)^2} - \frac{1}{(15)^3} + \cdots\right) = 60\left(\frac{1}{1+\frac{1}{15}}\right)$$
$$= \frac{900}{16} = \underline{56.25\text{ V}}$$

$$(+) \text{ wave Current} = 1 - \frac{1}{15} + \frac{1}{(15)^2} - \frac{1}{(15)^3} + \cdots$$
$$= \frac{1}{1+\frac{1}{15}} = \frac{15}{16} = \underline{0.9375\text{ A}}$$

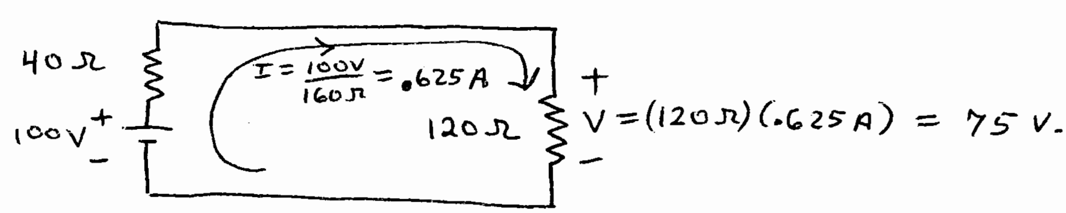
$$(-) \text{ wave Voltage} = 20 - \frac{4}{3} + \frac{4}{45} - \frac{4}{675} + \cdots$$
$$= 20\left(1 - \frac{1}{15} + \frac{1}{(15)^2} - \cdots\right) = 20\left(\frac{1}{1+\frac{1}{15}}\right) = \frac{300}{16} = \underline{18.75\text{ V}}$$

$$(-) \text{ wave Current} = -\frac{1}{3} + \frac{1}{45} - \frac{1}{675} + \cdots$$
$$= -\frac{1}{3}\left(1 - \frac{1}{15} + \frac{1}{(15)^2} - \cdots\right) = -\frac{1}{3}\left(\frac{1}{1+\frac{1}{15}}\right) = \frac{-5}{16} = \underline{-0.3125\text{ A}}$$

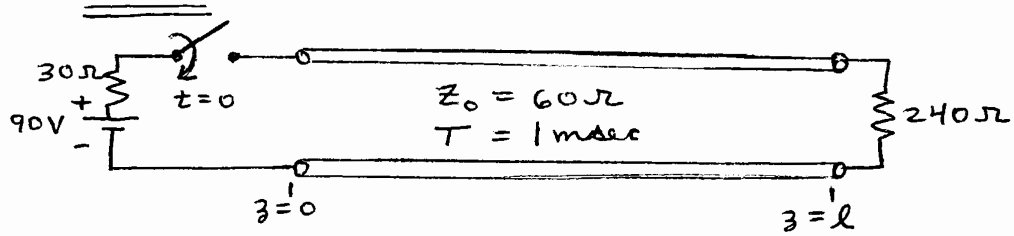
Total Line Voltage $= 56.25 + 18.75 = 75$ Volts

Total Line Current $= 0.9375 - 0.3125 = .625$ Amps

<u>DC CKt Analysis Verification:</u>



<u>HW # 3</u>

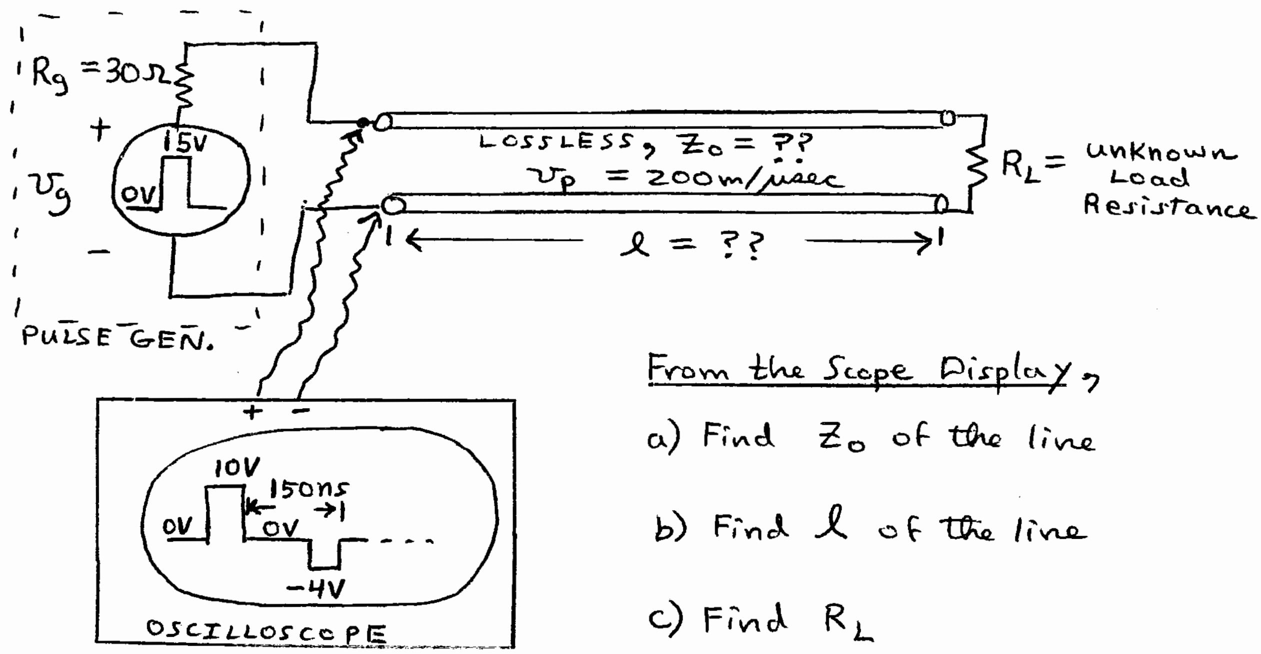


HW 3 (Cont'd) BOUNCE DIAGRAM ANALYSIS

1. a. Draw the voltage and current bounce diagrams for the system above.
   b. Sketch line voltage and line current as functions of time at $z = 0$, $z = \ell$, $z = \ell/2$
   c. Sketch line voltage and line current as functions of z for $t = 2.5$ msec and $t = 4.5$ msec.

2. TDR (Time Domain Reflectometer) Measurements

Imagine the experimental setup shown below:



From the Scope Display,

a) Find Z_0 of the line

b) Find ℓ of the line

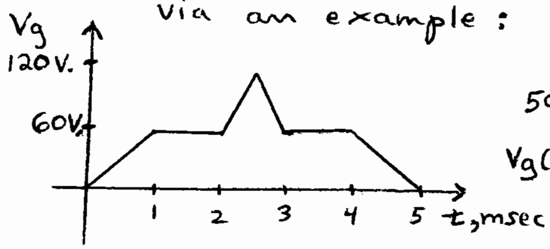
c) Find R_L

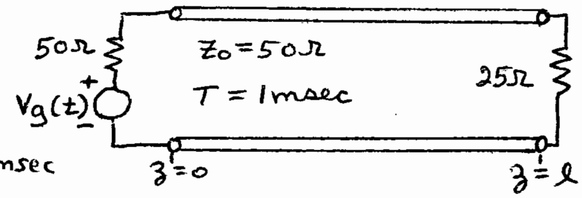
Hint: In finding R_L, you should note that this example differs from that of pages 2-8 thru 2-10 in that the −4V pulse consists of just V^- and V^{-+} components ... the original V^+ wave is no longer present, since a pulse generator drives the line, Not a step generator, as on p. 2-8.

scrambled answers to prob 2: (15, 15, 60)

F. Bounce Diagrams for Transient Waveshapes

Recall that the previous section dealt with constructing bounce diagrams for a transmission line driven by a switched DC source. This technique shall now be extended to include voltage sources of finite duration and arbitrary waveshape. Again, this shall be done via an example:



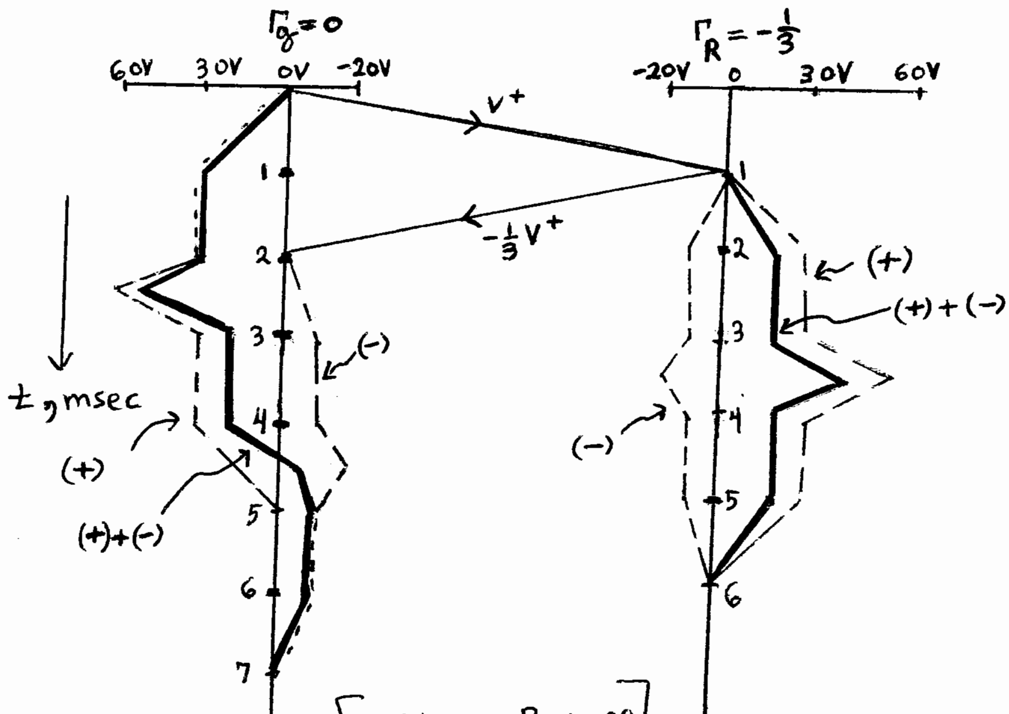


Note: $V^+\big|_{z=0} = V_g(t)\left(\frac{50}{50+50}\right) = \frac{1}{2}V_g(t)$

@load, $\Gamma_R = \frac{25-50}{25+50} = \frac{-25}{75} = -\frac{1}{3}$

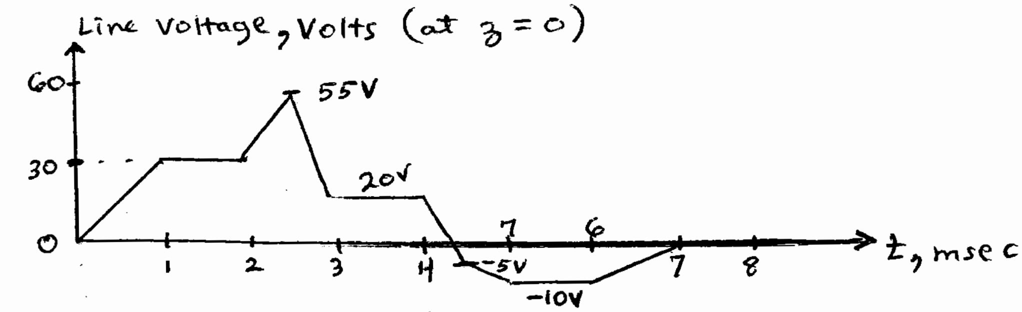
@source $\Gamma_g = \frac{50-50}{50+50} = 0$

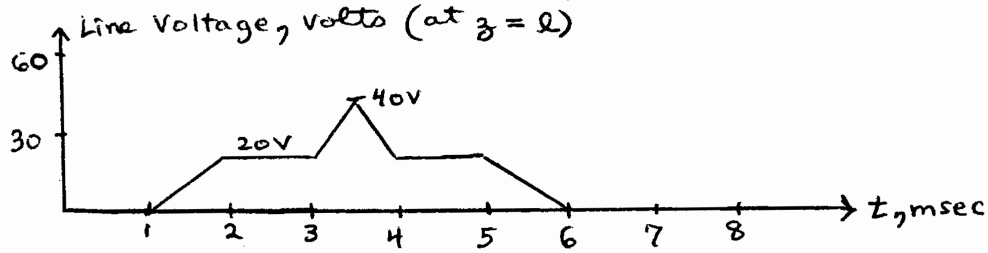
This will permit only a (+) and a (−) wave on the line, greatly simplifying the example!

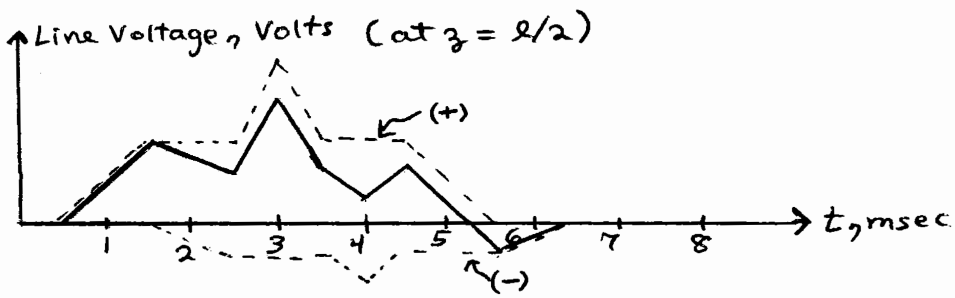


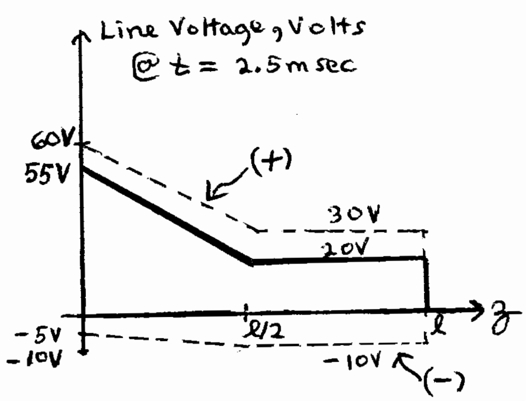
[Voltage Bounce Diagram]

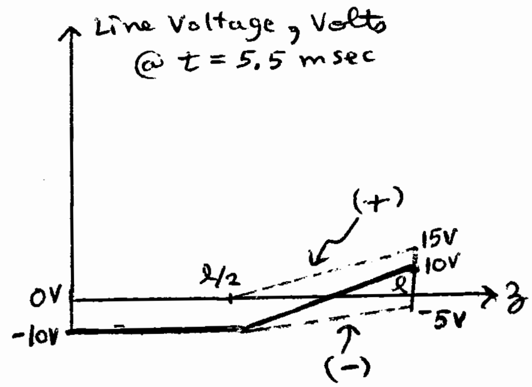
From the voltage bounce diagram, we may sketch line voltage as a function of time at any value of z and line voltage as a function of z for any value of t.





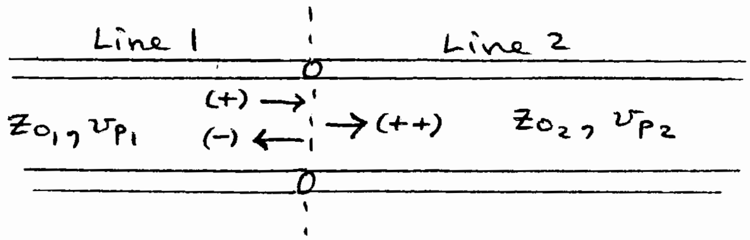




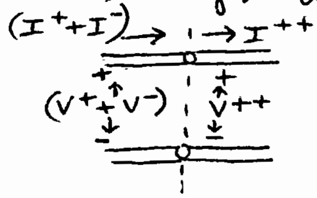


G. Transmission Line Discontinuity

Consider the junction between two transmission lines having different parameters Z_o and v_p. Assuming that a (+) wave is incident upon the junction from the left, we will find that the (+) wave alone cannot satisfy the boundary conditions at the junction, since the voltage-to-current ratio is Z_{01} in line 1, whereas the characteristic impedance in line 2 is $Z_{02} \neq Z_{01}$.



Hence a reflected wave (−) and a transmitted wave (++) are set up. We have the following situation at the junction:



$$\left.\begin{aligned} V^{++} &= V^{+} + V^{-} \\ I^{++} &= I^{+} + I^{-} \end{aligned}\right\} \text{Boundary Conditions}$$

But $\quad I^{+} = \dfrac{V^{+}}{Z_{o_1}}, \quad I^{-} = \dfrac{-V^{-}}{Z_{o_1}}, \quad I^{++} = \dfrac{V^{++}}{Z_{o_2}}$

Thus
$$V^{++} = V^{+} + V^{-}$$

$$\frac{V^{++}}{Z_{o_2}} = \frac{V^{+}}{Z_{01}} - \frac{V^{-}}{Z_{01}} \quad \Rightarrow \quad V^{++} = \frac{Z_{o2}}{Z_{o_1}}\left(V^{+} - V^{-}\right)$$

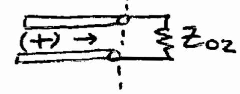
$$V^{+} + V^{-} = \frac{Z_{o2}}{Z_{o1}}\left(V^{+} - V^{-}\right)$$

$$V^{-}\left(1 + \frac{Z_{o2}}{Z_{o1}}\right) = V^{+}\left(\frac{Z_{o2}}{Z_{o1}} - 1\right)$$

$$V^{-} = V^{+}\left(\frac{Z_{o2} - Z_{o1}}{Z_{o2} + Z_{o1}}\right)$$

Voltage Reflection Coefficient $\Gamma_V = \frac{V^-}{V^+} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$

Thus, to the incident wave, the transmission line to the right looks like its characteristic impedance, Z_{02}.



<u>We now define</u>

Voltage transmission coefficient, $\mathcal{T}_V \triangleq \frac{V^{++}}{V^+} = \frac{V^+ + V^-}{V^+} = 1 + \Gamma_V$

current transmission coefficient, $\mathcal{T}_I \triangleq \frac{I^{++}}{I^+} = \frac{I^+ + I^-}{I^+} = 1 - \Gamma_V$

Note that the transmitted voltage can be <u>greater than</u> the incident voltage if Γ_V is positive, i.e. $Z_{0_2} > Z_{01}$. This is not a violation of energy conservation, since the transmitted current will be less than the incident current. Of course, it must always end up that the transmitted power is $\leq$ the incident power. <u>Verification follows</u>:

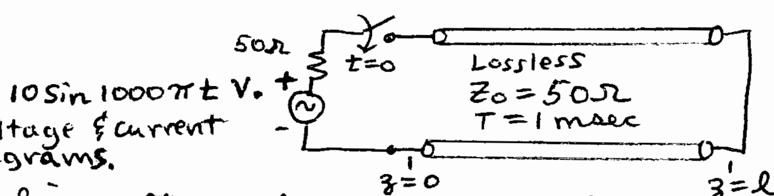
Transmitted power, $P^{++} = V^{++} \, I^{++}$

$$= [(1+\Gamma_V)V^+]\cdot[(1-\Gamma_V)I^+]$$

$$= (1-\Gamma_V^2)V^+I^+ = (1-\Gamma_V^2)P^+$$

$$= (1-\Gamma_V^2)\cdot(\text{Incident power})$$

<u>Homework #4</u>



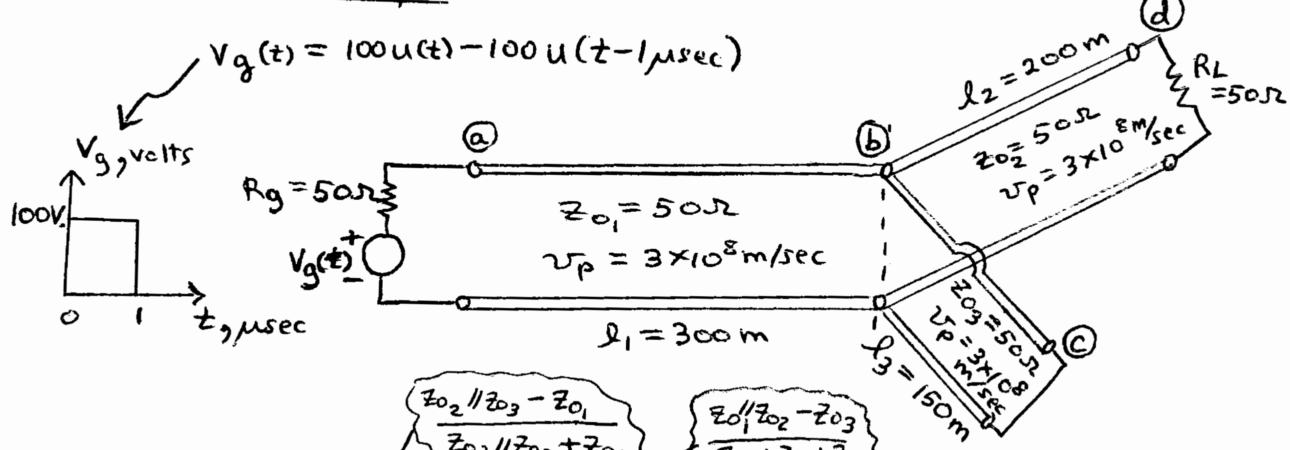
a) Sketch the voltage & current bounce diagrams.

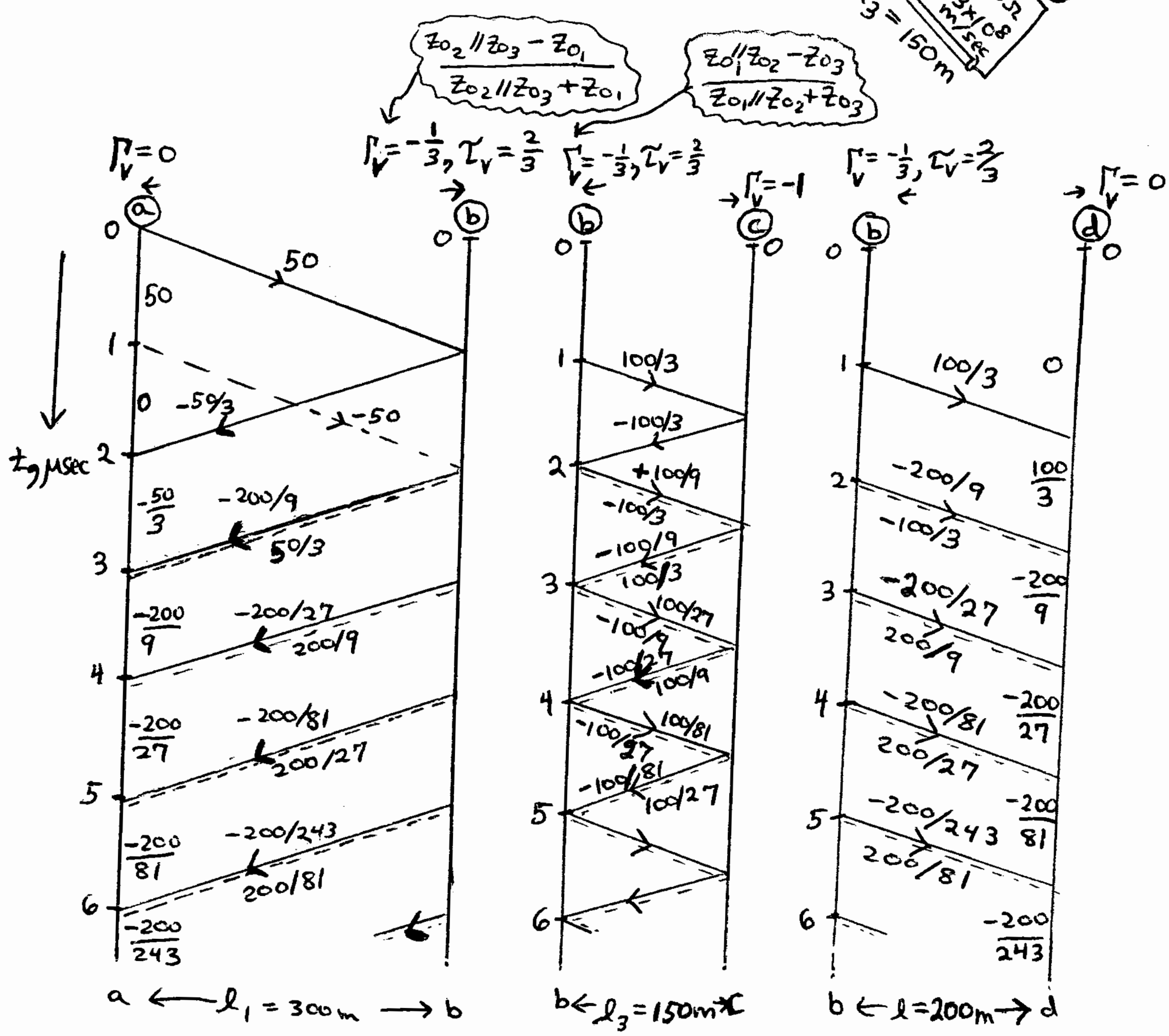
b) Sketch the line voltage at $z=0$ versus time.

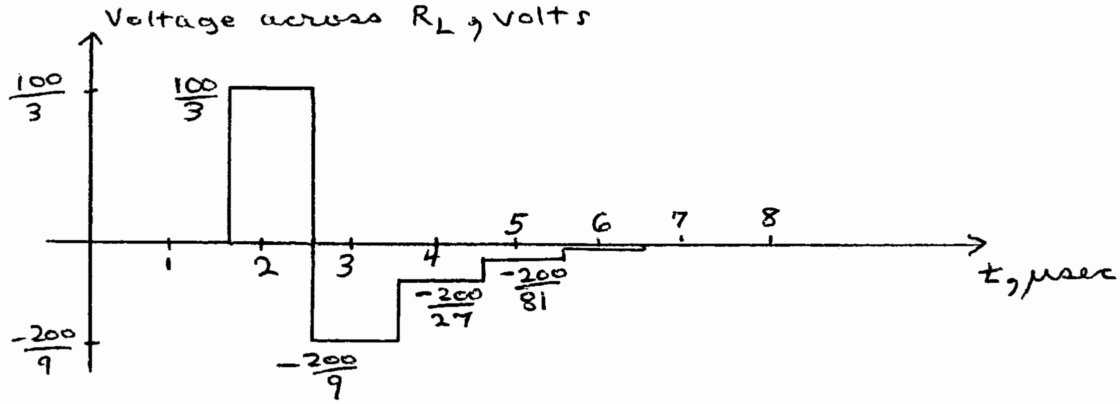
c) Sketch the line current at $z=0$ and $z=\ell$ versus time.

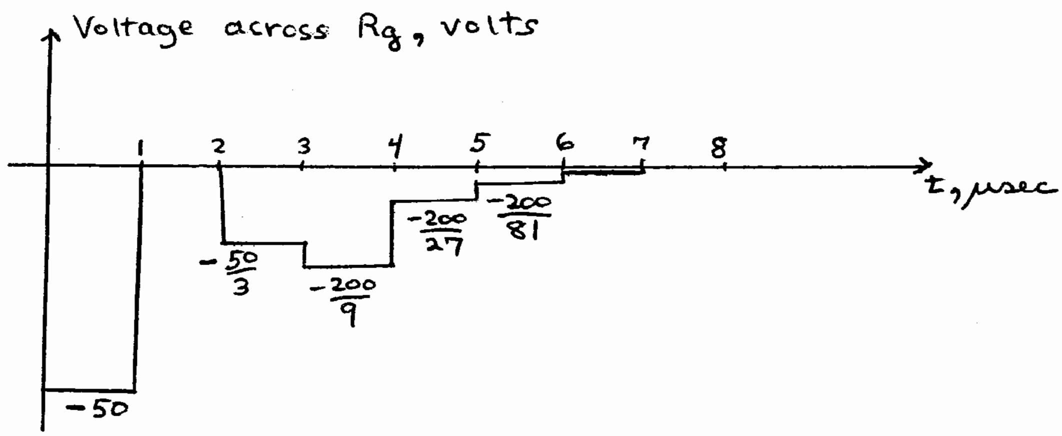
d) Sketch the line current and voltage as functions of z for $t = 2T, 2.5T, 3T$; where T = line propagation delay.

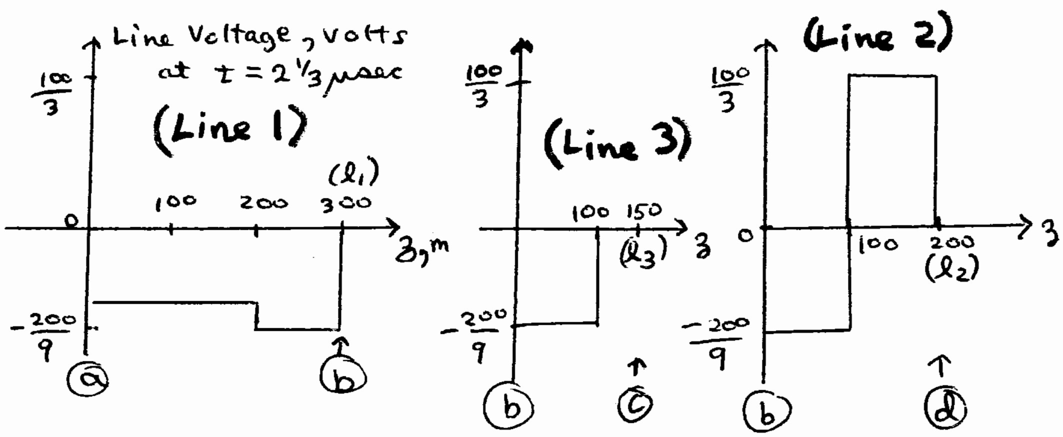
A 3-Line Example:






Voltage across R_L, volts
100/3
-200/9
-200/27
-200/81
1 2 3 4 5 6 7 8
t, μsec


Voltage across R_g, volts
1 2 3 4 5 6 7 8
t, μsec
-50
-50/3
-200/9
-200/27
-200/81


Line Voltage, volts
at t = 2 1/3 μsec
(Line 1)
(Line 3)
(Line 2)
100/3
-200/9
0
100 200 300
(l_1)
z, m
100 150
(l_3)
100 200
(l_2)
a
b
c
d

Check of energy balance

Total energy dissipated in $R_L = \int_0^\infty \frac{V_{R_L}^2}{R_L}\,dt$

$$= \int_{1\frac{2}{3}}^{2\frac{2}{3}} \frac{10^4}{9\times 50}\,dt + \int_{2\frac{2}{3}}^{3\frac{2}{3}} \frac{4\times 10^4}{(9)^2\times 50}\,dt + \int_{3\frac{2}{3}}^{4\frac{2}{3}} \frac{4\times 10^4}{(27)^2\times 50}\,dt + \int_{4\frac{2}{3}}^{5\frac{2}{3}} \frac{4\times 10^4}{(81)^2\times 50}\,dt + \cdots$$

$$= \frac{10^4}{9\times 50} + \frac{4\times 10^4}{(9)^2\times 50}\left[1 + \frac{1}{9} + \frac{1}{9^2} + \frac{1}{9^3} + \cdots\right]$$

$$= \frac{200}{9} + \frac{800}{81}\left[\frac{1}{1-1/9}\right] = \frac{200}{9} + \frac{100}{9} = \underline{33\,1/3\,\mu\text{ joules}}$$

Total energy dissipated in $R_g = \int_0^\infty \frac{V_{R_g}^2}{R_g}\,dt$

$$= \int_0^1 \frac{(50)^2}{50}\,dt + \int_1^2 0\,dt + \int_2^3 \frac{(50)^2}{9\times 50}\,dt + \int_3^4 \frac{4\times 10^4}{9^2\times 50}\,dt + \int_4 \frac{4\times 10^4}{(9)^3\times 50}\,dt + \cdots$$

$$= 50 + \frac{50}{9} + \frac{4\times 10^4}{9^2\times 50}\left[1 + \frac{1}{9} + \frac{1}{9^2} + \cdots\right]$$

$$= 50 + \frac{50}{9} + \frac{800}{81}\left[\frac{1}{1-1/9}\right] = \underline{66\,2/3\,\mu\text{joules}}$$

Total energy supplied by source $= \int_0^\infty V_g I_g\,dt$

$$= \int_0^1 100\times 1\,dt = \underline{100\,\mu\text{joules}}$$

Thus $\quad P_{source} = P_{R_L} + P_{R_g}$

$$100\,\mu\text{ joules} = 33\,1/3\,\mu\text{ joules} + 66\,2/3\,\mu\text{joules}$$

HW # 5

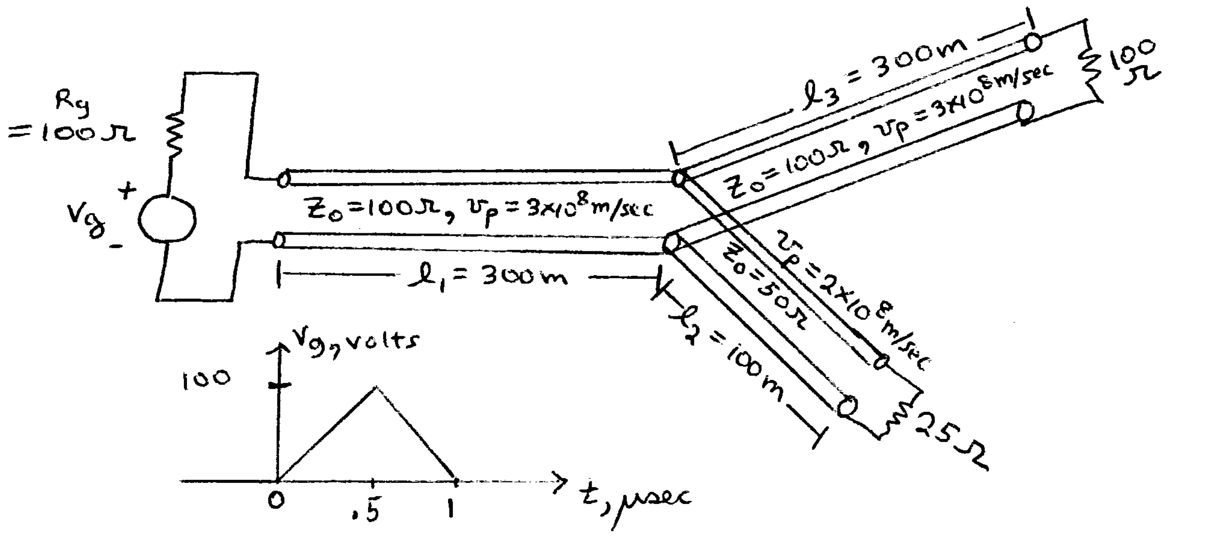
In the system shown below, a pulse generator supplies a triangular pulse of duration 1 μsec to a transmission line feeding two other parallel-connected lines, having the characteristic impedances and propagation velocities indicated.

(a) Sketch the voltage bounce diagrams for all 3 lines.

(b) Sketch the voltages across all 3 resistors as functions of times from $t = 0$ to $t = \infty$.

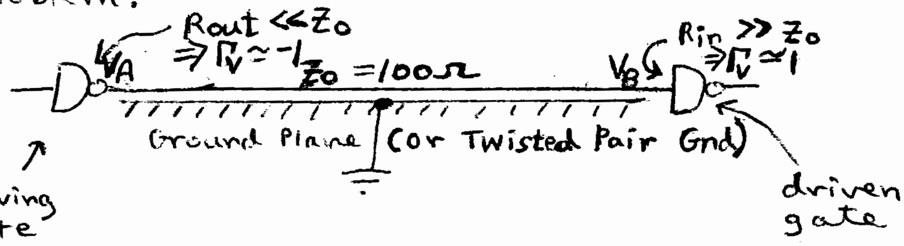
(c) Sketch the voltages along all 3 lines as functions of distance at $t = 2\,\mu sec$.

(d) Show that the energy supplied by the voltage generator is equal to the sum of the total energies dissipated in these 3 resistors.

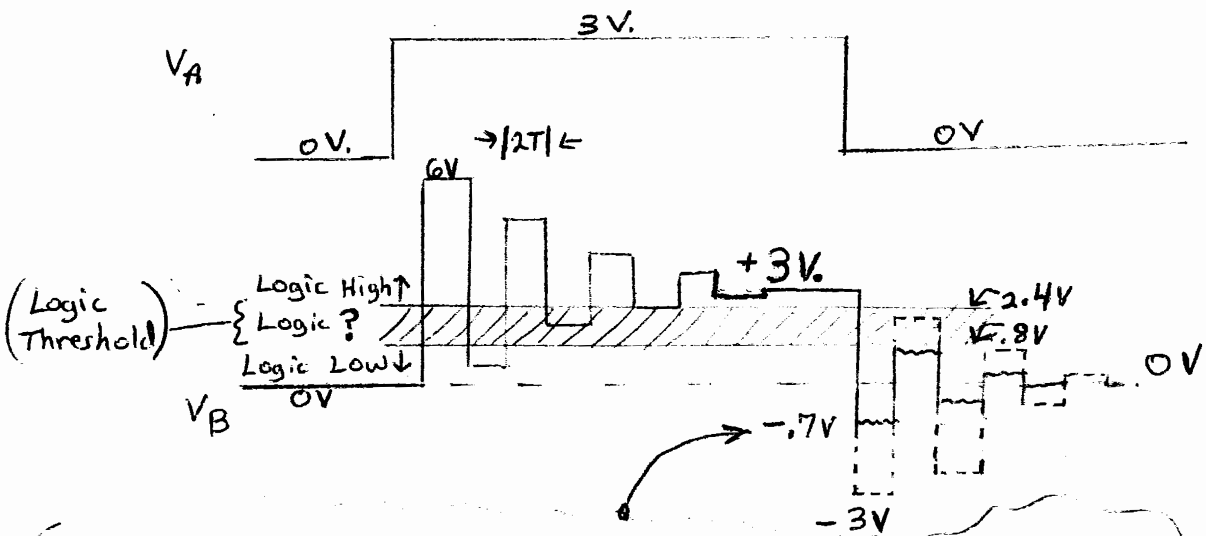


H. Nonlinear Termination: TTL Gate-Driven Line

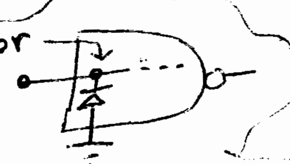
Whenever 10ns risetime digital (TTL) signals must be "bussed" over a distance exceeding several meters, transmission transients can cause intolerable distortion ("ringing") in the transmitted signal if no attention is paid to the transmission line problem:



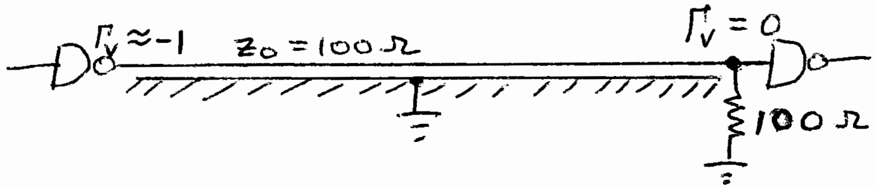
In the "unterminated" situation above, where no attention is paid to transmission line impedance matching, the output impedance of the driving gate is ≈ 7Ω << Z_0, resulting in a reflection coefficient of nearly −1. At the input of the driven gate, the input impedance is ≈ 2.5 kΩ >> Z_0, resulting in a reflection coefficient of nearly +1.



Almost all TTL gate designs have an interior "transient suppression diode" that "clamps" the negative-going transients at −.7V. This suppresses "ringing" during high → low input transitions, But does nothing for ringing during low → high transitions.



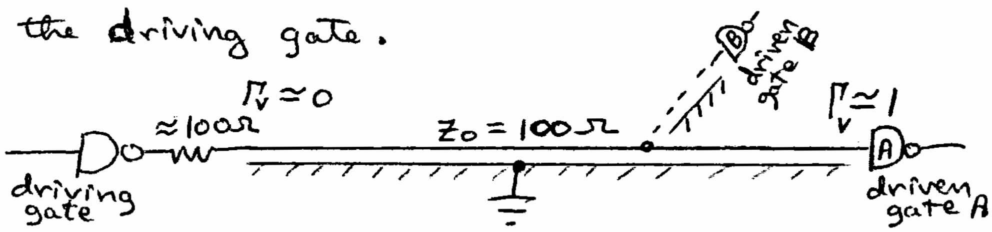
The obvious way to fix this problem is to place a resistor of Z_0 ohms in parallel with the driven gate:



Now there are no reflections, but standard TTL "line drivers" cannot drive such a line -- The steady state current for logical "High" drive $\approx \frac{V_{OH}}{100\,\Omega} = \frac{2.5\,V}{100\,\Omega} = \underline{25\,mA}$!

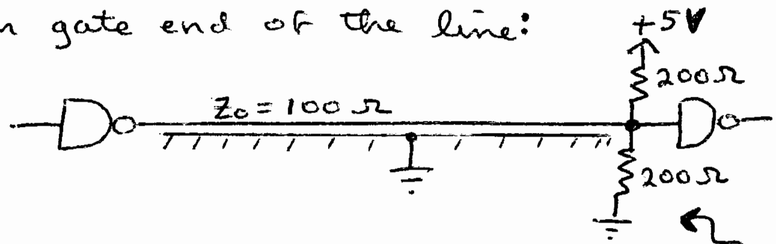
TTL may often not be able to source more than 0.5 mA during Logical High drive. (Consult spec sheet!)

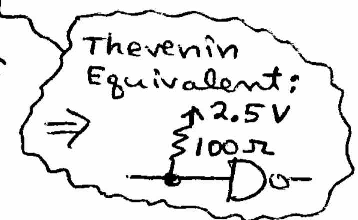
A second possible approach involves "back matching" by placing a resistor of Z_0 ohms in series with the output of the driving gate.



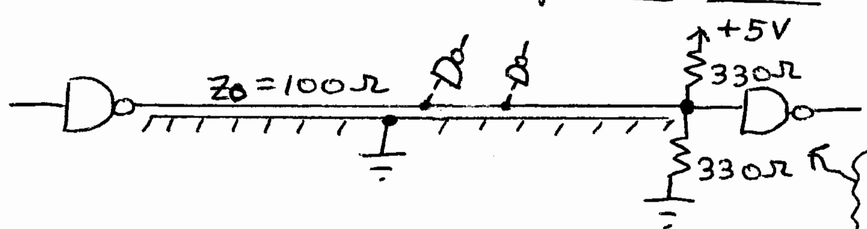
There is now a reflection from the driven end, but it is "gobbled up" at the driving end before it can bounce back and "pollute" the signal at the driven end. The problem with "back matching" is that the drive capability of the driving gate is severely limited by the series resistance, making it possible to drive only 1 or 2 gates. Furthermore, if a second gate (B) is driven from an intermediate point, it may pick up undesired transients reflecting from driven gate A. Thus "back matching" is generally permissible <u>only</u> in situations involving just one driving and one driven gate.

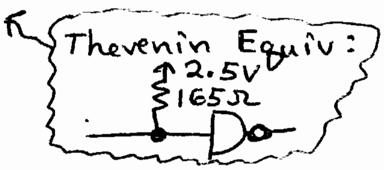
A third approach involves "active termination" at the driven gate end of the line:





Now, essentially no current is required for steadystate logic high drive! The reflection coefficient is zero. Unfortunately, approximately $\frac{2.5V}{100\Omega} = 25mA$ is required to be sunk by the driving gate during steady state logic low drive. This may be just beyond the capability of some TTL gates. The "ultimate" solution that we must often live with in practice is that of "partial active matching":





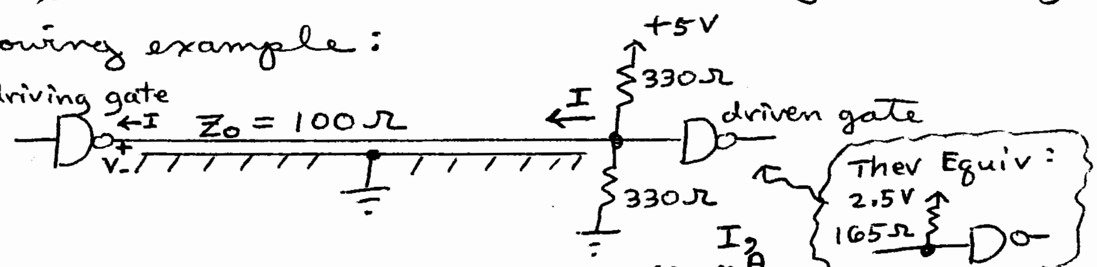
Again, essentially no current is required for steady state logic high drive; and now the current sinking requirement is $\frac{2.5V}{165\Omega} = 15mA$.....just within the capability of most TTL gates! The reflection coefficient $\approx +\frac{1}{4}$, which is usually sufficient to reduce the transients below the troublesome (TTL threshold-crossing level.

Any additional gates that are to be driven at intermediate points along the line may be tapped across the line (without termination) as "high impedance bridging connections," so as not to introduce line discontinuities and associated transient reflections, using short bridging wires.

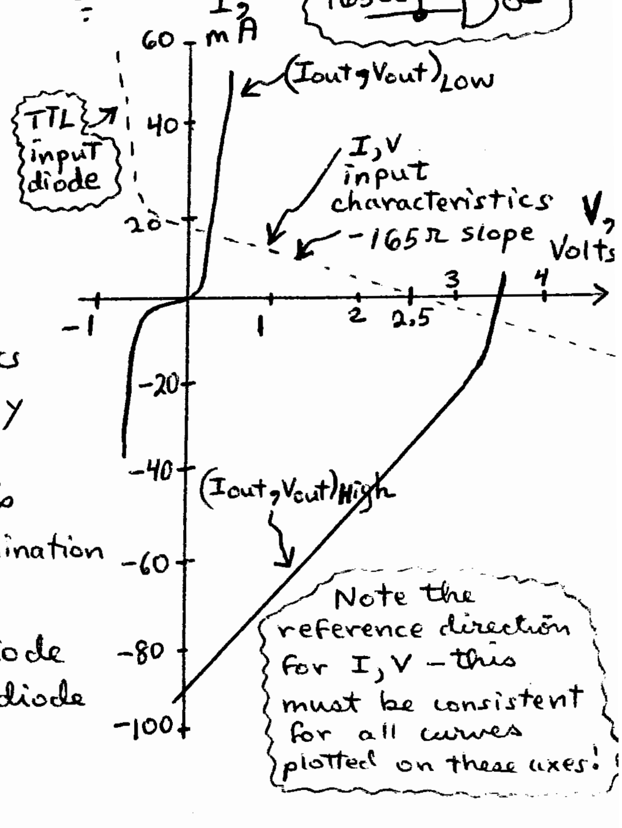
Reflection Diagram Analysis

Recall that we have made the claim that TTL gates exhibit an output impedance of 7Ω and an input impedance of 2.5kΩ. Actually, this is only an approximation — a more careful study would reveal that the I_{out} versus V_{out} characteristics for a TTL gate are nonlinear, with different characteristic curves for logic low and logic high states! Nevertheless, such problems are becoming important in predicting the performance of high-speed digital circuits. Fortunately this rather sticky problem can be solved in a straightforward fashion using the "Reflection Diagram."

The steps in the method will be illustrated by considering the following example:



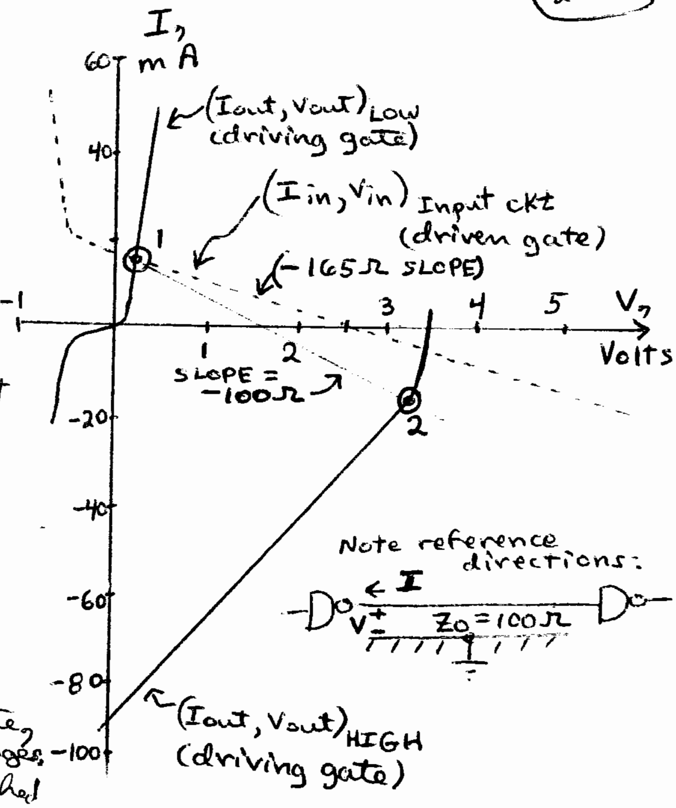
STEPS



First sketch the $(I_{out} \text{ vs } V_{out})_{\text{Logic High}}$ and the $(I_{out} \text{ vs } V_{out})_{\text{Logic Low}}$ output characteristics of the driving gate on the same set of I, V axes.

Next Superimpose the I-V characteristics of the driven gate input and any termination network (if present). Note that in our example, this curve is dominated by the termination network for V > −.7V, and then follows the input clamping diode curve for V < −.7V, as this diode begins to turn on.

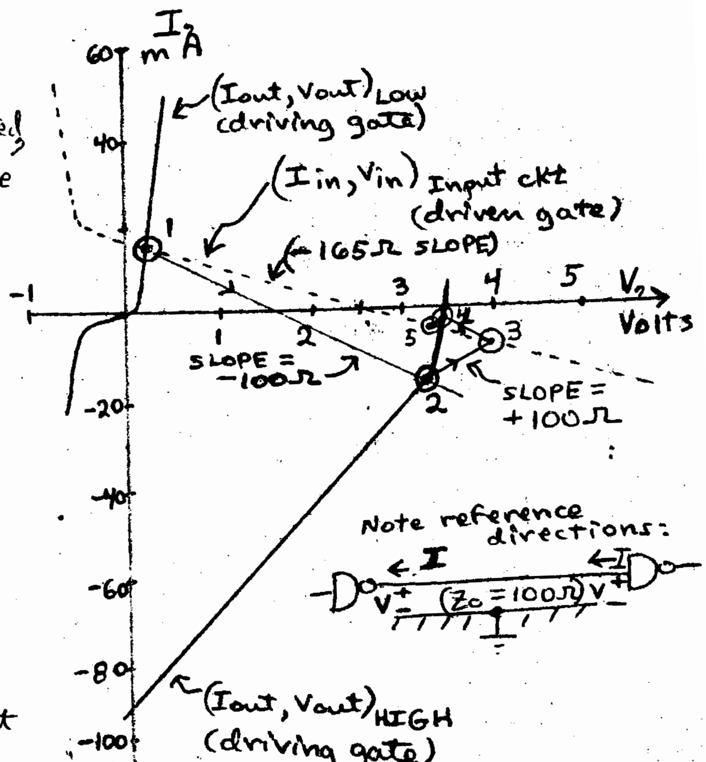
For Low → High Transition



1. Locate the intersection of the input characteristics of the driven gate and the driving gates' output low characteristics. This represents the steady state voltage & current existing on the line just before the Low → High transition occurs. (Here quiescent $V_{out} = .2V$)

2. Once the transition from low to high occurs at the driving gate, the driving gates output IV curve changes. ∴ The boundary conditions established by the DC "steady-state" solution (point 1) are no longer valid, and a (+) wave will be generated which "sees" the 100 Ω characteristic impedance of the line. This (+) wave, when added to the original initial conditions on the line must add together to satisfy the new driving gate device characteristics, $(I_{out}, V_{out})_{HIGH}$. Thus, the new operating point at the driving gate is found by passing a line with slope $-Z_0$ (minus because, even though this is a (+) wave, the current reference direction is opposite the way we normally define it for a transmission line .... if the device IV curves had been defined with I flowing out of the driving gate, then we would have worked with a $+Z_0$ slope!) through point 1 (to effectively "add" the (+) wave to the existing DC conditions on the line). The point where this line intersects the $(I_{out}, V_{out})_{HIGH}$ defines the new driving gate operating point, representing the sum of the initial DC conditions,

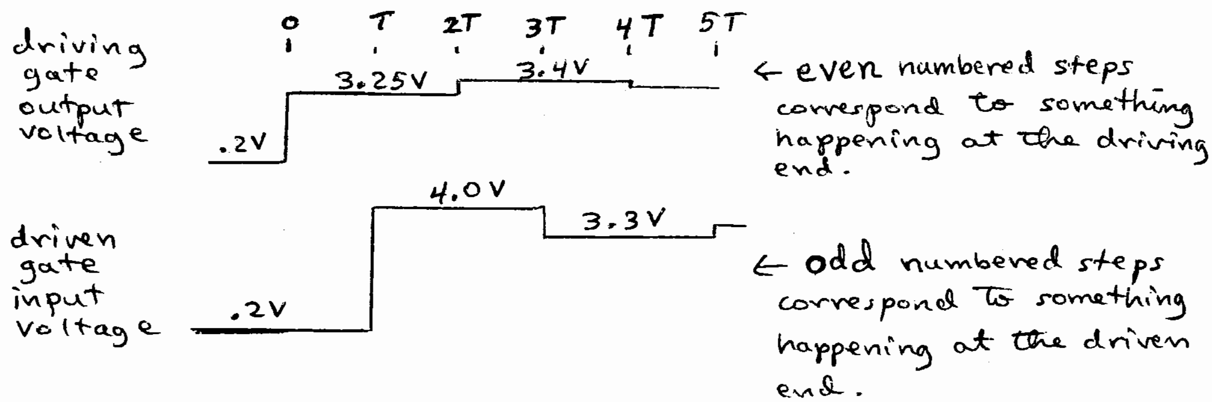
and the (+) wave needed to satisfy the new driving gate I,V characteristics, subject to the Transmission line constraint that $V^+ = Z_0 I^+$. Note that in this example, the net voltage at the driver has stepped from .2V → 3.25V.



3. When the (+) wave hits the driven gate, T seconds later, a (−) wave now must be generated which, when added to the (+) wave and original initial conditions, which are also present, must satisfy the boundary condition imposed by the (I_{in}, V_{in}) Input characteristics of the driven gate/resistor termination network. Because this (−) wave addition is subject to the transmission line constraint $V^- = -Z_0 I^-$, we find the new operating point at the driven gate by passing a line of slope $+Z_0$ through point 2 to where it intersects the (I_{in}, V_{in}) characteristic curve. Thus we see the voltage at the driven end steps from the DC value of .2V to a value of 4 V.

4, 5. This process repeats, alternately using slopes of $-Z_0$ and $+Z_0$, as the successive bounces contribute less and less to the net line voltage, rapidly converging to a new steady state condition that is found at the intersection of the $(I_{out}, V_{out})_{HIGH}$ driving gate output curve and the (I_{in}, V_{in}) driven gate/termination curves. Here we see the next step at the driving end is 3.4V (T seconds later). The next step at the driven end is 3.3V (2T seconds after the intial transition, etc.

In Summary, we may plot the line voltage at the driver and receiver end for a low → high transition:



HW#6

Trace over the driving gate output characteristics (both high and low), the driven gate/termination input characteristics, and the V, I axes on a fresh sheet of paper. Now consider the behavior of the system in the example above for a high ⟶ low logic transition at the driving gate. Show your graphical construction! Briefly explain each (numbered) point on your construction. Plot driving gate and driven gate voltage waveforms as done in the example above.

≈ Answers driver output: 3.4 → .3 → .2 V.
driven gate: 3.4 → −.5 → .3 V