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Introduction to Bipolar Junction Transistors

(Read Chapter 3 of Text)

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Bipolar Junction Transistor

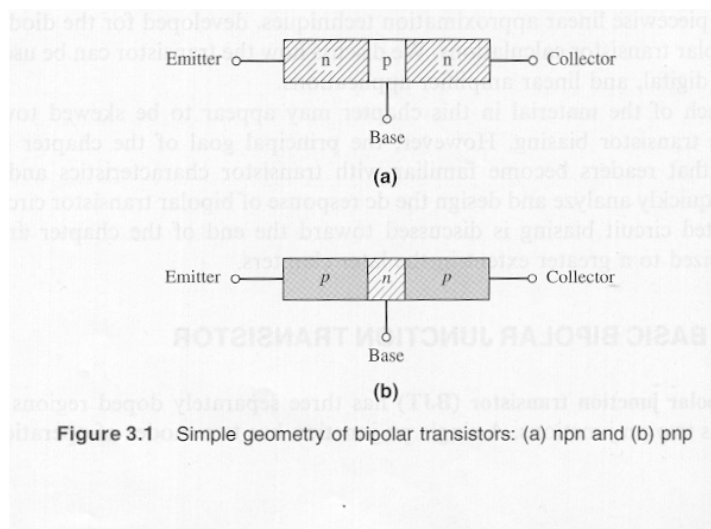
- Current-controlled current source
- Made by sandwiching thin N-type Si between two P-type Si (PNP BJT)
- Or by sandwiching thin P-type Si between two N-type Si (NPN BJT)
- Leads called Base (B), Collector (C) and Emitter (E). Control current " I_B " flows from B to E. Resulting current " I_C " is "pumped" from C to E.

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NPN and PNP BJTs



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Electron Current in Forward-Active NPN BJT

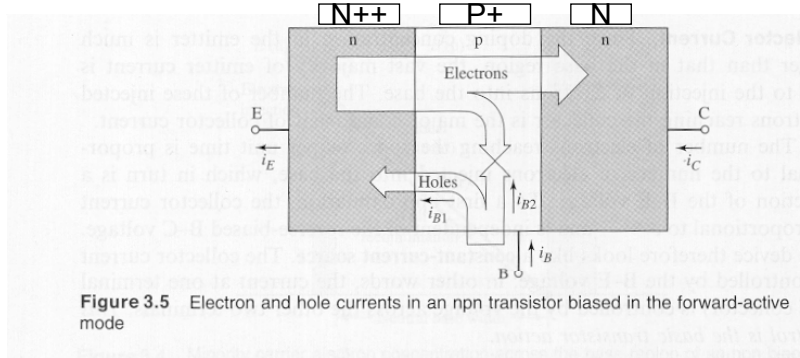


Figure 3.5 Electron and hole currents in an npn transistor biased in the forward-active mode

Electrons from the highly-doped emitter “n⁺⁺” region are injected into base region through the forward-biased BE junction. They diffuse through the VERY THIN base region, with only a few of these electrons having a chance to recombine with holes in the thin “p⁺” base region. The vast majority of the injected electrons (perhaps $\alpha \approx 0.99$ of the injected electrons) diffuse to the edge of the base region near the reverse-biased BC junction, where the intense electric field associated with the reverse-biased BC junction (directed from N to P regions) sweeps these electrons into the “n” collector region, where they constitute collector current. The remaining “ $1 - \alpha$ ” = $1 - 0.99 \approx 0.01$ of the injected electrons that recombine with holes in the base region constitute a portion of the small base current “ i_{B1} ”.

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Hole Current in Forward-Active NPN BJT

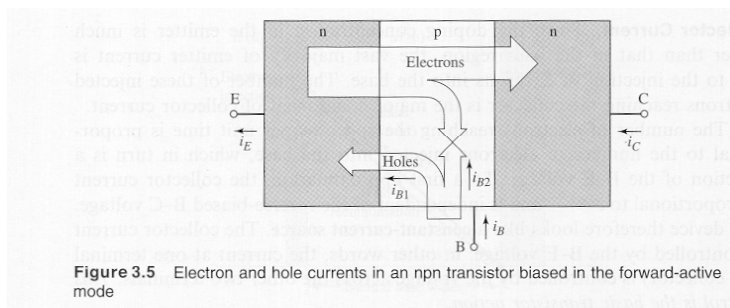


Figure 3.5 Electron and hole currents in an npn transistor biased in the forward-active mode

We shall see that the electron current diffusing across the forward-biased B-E junction from E to B regions, as discussed on the previous slide, results in desirable transistor action. However, the hole current that flows from B to E region adds an UNDESIRABLE contribution “ i_{B2} ” to the total base current, $i_B = i_{B1} + i_{B2}$. We desire that the total base current “ i_B ” be as small as possible for a given collector current “ i_C ”. This results in a “forward current transfer ratio” (current gain) $\beta_F = i_C / i_B$ that is as large as possible. The undesired hole current component of i_B (i_{B2}) is made negligibly small by doping the E region heavily (n⁺⁺) and the base region about 100 times less heavily than the B region (p⁺).

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Relationship between Emitter, Base, and Collector Current in a Forward-Active BJT

- $I_E = I_S \exp(V_{BE}/V_T)$, since the BE junction is forward-biased. (Note $\eta = 1$ for IC BJTs.)
- $I_C = \alpha I_E$, because $\alpha \approx 0.99$ of the electrons injected into the THIN base region diffuse across the base and get swept across the reverse-biased BC junction.
- $I_B = (1 - \alpha) I_E$, because the remaining $(1 - \alpha) \approx 0.01$ of the electrons recombine with holes in the base region.

Relating β to α

- α_F , the “forward current transfer ratio” is defined as the ratio of collector to emitter current in a forward-active BJT

$$\text{Note: } I_C/I_E = \alpha_F I_E / I_E = \alpha_F$$

- β_F , the “transistor current gain” is defined as the collector current divided by the controlling base current in a forward-active BJT.
- Note $\beta_F = I_C/I_B = \alpha_F I_E / (1 - \alpha_F) I_E = \alpha_F / (1 - \alpha_F)$
- Thus if $\alpha_F = 0.99$, $\beta_F = 0.99 / (1 - 0.99) = 99$
- The subscript “F” is often dropped, if it is clear that we are talking about a forward-active BJT.

NPN BJT in “Common Emitter” Configuration

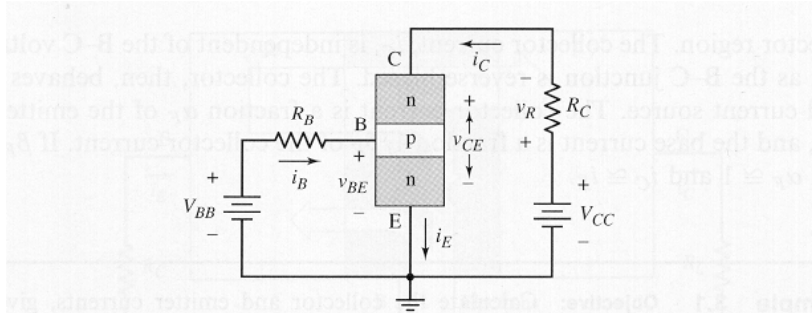


Figure 3.6 An npn transistor circuit in the common-emitter configuration

Voltage source V_{BB} forward-biases BE junction (assuming $V_{BB} > 0.7V$), making $V_{BE} \approx 0.7V$.

Voltage source V_{CC} reverse-biases BC junction. Thus BJT is biased into its “forward-active” region, and

$$I_B = (V_{BB} - 0.7V)/R_B, \quad I_C = \beta_F I_B, \quad \text{and} \quad I_E = I_C + I_B = (\beta_F + 1) I_B$$

Operating Modes of NPN BJT

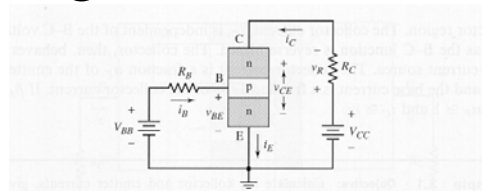


Figure 3.6 An npn transistor circuit in the common-emitter configuration

1. If $V_{BB} < 0.7V$, both BE and BC junctions are OFF, and BJT is “**cut off**”. No currents flow in a cut off BJT $I_C = I_E = I_B = 0$. The terminals of the BJT are essentially “open-circuited”.
2. If $V_{BB} > 0.7V$, $V_{CC} > 0.1V$, BE junction is forward-biased, and BC junction is reverse-biased. So BJT is “**forward active**”, where $I_C = \beta_F I_B$.
3. If, while in forward-active mode, V_{BB} is increased to a point where $V_{CE} = V_{CC} - \beta_F I_B R_C$ falls below about $0.1V$, $V_{BE} = 0.7V$ (on hard) and $V_{BC} = V_{BE} - V_{CE} = 0.7V - 0.1V = 0.6V$, and thus the BC junction turns on (lightly). Under this condition the BJT is said to be “**saturated**”. I_C no longer $= \beta_F I_B$. Instead, the BE junction acts like a $0.7V$ battery and the BC junction acts like a $0.6V$ battery.

4. Reverse Active Mode (BC fwd-biased, BE rev-biased)

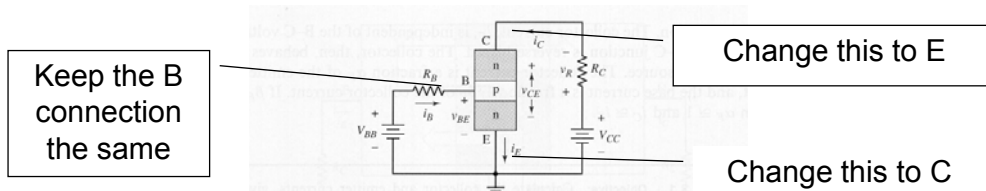


Figure 3.6 An npn transistor circuit in the common-emitter configuration

- If C and E terminals are interchanged in the circuit above, it would at first seem that the circuit will function exactly as it did before due to the apparent symmetry of the BJT. Now the BC junction will be forward-biased and the BE junction will be reverse-biased.
- However, we learned earlier that a modern BJT is NOT symmetric, since the C and E regions are different shape, and the doping level in the E region is much higher (n^{++}) than the doping level in the C region (n).
- Thus with C and E interchanged, the BJT operates in **reverse active** mode, which is similar to forward active mode, except the roles of I_E and I_C have been interchanged, and now the undesired hole current component of I_B (I_{B2}) is much larger and more injected electrons recombine in the base region, thereby reducing the forward transfer current ratio well below 1. Now $\alpha_R \sim 0.5$, and now $\beta_R = \alpha_R / (1 - \alpha_R) \sim 1$.
- Now $I_E = \beta_R I_B$ and $I_C = (\beta_R + 1)I_B$, since the roles of C and E are reversed.

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Electron and Hole Currents in a Forward-Active PNP BJT

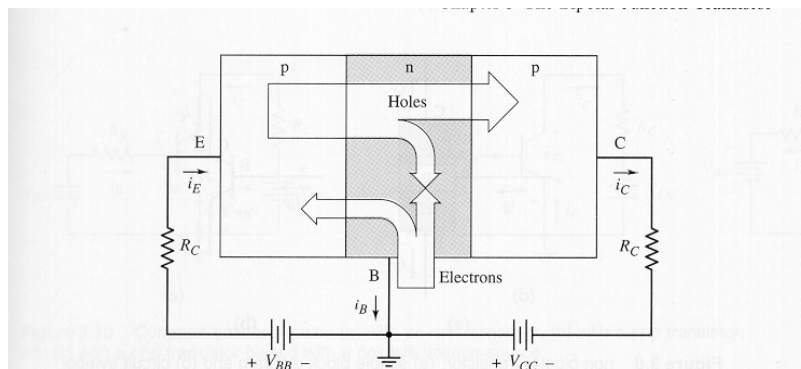


Figure 3.7 Electron and hole currents in a pnp transistor biased in the forward-active mode

Forward Active PNP BJT has SAME equations as before, just opposite current and voltage polarities! As before, BE junction is forward biased and BC junction is reverse biased. Now $(V_{EB})_{on} = 0.7 \text{ V}$.

$$I_C = \alpha_F I_E \quad I_C = \beta_F I_B \quad I_E = (\beta_F + 1) I_B$$

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NPN and PNP BJT Symbols

Note that emitter arrow indicates reference direction of emitter current, I_E

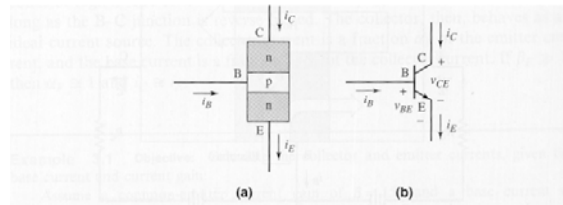


Figure 3.8 npn bipolar transistor: (a) simple block diagram and (b) circuit symbol

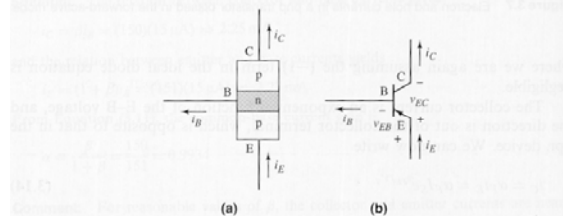


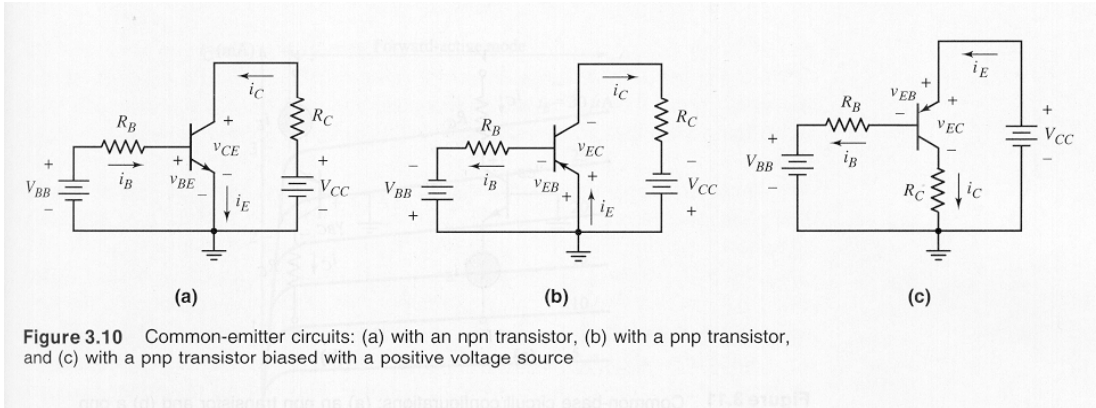
Figure 3.9 pnp bipolar transistor: (a) simple block diagram and (b) circuit symbol

Summary of NPN and BJT Fwd Active Equations

Table 3.1 Summary of the bipolar current–voltage relationships in the active region

npn	pnp
$i_E = I_S e^{v_{BE}/V_T}$	$i_E = I_S e^{v_{EB}/V_T}$
$i_C = \alpha_F i_E = \alpha_F I_S e^{v_{BE}/V_T}$	$i_C = \alpha_F i_E = \alpha_F I_S e^{v_{EB}/V_T}$
$i_B = \frac{i_C}{\beta_F} = \frac{\alpha_F I_S}{\beta_F} e^{v_{BE}/V_T}$	$i_B = \frac{i_C}{\beta_F} = \frac{\alpha_F I_S}{\beta_F} e^{v_{EB}/V_T}$
For both transistors	
$i_C = \beta_F i_B$	$\alpha_F = \frac{\beta_F}{1 + \beta_F}$
$i_E = (1 + \beta_F) i_B$	$\beta_F = \frac{\alpha_F}{1 - \alpha_F}$
$i_C = \left(\frac{\beta_F}{1 + \beta_F} \right) i_E = \alpha_F i_E$	

Common-Emitter NPN and PNP BJT Circuits



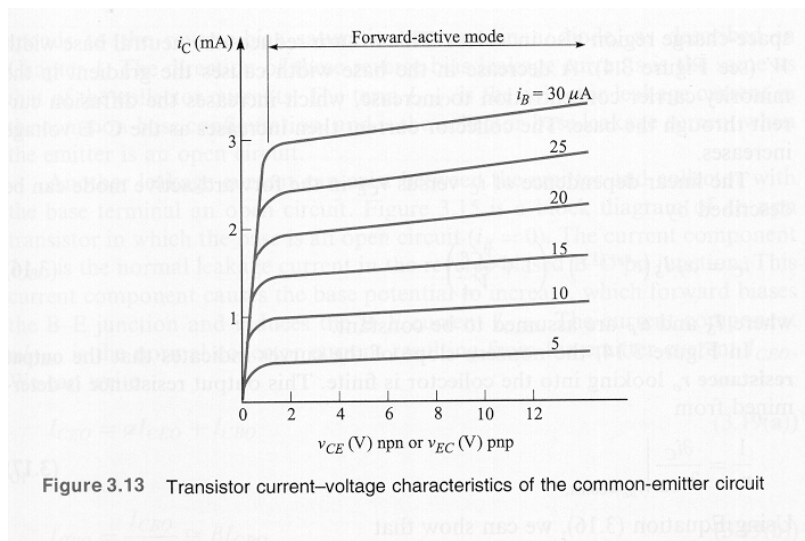
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Typical I vs V “Family of Curves” for the Common-Emitter NPN BJT Circuit

(Ideally, each curve should be horizontal, so $I_C = \beta_F I_B$ for any $V_{CE} > 0.1V$)



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Non-Ideality Parameters for the BJT: V_A and BV_{CEO}

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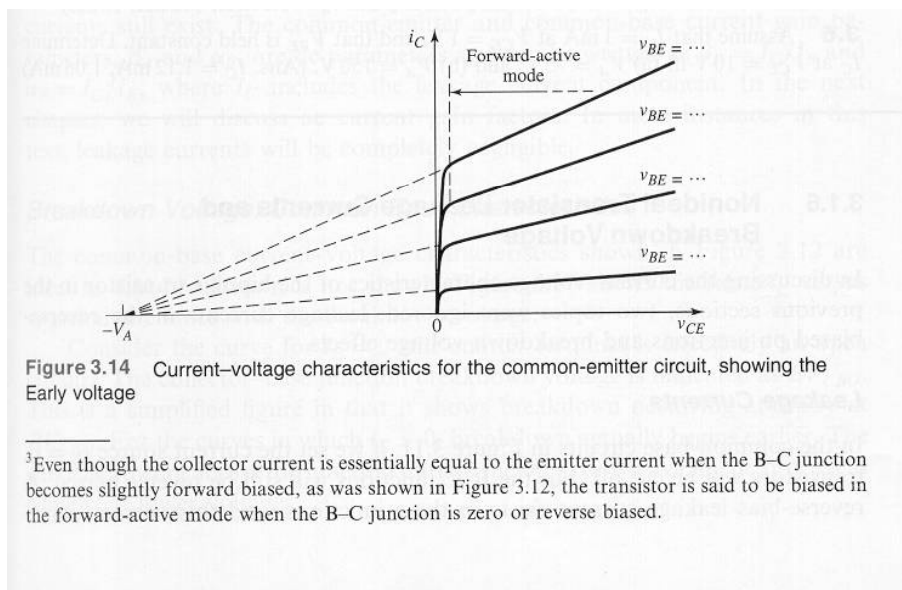
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Early Voltage (Base-Width Modulation) Parameter, V_A

$$50 \text{ V} < V_A < 300 \text{ V}$$

$$I_C = \alpha_F I_S \exp(V_{BE}/V_T)(1 + V_{CE}/V_A)$$



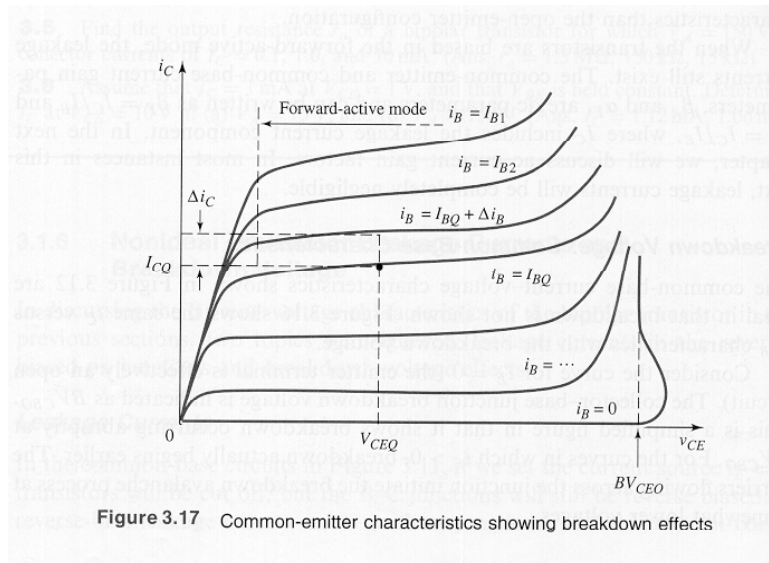
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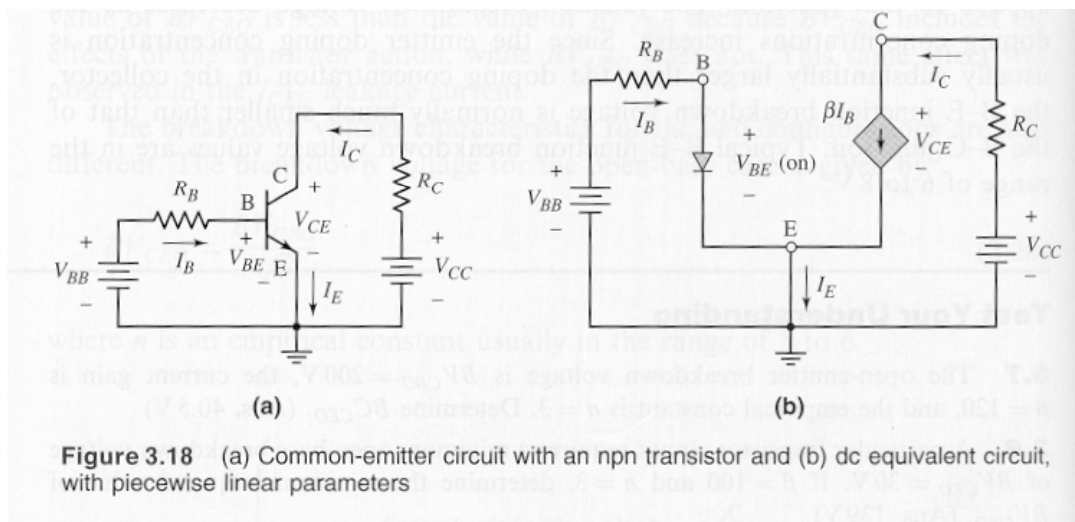
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BV_{CEO} Breakdown Voltage Parameter

V_{ce} gets so large that the reverse-biased BC junction breaks down, allowing I_C to increase dramatically, losing the $I_C = \beta_F I_B$ amplifying effect.



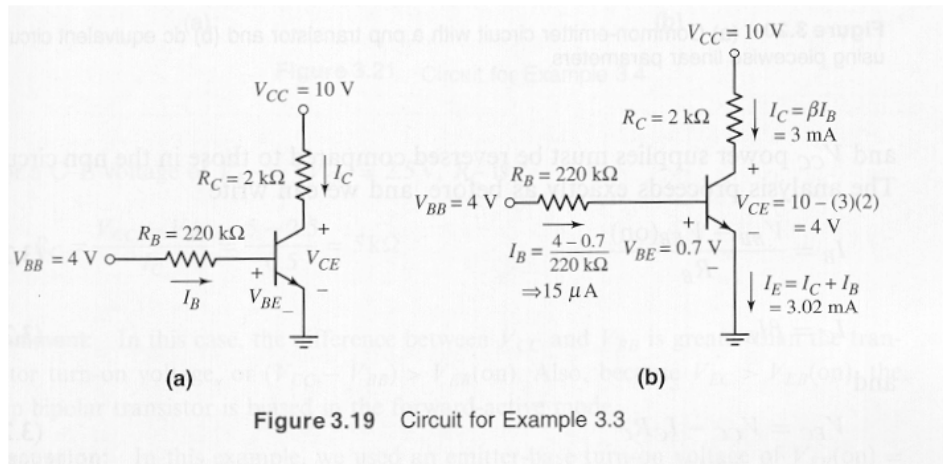
DC Equivalent Forward-Active Model of NPN BJT



NPN BJT DC Analysis

($\beta_F = 200$, $V_{BEON} = 0.7\text{ V}$, $V_{CESAT} = 0.1\text{ V}$)

Since $V_{BB} > V_{BEon}$, assume BJT is Forward Active.



Note: Since V_{CE} came out to be 4 V ($> (V_{CE})_{sat} = 0.1\text{ V}$), the BJT is indeed Forward-Active as initially assumed!

Load Line Analysis of Figure 3.19.

KVL Around Base Loop: $V_{BB} = I_B R_B + V_{BE} \Rightarrow I_B = -V_{BE}/R_B + V_{BB}/R_B$

KVL Around Collector Loop: $V_{CC} = I_C R_C + V_{CE} \Rightarrow I_C = -V_{CE}/R_C + V_{CC}/R_C$

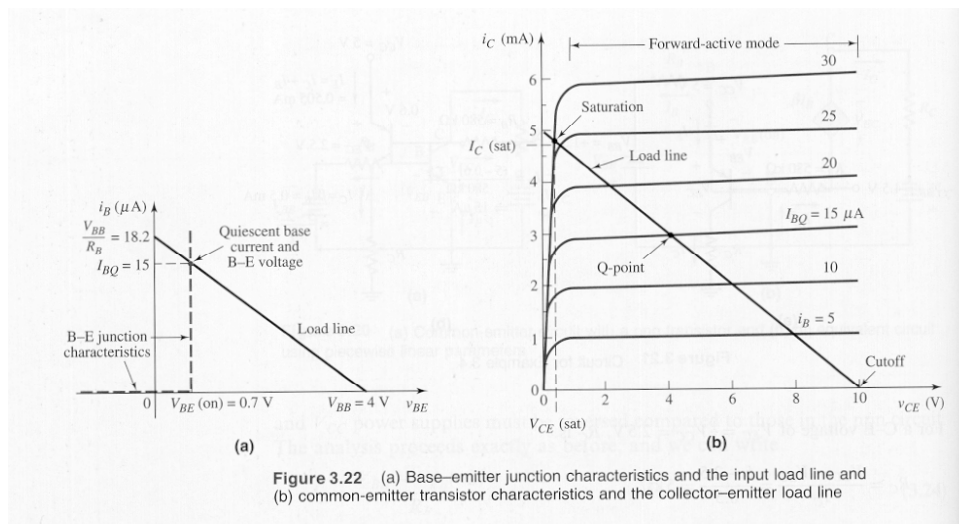
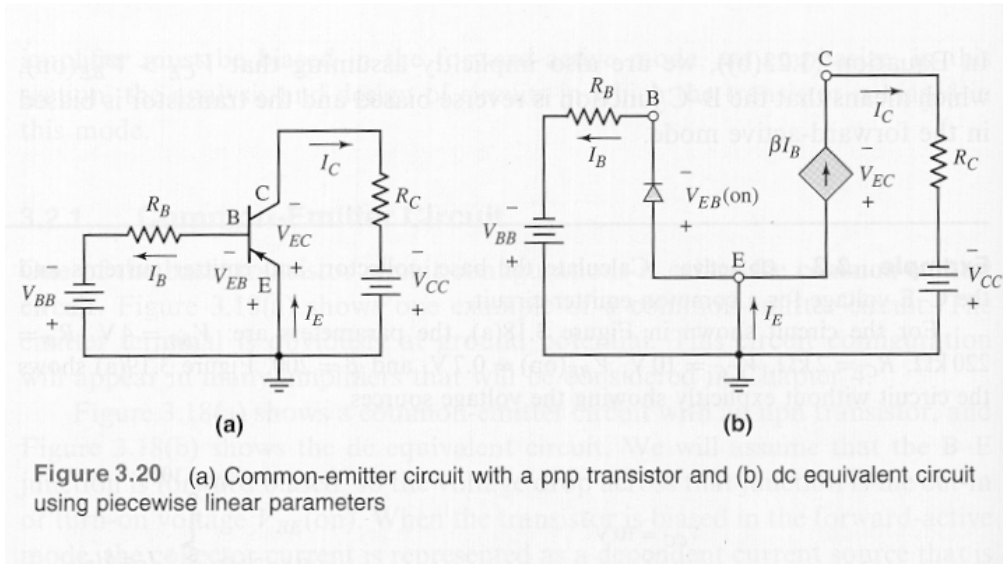


Figure 3.22 (a) Base-emitter junction characteristics and the input load line and (b) common-emitter transistor characteristics and the collector-emitter load line

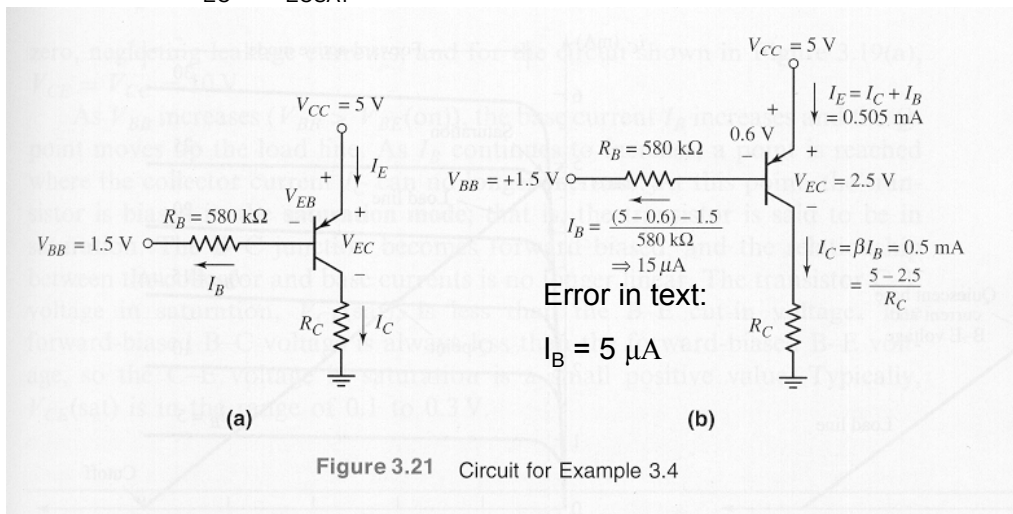
DC Equivalent Forward-Active Model of PNP BJT



PNP BJT DC Analysis

($\beta_F = 100$, $V_{EBON} = 0.6 \text{ V}$, $V_{ECSAT} = 0.2\text{V}$) **FIND: R_c so $V_{ec} = V_{cc}/2$**

Since $V_{EC} > V_{ECSAT}$, NPN BJT is Forward Active.



Final Result: $R_c = (V_{cc} - V_{ec})/I_c = (5 - 2.5)/0.5\text{mA} = 5 \text{ kohms}$.

NPN BJT Analysis Example when BJT Saturated

$$(\beta_F = 100, V_{BE(on)} = 0.7 \text{ V}, V_{CE(sat)} = 0.2 \text{ V})$$

Initially assume forward-active, but when we discover that this assumption leads to $V_{CE} = -3.28 \text{ V} < V_{CE(sat)}$, we must assume BJT is saturated.

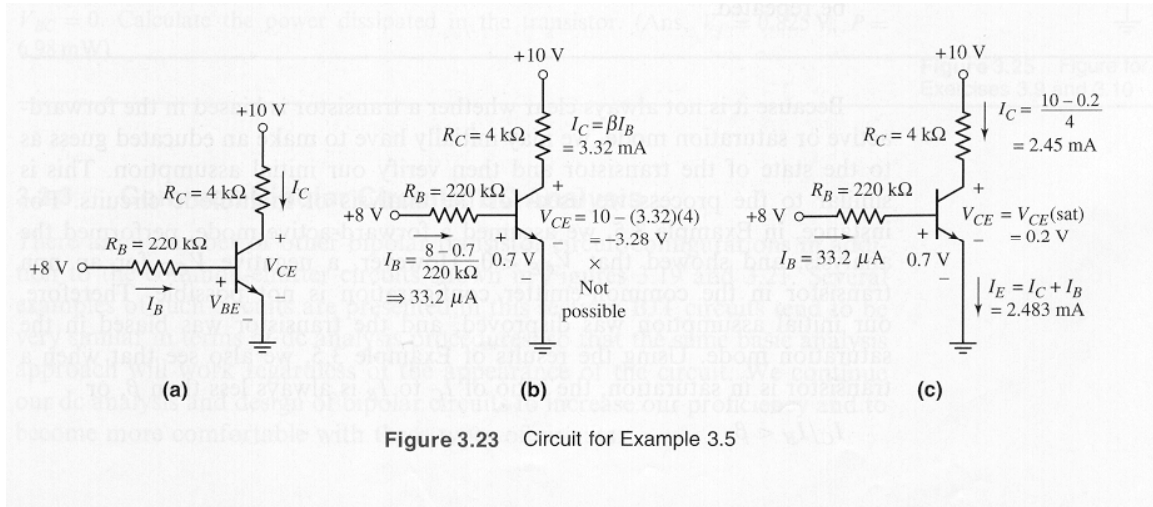


Figure 3.23 Circuit for Example 3.5

Problem-Solving Technique: Bipolar DC Analysis

Analyzing the dc response of a bipolar transistor circuit requires knowing the mode of operation of the transistor. In some cases, the mode of operation may not be obvious, which means that we have to guess the state of the transistor, then analyze the circuit to determine if we have a solution consistent with our initial guess. To do this, we can:

1. Assume that the transistor is biased in the forward-active mode in which case $V_{BE} = V_{BE(on)}$, $I_B > 0$, and $I_C = \beta I_B$.
2. Analyze the "linear" circuit with this assumption.
3. Evaluate the resulting state of the transistor. If the initial assumed parameter values and $V_{CE} > V_{CE(sat)}$ are true, then the initial assumption is correct. However, if $I_B < 0$, then the transistor is probably cut off, and if $V_{CE} < 0$, the transistor is likely biased in saturation.
4. If the initial assumption is proven incorrect, then a new assumption must be made and the new "linear" circuit must be analyzed. Step 3 must then be repeated.

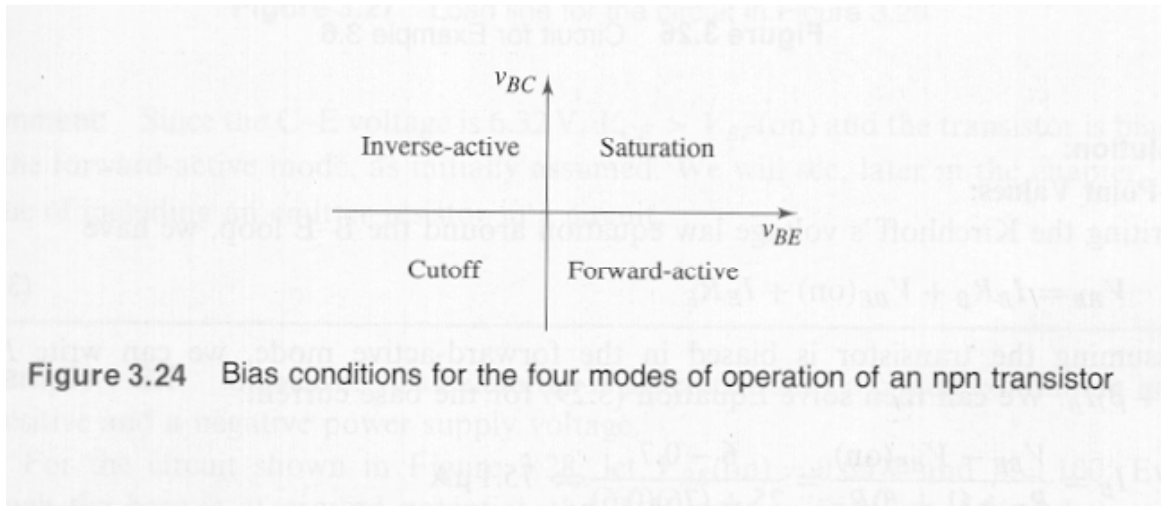


Figure 3.24 Bias conditions for the four modes of operation of an npn transistor

Example 3.6 Objective: Calculate the characteristics of a circuit containing an emitter resistor.

For the circuit shown in Figure 3.26(a), let $V_{BE(on)} = 0.7\text{ V}$ and $\beta = 75$.

Figure 3.26 Circuit for Example 3.6

Solution:
Q-Point Values:
 Writing the Kirchhoff's voltage law equation around the B-E loop, we have

$$V_{BB} = I_B R_B + V_{BE(on)} + I_E R_E \quad (3.29)$$

Assuming the transistor is biased in the forward-active mode, we can write $I_E = (1 + \beta)I_B$. We can then solve Equation (3.29) for the base current:

$$I_B = \frac{V_{BB} - V_{BE(on)}}{R_B + (1 + \beta)R_E} = \frac{6 - 0.7}{25 + (76)(0.6)} \Rightarrow 75.1 \mu\text{A}$$

Plot Load Line using KVL:

$$V_{CC} = R_C I_C + V_{CE} + (\beta_F + 1) (I_C / \beta_F) R_E$$

$$\Rightarrow I_C = -V_{CE} / (R_C + R_E (\beta_F + 1) / \beta_F) + V_{CC} / (R_C + R_E (\beta_F + 1) / \beta_F)$$

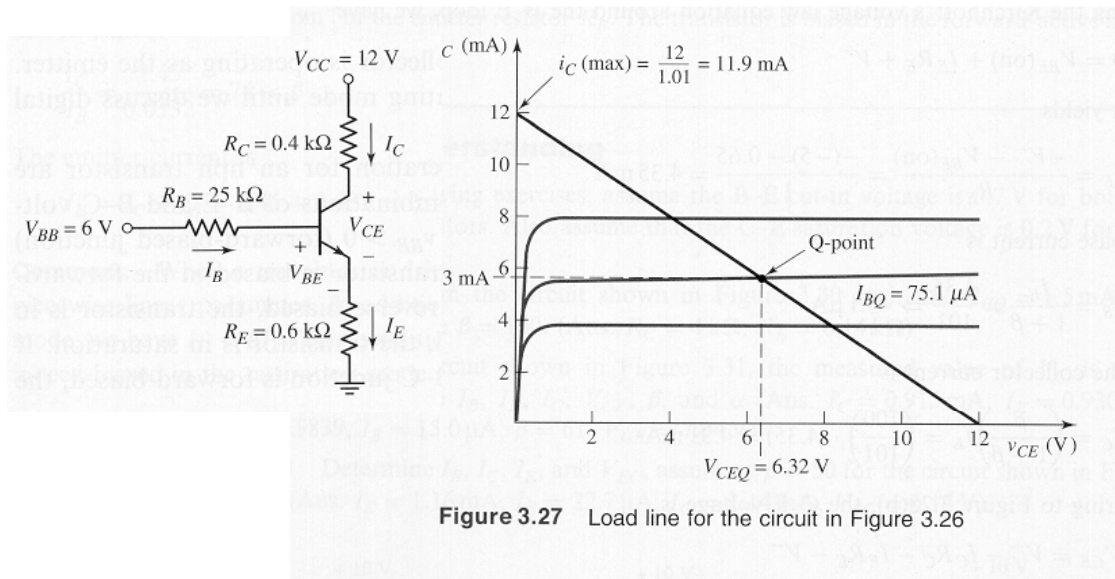


Figure 3.27 Load line for the circuit in Figure 3.26

Example 3.7 Objective: Calculate the characteristics of a circuit containing both a positive and a negative power supply voltage.

For the circuit shown in Figure 3.28, let $V_{BE(on)} = 0.65\text{ V}$ and $\beta = 100$. Even though the base is at ground potential, the B-E junction is forward biased through R_E and V^- .

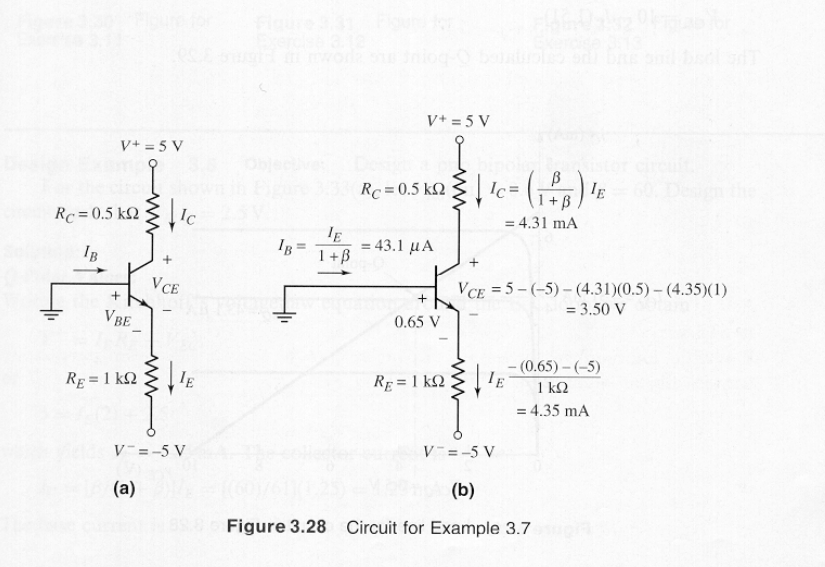


Figure 3.28 Circuit for Example 3.7

Solution:

Load Line:

The load line equation is

$$V_{CE} = (V^+ - V^-) - I_C \left[R_C + \left(\frac{1 + \beta}{\beta} \right) R_E \right] = (5 - (-5)) - I_C \left[0.5 + \left(\frac{101}{100} \right) (1) \right]$$

or

$$V_{CE} = 10 - I_C(1.51)$$

The load line and the calculated Q-point are shown in Figure 3.29.

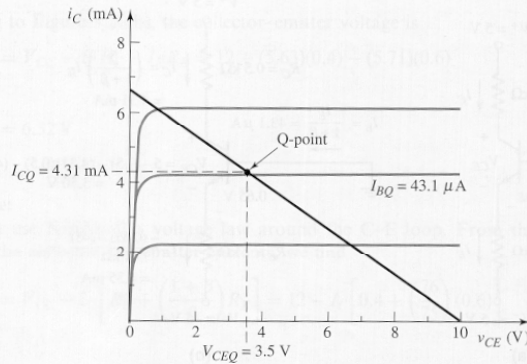
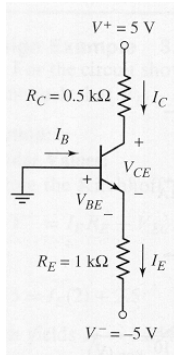


Figure 3.29 Load line for the circuit in Figure 3.28

Example 3.9 Objective: Calculate the characteristics of an npn bipolar circuit with a load resistance. The load resistance can represent a second transistor stage connected to the output of a transistor circuit.

For the circuit shown in Figure 3.36(a), the transistor parameters are: $V_{BE(on)} = 0.7$ V, and $\beta = 100$.

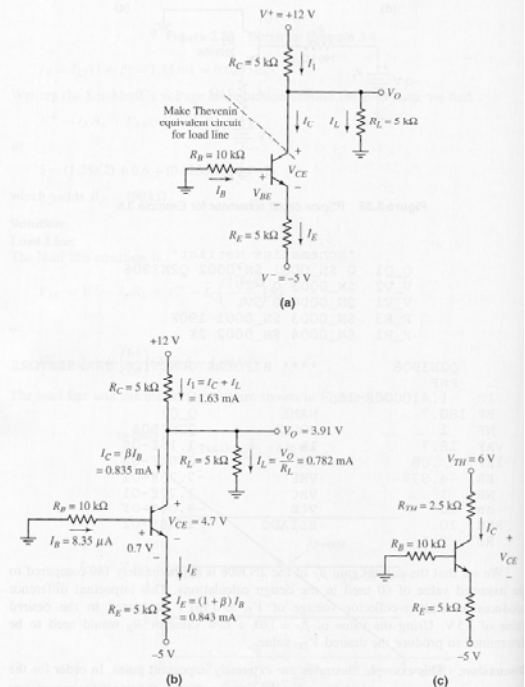


Figure 3.36 Circuit for Example 3.9

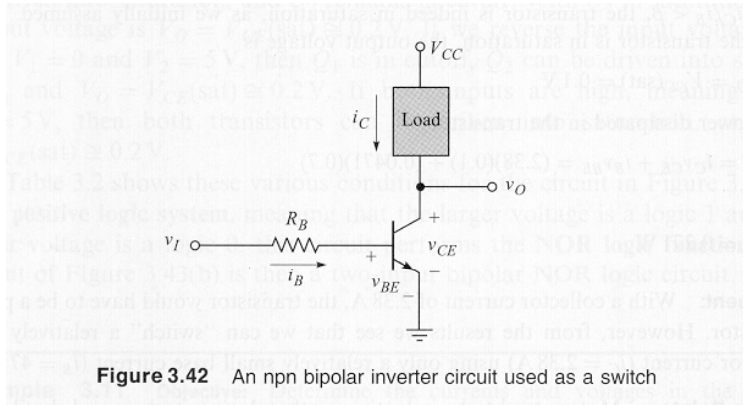


Figure 3.42 An npn bipolar inverter circuit used as a switch

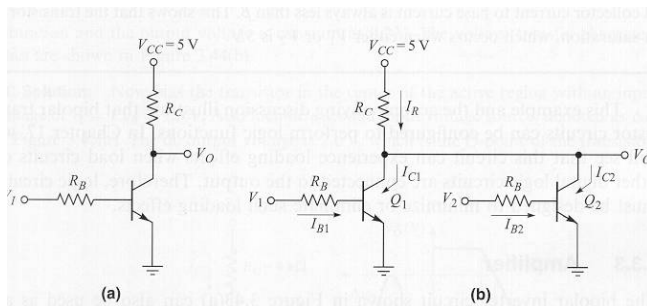


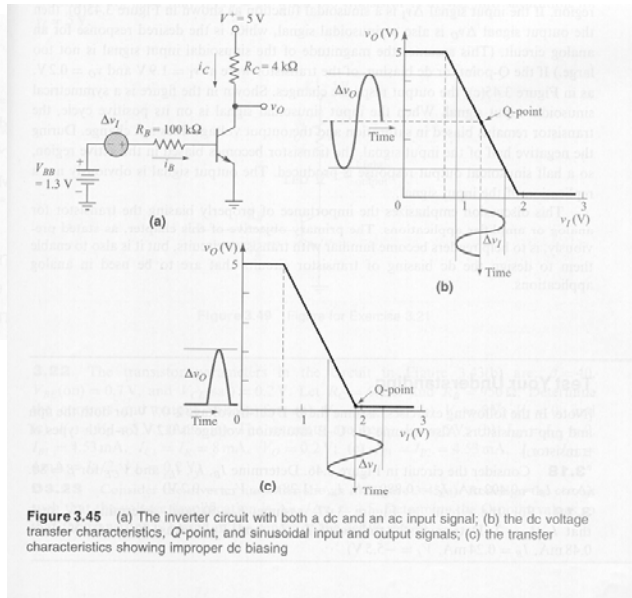
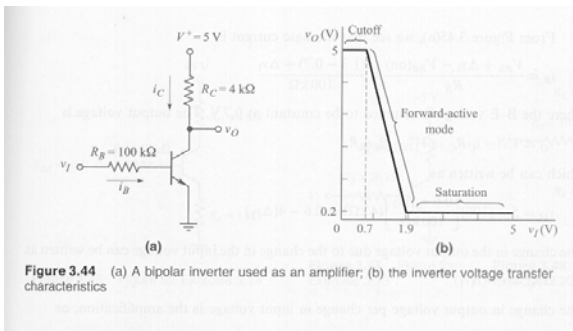
Figure 3.43 A bipolar (a) inverter circuit and (b) NOR logic gate

Table 3.2 The bipolar NOR logic circuit response

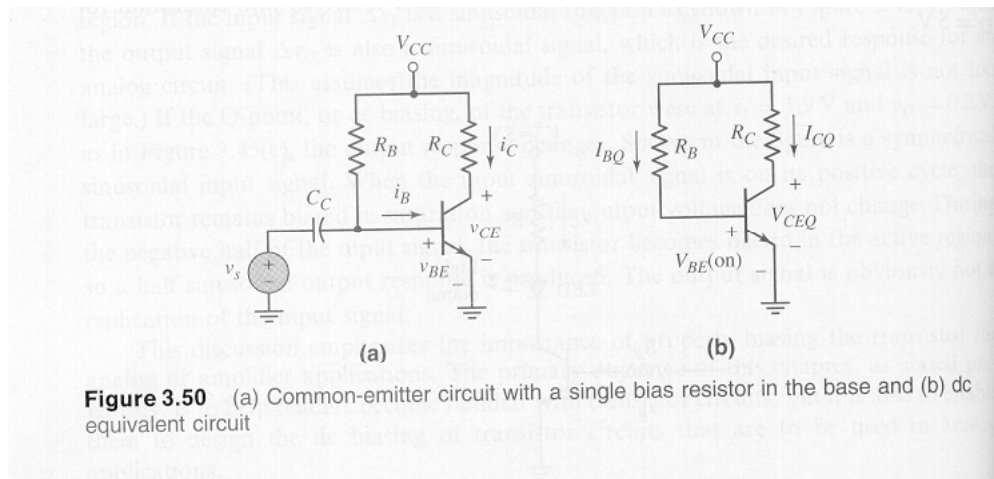
V_1 (V)	V_2 (V)	V_O (V)
0	0	5
5	0	0.2
0	5	0.2
5	5	0.2

Solution: The following table indicates the equations and results for this example.

Condition	V_O	I_R	Q_1	Q_2
$V_1 = 0, V_2 = 0$	5 V	0	$I_{B1} = I_{C1} = 0$	$I_{B2} = I_{C2} = 0$
$V_1 = 5 \text{ V}, V_2 = 0$	0.2 V	$\frac{5 - 0.7}{1} = 4.48 \text{ mA}$	$I_{B1} = \frac{5 - 0.7}{20} = 0.215 \text{ mA}$ $I_{C1} = I_R = 4.8 \text{ mA}$	$I_{B2} = I_{C2} = 0$
$V_1 = 0, V_2 = 5 \text{ V}$	0.2 V	4.8 mA	$I_{B1} = I_{C1} = 0$	$I_{B2} = 0.215 \text{ mA}$ $I_{C2} = I_R = 4.8 \text{ mA}$
$V_1 = 5 \text{ V}, V_2 = 5 \text{ V}$	0.2 V	4.8 mA	$I_{B1} = 0.215 \text{ mA}$ $I_{C1} - I_R = 2.4 \text{ mA}$	$I_{B2} = 0.215 \text{ mA}$ $I_{C2} = \frac{I_R}{2} = 2.4 \text{ mA}$



Single Base Bias Resistor Circuit



Design Example 3.13 Objective: Design the circuit shown in Figure 3.50 to yield a given I_{CQ} and V_{CEQ} . Assume that $V_{CC} = 12\text{ V}$, $\beta = 100$, and $V_{BE(\text{on})} = 0.7\text{ V}$. The Q -point values are $I_{CQ} = 1\text{ mA}$ and $V_{CEQ} = 6\text{ V}$.

Solution: The collector resistance can be found from

$$R_C = \frac{V_{CC} - V_{CEQ}}{I_{CQ}} = \frac{12 - 6}{1} = 6\text{ k}\Omega$$

The base current must then be

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{1\text{ mA}}{100} \Rightarrow 10\text{ }\mu\text{A}$$

and the base resistance is determined to be

$$R_B = \frac{V_{CC} - V_{BE(\text{on})}}{I_{BQ}} = \frac{12 - 0.7}{10\text{ }\mu\text{A}} = 1.13\text{ M}\Omega$$

Load Line:

$$\text{KVL: } V_{CC} = I_C R_C + V_{CE}$$

$$\Rightarrow I_C = -V_{CE}/R_C + V_{CC}/R_C$$

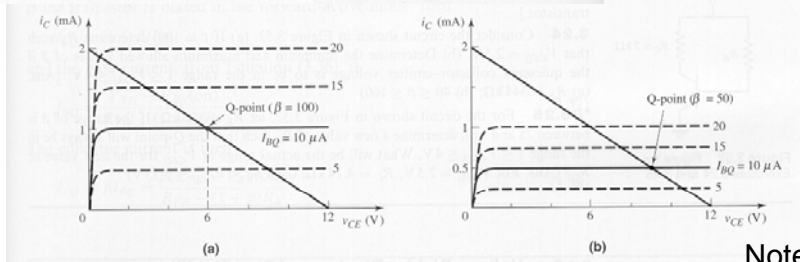


Figure 3.51 Transistor characteristics and load line for the circuit in Example 3.13 when (a) $\beta = 100$ and (b) $\beta = 50$

Note the **LARGE**

VARIATION in I_{CQ} and V_{CEQ} as β is halved!

3-Resistor “Voltage Divider” DC Bias Network

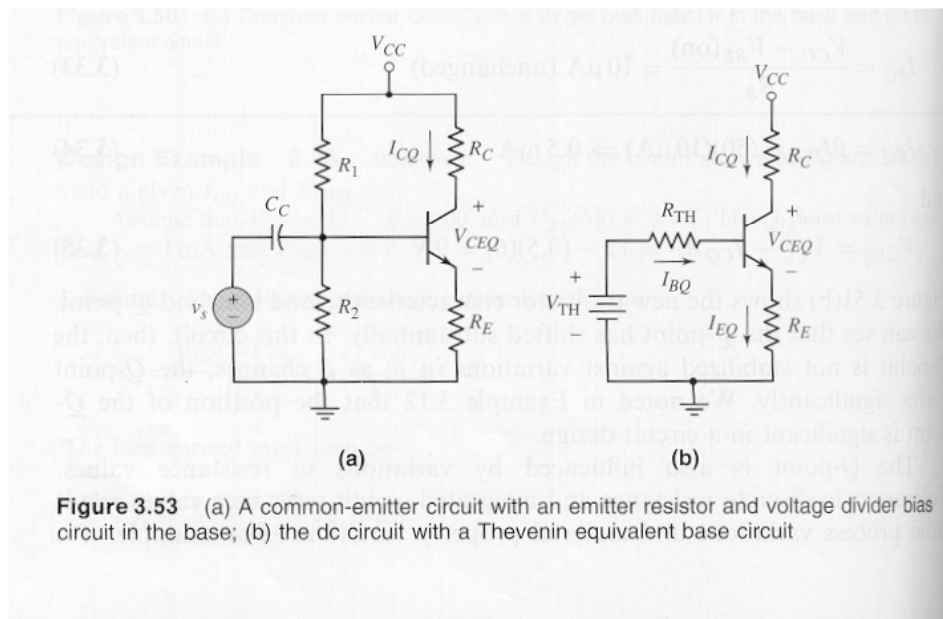


Figure 3.53 (a) A common-emitter circuit with an emitter resistor and voltage divider bias circuit in the base; (b) the dc circuit with a Thevenin equivalent base circuit

Analysis of 3-Resistor Bias Network

$$V_{th} = R_2/(R_1+R_2)V_{cc}$$

$$R_{th} = R_1 // R_2 = R_1 * R_2 / (R_1 + R_2)$$

KVL around base loop:

$$V_{th} = I_{BQ} * R_{TH} + V_{BEon} + (1 + \beta) I_{BQ} * R_E$$

$$\Rightarrow I_{BQ} = (V_{TH} - V_{BEON}) / \{R_{TH} + (1 + \beta)R_E\}$$

$$I_{CQ} = \beta I_{BQ} = \beta(V_{TH} - V_{BEON}) / \{R_{TH} + (1 + \beta)R_E\}$$

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Design for "Bias Stability" w.r.t. β

Problem: β varies over a wide range. For the 2N2222, $80 < \beta < 300$). How do we keep the dc bias current and voltage from changing as β changes?

Solution: For the 3-resistor bias network, we found on the previous slide

$$I_{CQ} = \beta(V_{TH} - V_{BEon}) / \{R_{TH} + (1 + \beta)R_E\}$$

If we make $R_{TH} \ll (1 + \beta)R_E$ then

$$I_{CQ} \approx \beta(V_{TH} - V_{BEon}) / (1 + \beta)R_E$$

Since $\beta / (1 + \beta) \approx 1$ (since β is typically > 100)

$$I_{CQ} \approx (V_{TH} - V_{BEon}) / R_E$$

Thus we can make I_{CQ} approximately independent of β variation, simply by choosing component values so that

$$(1 + \beta)R_E = 10R_{TH}$$

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BJT Common-Emitter (CE) Audio Amplifier Analysis and Design

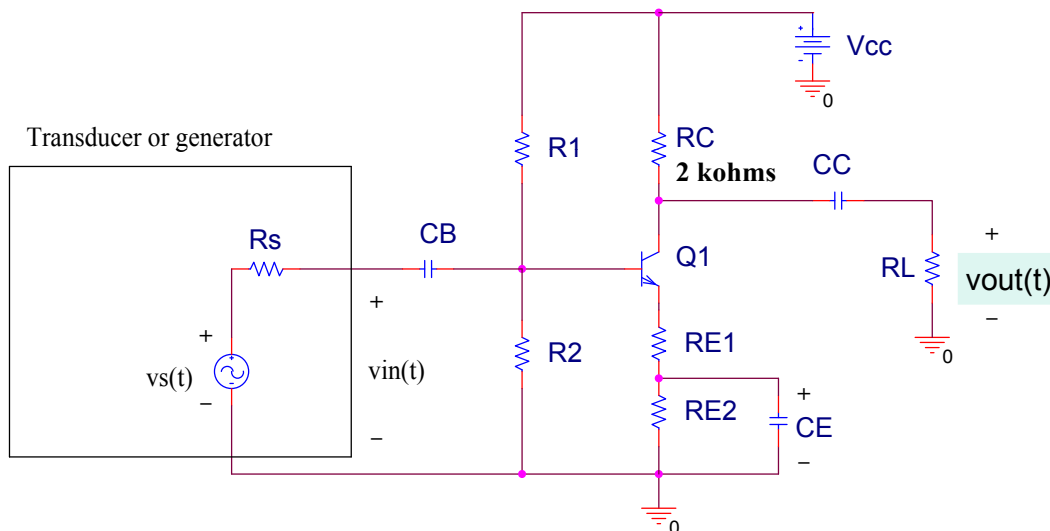
We will start with the complete circuit of a CE amplifier, then we will proceed to its DC model to find its Q-point (I_{cQ} , V_{ceQ}), then its AC model to find its input resistance, output resistance, and its small-signal gain
$$A_v = v_{out}(t) / v_{in}(t)$$

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BJT Common-Emitter (CE) Amplifier



This is the complete CE amplifier circuit that we will analyze using the principle of Linear Superposition. First we shall find the dc bias (quiescent) portion of I_b , V_{ce} , etc. due to the dc source (V_{cc}) acting alone, with $v_s(t)$ set to 0. Then we will find the ac portion of the response due to $v_s(t)$ acting alone with V_{cc} set to 0.

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DC Bias Point Design Problem

In a design problem, you are given the desired Q-point, and you must choose the component values needed to “make this Q-point happen”

We shall follow two “design rules of thumb” to promote bias stability:

1. Choose component values so that $(\beta + 1)R_E \gg R_{TH}$,
Let us make

$$(\beta + 1)R_E = 10R_{TH}$$

2. V_E should be the same order of magnitude as $V_{be(on)}$, so we let us make

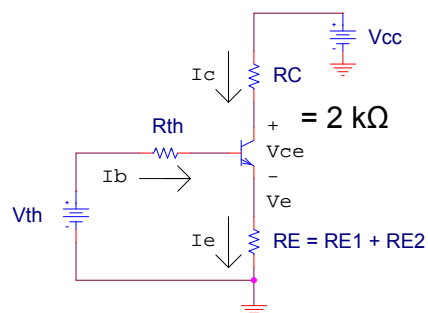
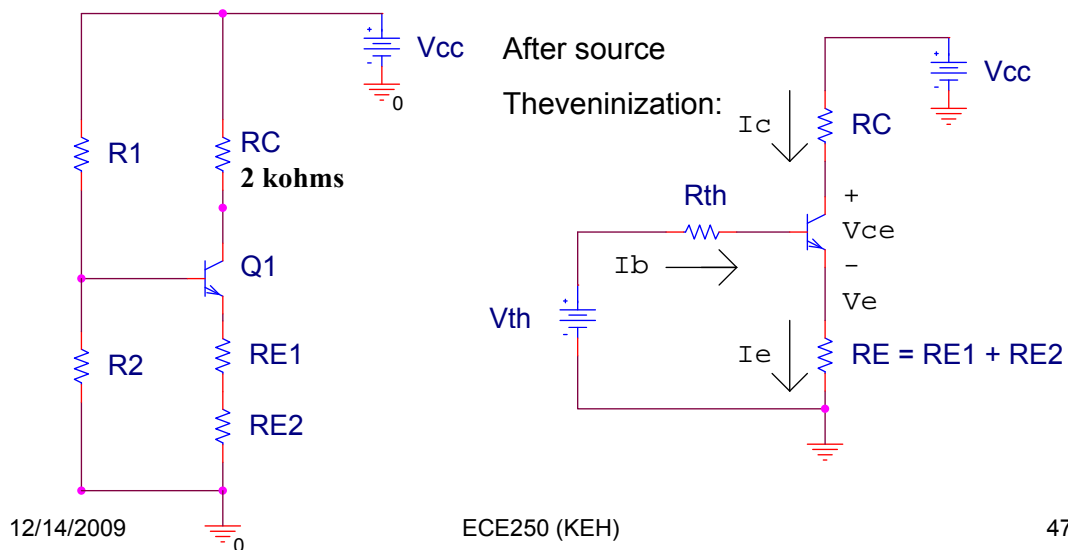
$$V_E = 1.0 \text{ V.}$$

DC Bias Point Design Example

Given: $R_C = 2 \text{ kohms}$, $\beta = 100$, $V_{be(on)} = 0.7 \text{ V}$

Find: R_1 , R_2 , $R_E = R_{E1} + R_{E2}$, and V_{cc} such that BJT is biased at the following Q-Point: ($V_{ceq} = 2 \text{ V}$, $I_{cq} = 1 \text{ mA}$)

Begin by constructing the dc model of the circuit. Set $v_s(t) = 0$, replace all capacitors by open circuits. The dc model becomes:



Note: $V_{th} = (5 \text{ V})(R_2/(R_1+R_2))$

$R_{th} = R_1 \parallel R_2$

But we cannot evaluate V_{th} or R_{th} , since we do not know the values of R_1 and R_2 yet!

We may now calculate R_E :

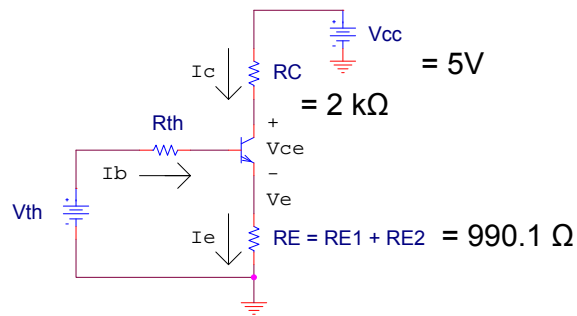
$$R_E = V_E / I_E = 1 \text{ V} / (I_C(\beta+1)/\beta) = 1 \text{ V} / (1 \text{ mA} \cdot (101/100)) = \underline{990.1 \Omega}$$

We require that $I_C = I_{CQ} = 1 \text{ mA}$

$$V_{CE} = V_{CEQ} = 2 \text{ V}$$

also that $V_E = 1.0 \text{ V}$ (rule of thumb)

$$\begin{aligned} \text{Thus } V_{CC} &= I_C \cdot R_C + V_{CEQ} + V_E \\ &= 1 \text{ mA} \cdot 2 \text{ k}\Omega + 2 + 1 \\ &= 5 \text{ V} \end{aligned}$$



Note: $V_{th} = (5\text{ V})(R_2/(R_1+R_2))$

$$R_{th} = R_1 \parallel R_2$$

KVL around the base loop =>

$$V_{th} = I_b R_{th} + 0.7\text{ V} + (\beta+1)I_b R_E \Rightarrow I_b = (V_{th} - 0.7\text{ V})/(R_{th} + (\beta+1)R_E)$$

$$I_c = I_{cq} = \beta I_b = \beta(V_{th} - 0.7\text{ V})/(R_{th} + (\beta+1)R_E) = 1\text{ mA} \quad (\text{desired } I_{cq})$$

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Also, because of Design Rule of Thumb #1, we require

$$R_{th} = 0.1(\beta+1)R_E = 0.1(101)990.1\ \Omega = 10.0\text{ k}\Omega$$

Substituting this value of R_{th} into the equation for I_{cq} :

$$\beta(V_{th} - 0.7\text{ V})/(R_{th} + (\beta+1)R_E) = I_{cq}$$

$$100(V_{th} - 0.7\text{ V})/(10.0\text{ k}\Omega + (101)(990.1\ \Omega)) = 1.0\text{ mA}$$

$$\Rightarrow \underline{V_{th} = 1.80\text{ V}}$$

From the equations for R_{th} and V_{th} , we may solve for R_1 and R_2

$$R_{th} = R_1 R_2 / (R_1 + R_2) = 10\text{ k}\Omega \quad \text{and} \quad V_{th} = V_{cc} R_2 / (R_1 + R_2) = 1.80\text{ V}$$

$$\Rightarrow \underline{R_1 = 27.77\text{ k}\Omega} \quad \text{and} \quad \underline{R_2 = 15.625\text{ k}\Omega}$$

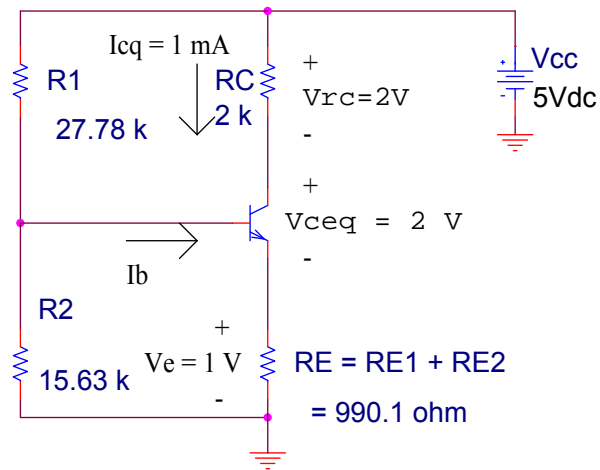
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Final dc bias network design for $\beta = 100$;

Q-Point: ($I_{CQ} = 1 \text{ mA}$, $V_{CEQ} = 2 \text{ V}$)



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DC Bias Point Analysis Example Problem

In an analysis problem, all of the component values are given, and you are to find the resulting Q-point

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DC Bias Point Analysis Example:

Given: The dc bias circuit designed above.

Find: the new Q-point if β is increased from 100 to 200. (Hopefully it will not change much, since we specifically designed the circuit to have its Q-point stable w.r.t. changes in β .)

Note: Since we have changed β and kept everything else the same, we can no longer assume the “rules of thumb” hold. That is, V_E no longer = 1 V, nor does $R_{th} = 0.1(\beta+1)R_E$.

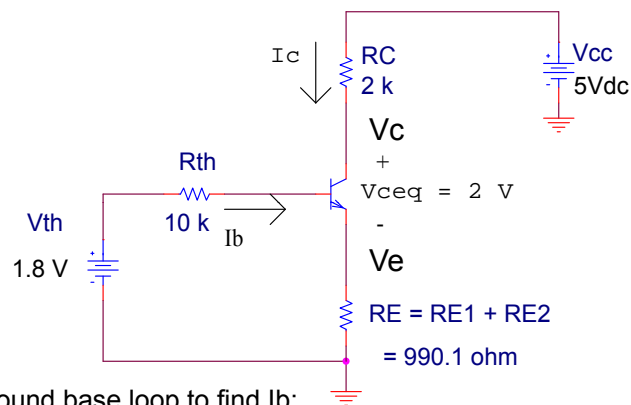
Solution: Begin by Theveninizing source

$$V_{th} = V_{cc}(R_2/(R_2+R_1)) = 1.8 \text{ V} \quad \text{and} \quad R_{th} = R_1 // R_2 = 10 \text{ k}\Omega$$

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Write KVL around base loop to find I_b :

$$V_{th} = I_b R_{th} + 0.7 \text{ V} + I_b (\beta + 1) R_E$$

$$\Rightarrow I_b = (V_{th} - 0.7 \text{ V}) / (R_{th} + (\beta + 1) R_E) = 5.263 \mu\text{A} \Rightarrow I_c = \beta I_b = \underline{1.053 \text{ mA}}$$

$$V_c = V_{cc} - \beta I_b R_C = 5 - 200(5.263 \mu\text{A})(2 \text{ k}\Omega) = 2.895 \text{ V}$$

$$V_e = (\beta + 1) I_b R_E = 201(5.263 \mu\text{A})(990.1 \Omega) = 1.047 \text{ V}$$

$$\text{Thus } V_{ce} = V_c - V_e = 2.895 - 1.047 = \underline{1.847 \text{ V}}$$

Thus the Q-point corresponding to $\beta = 200$ ($I_{cQ} = 1.053 \text{ mA}$, $V_{ceQ} = 1.847 \text{ V}$)

is relatively close to original Q-point corresponding to $\beta = 100$ ($I_{cQ} = 1 \text{ mA}$, $V_{ceQ} = 2 \text{ V}$)

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AC Model of Forward-Active NPN BJT

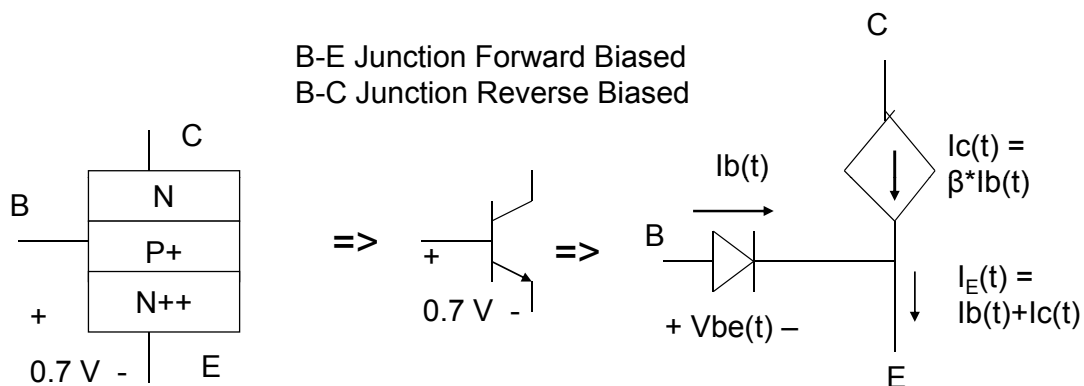
In this section, we shall derive a “linearized” ac small-signal model of the BJT that is analogous to the “rd” linearized ac small-signal model of the diode that was derived earlier.

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For a forward-active BJT, we know that the



For forward-biased diode junction,

$$I_b = I_s(\exp[V_{be}/(\eta V_T)] - 1)$$

$$I_b = I_s \cdot \exp[V_{be}/(\eta V_T)] \quad (\text{fwd-bias approx})$$

(Note: $\eta \approx 1$ for IC BJT, $\eta \approx 2$ for Discrete BJT)

The total base current and base-emitter voltage can be divided into a relatively large dc (quiescent) bias value and a relatively small ac signal

$$I_b(t) = I_{bq} + i_b(t)$$

$$V_{be}(t) = V_{bq} + v_{be}(t)$$

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$$\begin{aligned}
 I_b(t) &= I_{bq} + i_b(t) \\
 &= I_{bq} + \Delta I_b
 \end{aligned}$$

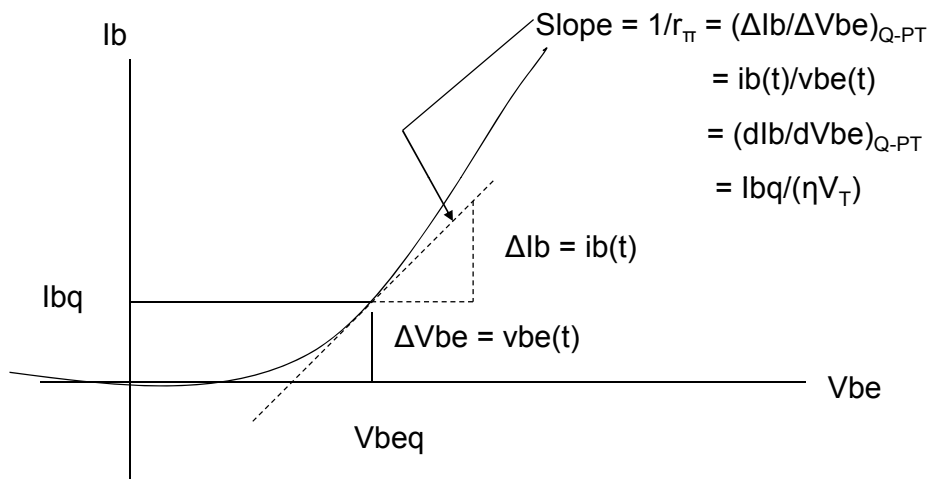
$$\begin{aligned}
 V_{be}(t) &= V_{bq} + v_{be}(t) \\
 &= V_{bq} + \Delta V_{be}
 \end{aligned}$$

$$I_b(t) = I_s \exp(V_{be}(t) / (\eta V_T))$$

Working as we did with the ac model of the diode, let us define r_{π} as the ratio $v_{be}(t)/i_b(t)$. Then the base-emitter diode junction of the BJT can be thought of as a (linear) resistor of value " r_{π} " that converts the ac component of base current $i_b(t)$ into the ac component of base-emitter voltage $v_{be}(t)$ via the equation " $v_{be}(t) = i_b(t) * r_{\pi}$ ".

Let us determine an expression for r_{π}

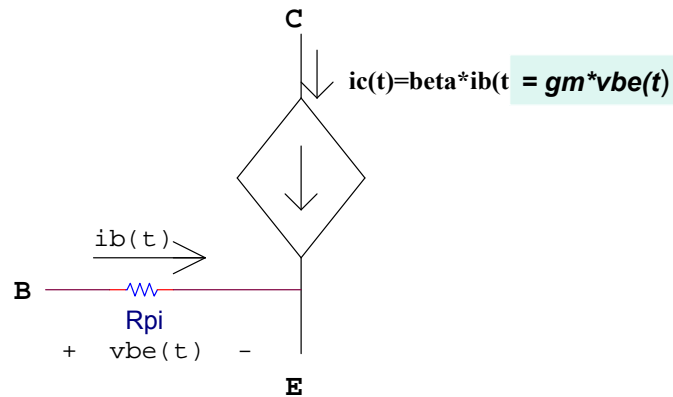
$$\begin{aligned}
 1/r_{\pi} &= i_b(t)/v_{be}(t) = (\Delta I_b / \Delta V_{be})_{Q-PT} = (dI_b/dV_{be})_{I_{bq}, V_{beq}} \\
 &= (1/(\eta V_T) * I_s \exp(V_{be}(t) / (\eta V_T)))_{Q-PT} \\
 &= (1/(\eta V_T)) * I_{bq} = I_{bq} / (\eta V_T)
 \end{aligned}$$



r_{π} is used in place of the B-E junction diode in the small-signal AC model of the BJT in order to change the ac portion of base current $i_b(t)$ into the ac portion of base-emitter voltage $v_{be}(t)$

$$v_{be}(t) = r_{\pi} * i_b(t)$$

AC Model of NPN BJT



Note: $i_c(t) = \beta i_b(t) = \beta \cdot (v_{be}(t)/r_{\pi}) = (\beta / r_{\pi}) v_{be}(t) = g_m \cdot v_{be}(t)$

Where g_m is the BJT's transconductance = (β / r_{π})

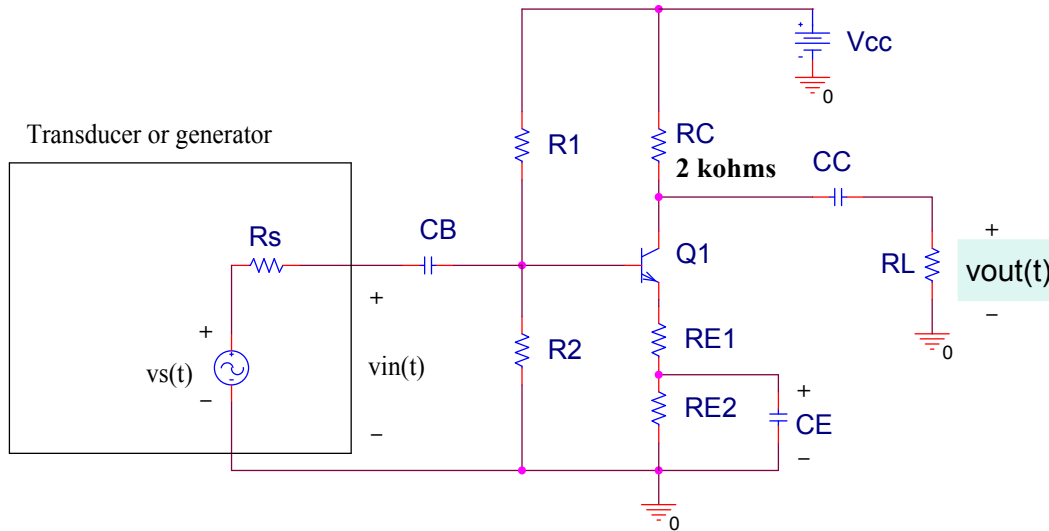
Note $g_m = i_c/v_{be}$ has units of $A/V = \text{mhos}$ or **Siemens**,

And $\beta = i_c/i_b$ which makes it a unitless quantity.

AC Model of BJT Amplifier

In this section, we shall construct the ac model of the CE amplifier circuit, and from it, we shall derive the small-signal ac voltage gain, input impedance, output impedance, etc.

BJT Common-Emitter (CE) Amplifier



This is the complete CE amplifier circuit that we will analyze using the principle of Linear Superposition. First we shall find the dc bias (quiescent) portion of I_b , V_{ce} , etc. due to the dc source (V_{cc}) acting alone, with $v_s(t)$ set to 0. Then we will find the ac portion of the response due to $v_s(t)$ acting alone with V_{cc} set to 0.

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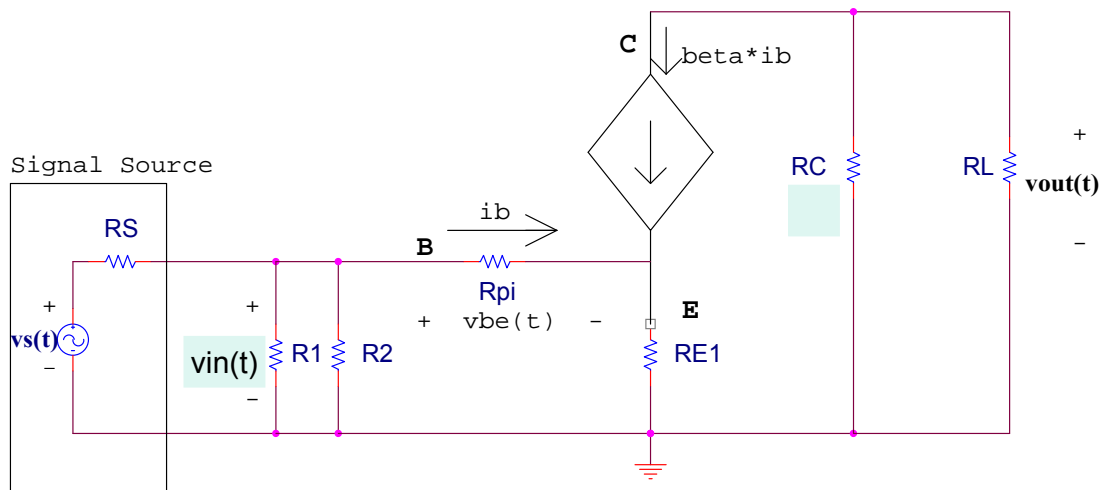
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AC Model of CE Amplifier Circuit

Set $V_{cc} = 0$, replace all capacitors by short circuits.

(We assume that all capacitors are sufficiently high in value so that the magnitude of the impedance of each capacitor ($1/(2\pi f C)$) is \ll than the surrounding resistor values.)



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AC Transducer Voltage Gain Calculations

Define the Transducer Voltage Gain $A_{vT} = v_{out} / v_{in}$

First find i_b by writing a KVL equation around the base loop:

$$v_{in} = i_b \cdot r_{\pi} + (\beta \cdot i_b + i_b) \cdot R_{E1}$$
$$\Rightarrow i_b = \frac{v_{in}}{[r_{\pi} + R_{E1} \cdot (\beta + 1)]}$$

Now find $v_{out}(t)$ by writing a KVL equation around the collector loop

$$v_{out} = -\beta \cdot i_b \cdot \frac{(R_C \cdot R_L)}{R_C + R_L}$$
$$v_{out} = -\beta \cdot \frac{v_{in}}{[r_{\pi} + (\beta + 1) \cdot R_{E1}]} \cdot \frac{R_C \cdot R_L}{(R_C + R_L)}$$
$$A_{vT} = \frac{v_{out}}{v_{in}} = \frac{-\beta}{r_{\pi} + (\beta + 1) \cdot R_{E1}} \cdot \frac{R_C \cdot R_L}{(R_C + R_L)}$$

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General Equation for transducer voltage gain:

$$A_{vT} = \frac{v_{out}}{v_{in}} = \frac{-\beta}{r_{\pi} + (\beta + 1) \cdot R_{E1}} \cdot \frac{R_C \cdot R_L}{(R_C + R_L)}$$

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Special Case 1: If RE is fully bypassed (CE is connected across entire emitter resistor, so RE1 = 0), the equation for AvT reduces to:

$$A_{vT} = \frac{-\beta}{r_{\pi}} \cdot \frac{RC \cdot RL}{(RC + RL)}$$

This results in the highest possible voltage gain, but the gain is very dependent upon β , and thus the gain cannot be tightly controlled from one circuit board to the next, since β varies from one BJT to another, even if they are of the same type.

Special Case 2: If $(\beta+1)RE1 \gg r_{\pi}$, the general equation for AvT reduces to:

$$A_{vT} = \frac{-\beta}{(\beta + 1) \cdot RE1} \cdot \frac{RC \cdot RL}{(RC + RL)}$$

For β sufficiently large, $\beta/(\beta+1) \approx 1$, so

$$A_{vT} = \frac{-1}{RE1} \cdot \frac{RC \cdot RL}{RC + RL}$$

Note in this case, AvT is **independent of β** , and is set solely by the resistor ratio $-(RC // RL) / RE1$. Unfortunately, this usually results in a relatively small AvT.

In our design example, let $R_L = 4 \text{ k}\Omega$

Given:

$$\eta := 2 \quad (\text{This is a discrete BJT}) \quad \beta_{\text{low}} := 100 \quad \beta_{\text{high}} := 200$$

$$R_1 := 27.78 \cdot \text{k}\Omega \quad R_2 := 15.63 \cdot \text{k}\Omega \quad R_C := 2 \cdot \text{k}\Omega \quad R_L := 4 \cdot \text{k}\Omega$$

Find AC model B-E resistance " r_π " for $\beta=100$ and for $\beta=200$:

$$r_{\pi 100} := \frac{\eta \cdot 26 \cdot \text{mV}}{\left(\frac{1.0 \cdot \text{mA}}{\beta_{\text{low}}} \right)} \quad r_{\pi 100} = 5.2 \times 10^3 \Omega$$

$$r_{\pi 200} := \frac{\eta \cdot 26 \cdot \text{mV}}{\left(\frac{1.053 \cdot \text{mA}}{\beta_{\text{high}}} \right)} \quad r_{\pi 200} = 9.877 \times 10^3 \Omega$$

Case 1: Fully Bypassed Case (CE across entire RE)

$$\Rightarrow \quad R_{E2} := 990.1 \cdot \Omega \quad R_{E1} := 0 \cdot \Omega$$

$$A_{vT200} := \frac{-\beta_{\text{high}}}{r_{\pi 200} + (\beta_{\text{high}} + 1) \cdot R_{E1}} \cdot \frac{R_C \cdot R_L}{(R_C + R_L)} \quad A_{vT200} = -27$$

$$A_{vT100} := \frac{-\beta_{\text{low}}}{r_{\pi 100} + (\beta_{\text{low}} + 1) \cdot R_{E1}} \cdot \frac{R_C \cdot R_L}{(R_C + R_L)} \quad A_{vT100} = -25.641$$

Case 2: Partially Bypassed Case (CE across all but 50 ohms of RE)

$$\Rightarrow \quad RE2 := 940.1 \cdot \Omega \quad RE1 := 50 \cdot \Omega$$

$$A_{vT200} := \frac{-\beta_{high}}{r_{\pi 200} + (\beta_{high} + 1) \cdot RE1} \cdot \frac{RC \cdot RL}{(RC + RL)} \quad A_{vT200} = -13.382$$

$$A_{vT100} := \frac{-\beta_{low}}{r_{\pi 100} + (\beta_{low} + 1) \cdot RE1} \cdot \frac{RC \cdot RL}{(RC + RL)} \quad A_{vT100} = -13.008$$

Case 3: Unbypassed Case (CE not present)

$$\Rightarrow \quad RE2 := 0 \cdot \Omega \quad RE1 := 990.1 \cdot \Omega$$

$$A_{vT200} := \frac{-\beta_{high}}{r_{\pi 200} + (\beta_{high} + 1) \cdot RE1} \cdot \frac{RC \cdot RL}{(RC + RL)} \quad A_{vT200} = -1.277$$

$$A_{vT100} := \frac{-\beta_{low}}{r_{\pi 100} + (\beta_{low} + 1) \cdot RE1} \cdot \frac{RC \cdot RL}{(RC + RL)} \quad A_{vT100} = -1.267$$

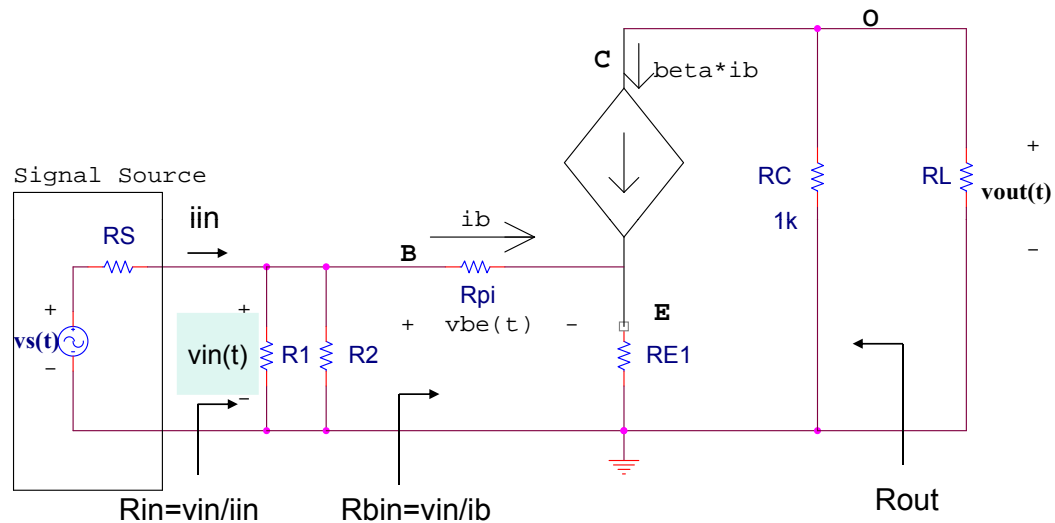
Input Impedance Rin

Input Impedance (R_{in}) is the impedance seen looking into the input terminals, R_{in} is the ratio of the input current to the input voltage (i_{in}/v_{in})

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$$\text{KVL base loop} \Rightarrow v_{in} = i_b \cdot r_{\pi} + (\beta + 1) \cdot i_b \cdot R_{E1}$$

$$\Rightarrow R_{bin} = v_{in} / i_b = r_{\pi} + (\beta + 1) \cdot R_{E1}$$

$$R_{in} = v_{in} / i_{in} = R_{bin} \parallel R_1 \parallel R_2$$

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Output Impedance Rout

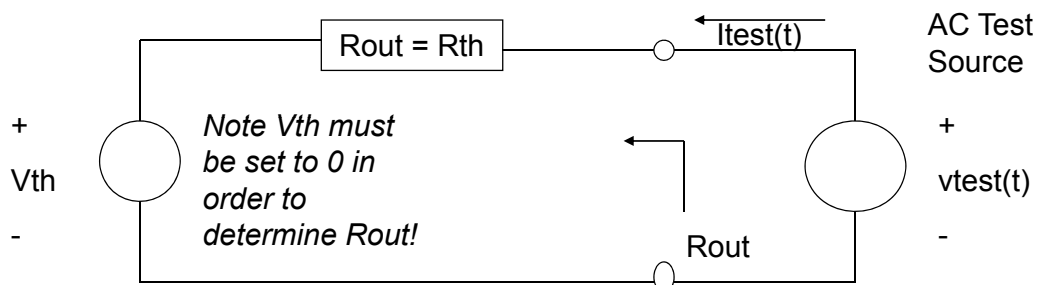
Rout is the Thevenin Equivalent resistance seen looking into the output terminals

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Imagine that the Thevenin Equivalent looking into the output terminals is given by V_{th} , R_{out} below:



R_{out} can be determined by connecting an ac small-signal test source, $v_{test}(t)$ across the output terminals **AND BY SETTING $V_{th} = 0$** . Then

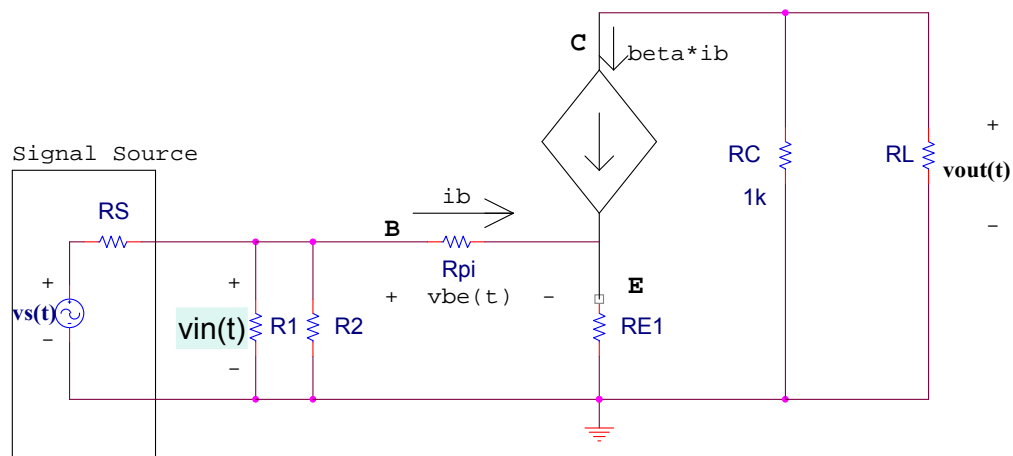
$$R_{out} = \{v_{test}(t)/i_{test}(t)\}_{V_{th} = 0}$$

V_{th} represents the effects of all independent sources in the circuit. Therefore, you must set all sources in the actual circuit = 0. In this case, you must set $v_s(t) = 0$.

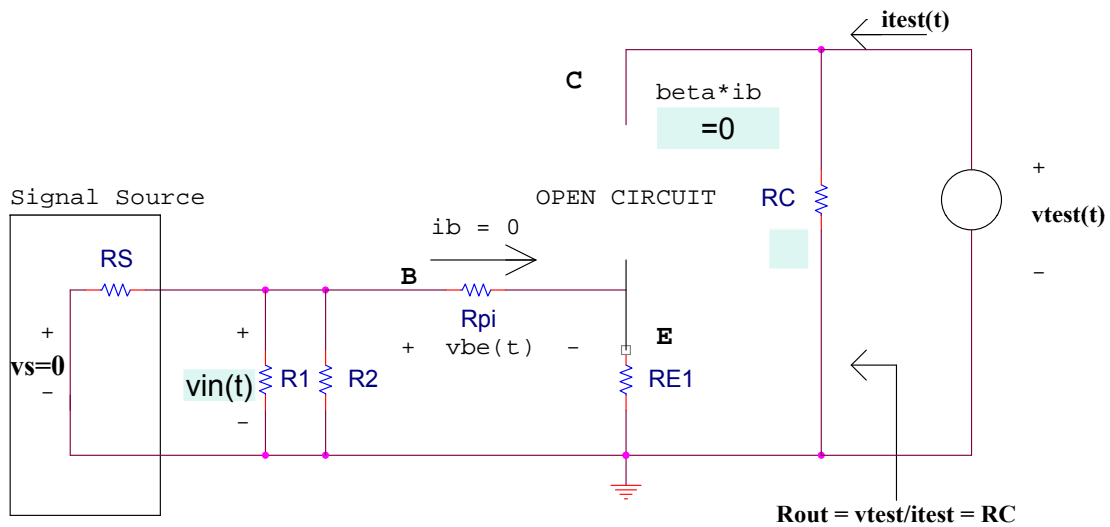
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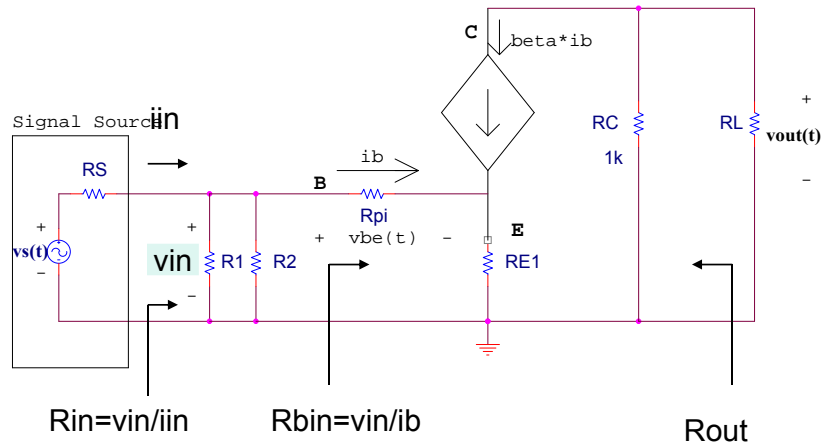


If $v_s(t)$ is set to 0 in this circuit, i_b will = 0, and so $i_c = \beta i_b = 0$, thus the controlled current source may be erased in the diagram above, resulting in....



It should be apparent, by inspection, that

$$\mathbf{R_{out} = v_{test}/i_{test} = R_C}$$



For Complete Bypass Example (Case 1, $R_{E1} = 0, \beta = 100$)

$$R_{bin} := r_{\pi}100 \qquad R_{bin} = 5.2 \times 10^3 \Omega$$

$$R_{in} := \frac{1}{\frac{1}{R_{bin}} + \frac{1}{R_1} + \frac{1}{R_2}} \qquad R_{in} = 3.421 \times 10^3 \Omega$$

$$R_{out} := R_C \qquad R_{out} = 2 \times 10^3 \Omega$$

For Partial Bypass Example (Case 2, $R_{E1} = 50 \Omega, \beta = 100$)

$$R_{bin} := r_{\pi}100 + (\beta_{low} + 1) \cdot 50 \cdot \Omega \qquad R_{bin} = 1.025 \times 10^4 \Omega$$

$$R_{in} := \frac{1}{\frac{1}{R_{bin}} + \frac{1}{R_1} + \frac{1}{R_2}} \qquad R_{in} = 5.062 \times 10^3 \Omega$$

$$R_{out} := R_C \qquad R_{out} = 2 \times 10^3 \Omega$$

For unbypassed Example (Case 3, $R_{E1} = 990.1 \Omega, \beta = 100$)

$$R_{bin} := r_{\pi}100 + (\beta_{low} + 1) \cdot 990.1 \cdot \Omega \qquad R_{bin} = 1.052 \times 10^5 \Omega$$

$$R_{in} := \frac{1}{\frac{1}{R_{bin}} + \frac{1}{R_1} + \frac{1}{R_2}} \qquad R_{in} = 9.134 \times 10^3 \Omega$$

$$R_{out} := R_C \qquad R_{out} = 2 \times 10^3 \Omega$$

Conclusions:

- **Full Emitter Bypassing:** Highest A_{vT} but most dependent upon β . Lowest R_{in} .
- **No Emitter Bypassing:** Smallest A_{vT} but least dependent upon β . Highest R_{in}
- **Partial Emitter Bypassing:** Yields a tradeoff between size of A_{vT} and β stability. Moderately high R_{in} .

General Voltage Amplifier Model

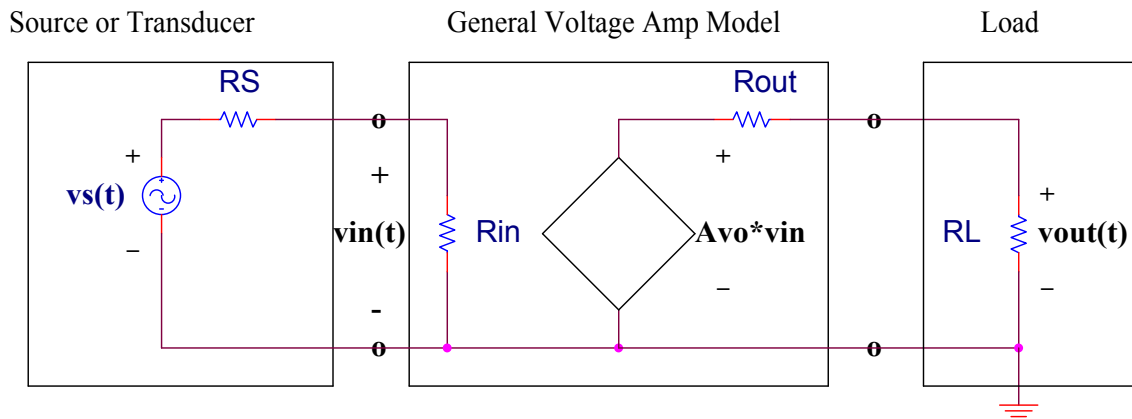
This model holds for all voltage amplifiers, be they BJT, FET, Vacuum Tube, OP-AMP. This model is DIFFERENT from the BJT model we have used thus far. Its parameters are R_{in} , R_{out} , and A_{vo} (unloaded voltage gain).

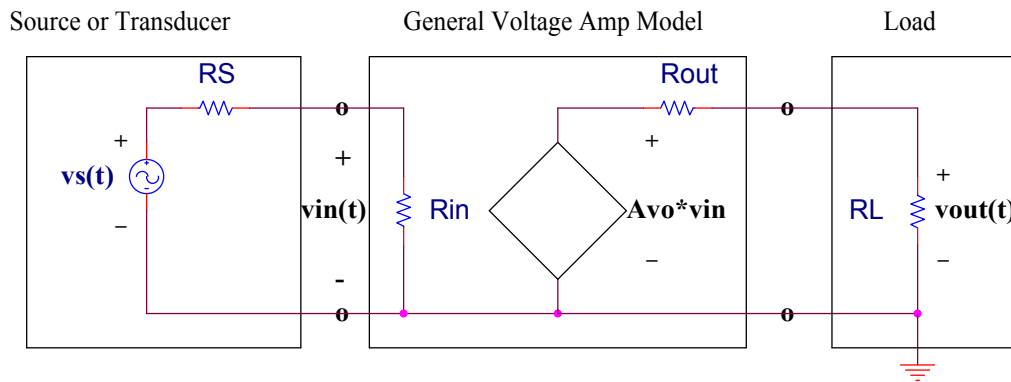
Note that this model uses a voltage-controlled voltage source, rather than a current-controlled current source, as in the BJT model.

We shall see that this model allows us to very easily find $A_v = v_{out}/v_s$ or $A_{v_T} = v_{out}/v_{in}$ for an amplifier with arbitrary source termination (R_S) and load termination (R_L) by application of the voltage divider equation.

Let us define A_{vo} as the “unloaded”, or “open circuit” voltage gain
 $A_{vo} = v_{out}/v_{in})_{R_L=\infty}$

General Voltage Amplifier with Source termination R_S and Load termination R_L





Overall Voltage Gain:

$$\begin{aligned} \mathbf{A_v} &= \mathbf{v_{out}/v_s} = (\mathbf{v_{in}/v_s}) \cdot (\mathbf{A_{vo} \cdot v_{in}}) / \mathbf{v_{in}} \cdot (\mathbf{v_{out}/A_{vo} \cdot v_{in}}) \\ &= \mathbf{R_{in}/(R_S + R_{in}) \cdot A_{vo} \cdot R_L / (R_{out} + R_L)} \end{aligned}$$

Transducer Voltage Gain (directly measurable in laboratory):

$$\begin{aligned} \mathbf{A_{v_T}} &= \mathbf{v_{out}/v_{in}} = (\mathbf{A_{vo} \cdot v_{in}}) / \mathbf{v_{in}} \cdot (\mathbf{v_{out}/A_{vo} \cdot v_{in}}) \\ &= \mathbf{A_{vo} \cdot R_L / (R_{out} + R_L)} \end{aligned}$$

Returning to our Fully-Bypassed CE Amplifier Example

Let us assume that

$$\begin{aligned} R_{E1} &= 0 \, \Omega, \quad R_1 = 27.8 \, \text{k}\Omega, \quad R_2 = 15.63 \, \text{k}\Omega \\ R_E &= 990 \, \Omega, \quad R_C = 2 \, \text{k}\Omega, \quad R_L = 4 \, \text{k}\Omega, \\ R_S &= 1 \, \text{k}\Omega, \quad \text{and} \quad \beta = 100 \end{aligned}$$

For the Fully Bypassed Example ($R_{E1} = 0 \Omega$)

First find the unloaded voltage gain (with R_L set to infinity)

$$A_{vo} := \frac{-\beta}{r_{\pi}} \cdot R_C \quad A_{vo} = -38.462$$

$$R_{bin} := r_{\pi} \quad R_{in} := \frac{1}{\frac{1}{R_{bin}} + \frac{1}{R_1} + \frac{1}{R_2}}$$
$$R_{in} = 3.421 \times 10^3 \Omega$$

$$R_{out} := R_C \quad R_{out} = 2 \times 10^3 \Omega$$

$$A_v := \frac{R_{in}}{R_S + R_{in}} \cdot A_{vo} \cdot \frac{R_L}{R_L + R_{out}} \quad A_v = -19.842$$

$$A_{vT} := A_{vo} \cdot \frac{R_L}{R_L + R_{out}} \quad A_{vT} = -25.641$$

For the partially-bypassed example $R_{E1} := 50 \cdot \Omega$

$$A_{vo} := \frac{-\beta}{r_{\pi} + (\beta + 1) \cdot R_{E1}} \cdot R_C \quad A_{vo} = -19.512$$

$$R_{bin} := r_{\pi} + (\beta + 1) \cdot R_{E1} \quad R_{in} := \frac{1}{\frac{1}{R_{bin}} + \frac{1}{R_1} + \frac{1}{R_2}}$$
$$R_{in} = 5.062 \times 10^3 \Omega$$

$$R_{out} := R_C \quad R_{out} = 2 \times 10^3 \Omega$$

$$A_v := \frac{R_{in}}{R_S + R_{in}} \cdot A_{vo} \cdot \frac{R_L}{R_L + R_{out}} \quad A_v = -10.862$$

$$A_{vT} := A_{vo} \cdot \frac{R_L}{R_L + R_{out}} \quad A_{vT} = -13.008$$

DC and AC Load Lines

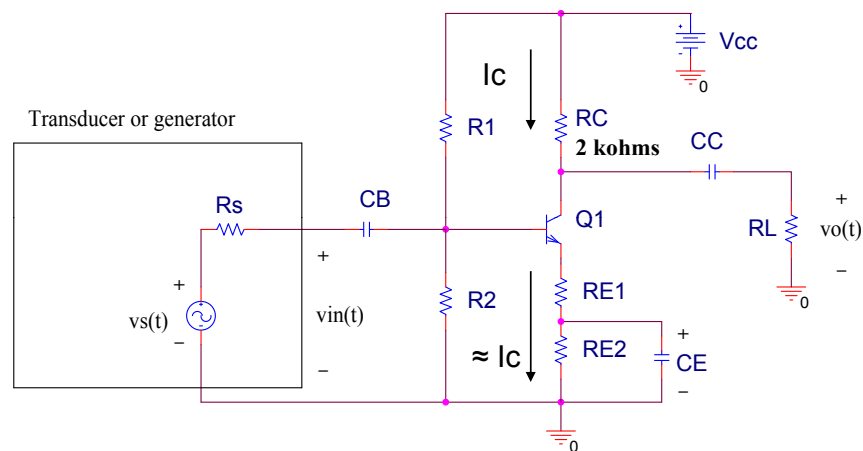
Maximum Symmetrical $V_{ce}(t)$ Output Voltage Swing

-Finding the largest permissible sinusoidal output voltage swing

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From DC Model, a KVL equation around collector loop yields

$$V_{cc} = I_c R_C + V_{ce} + (I_c / \beta) (\beta + 1) R_E \approx I_c R_C + V_{ce} + I_c R_E \quad (\text{Recall } R_E = R_{E1} + R_{E2})$$

$$V_{cc} = I_c (R_C + R_E) + V_{ce}$$

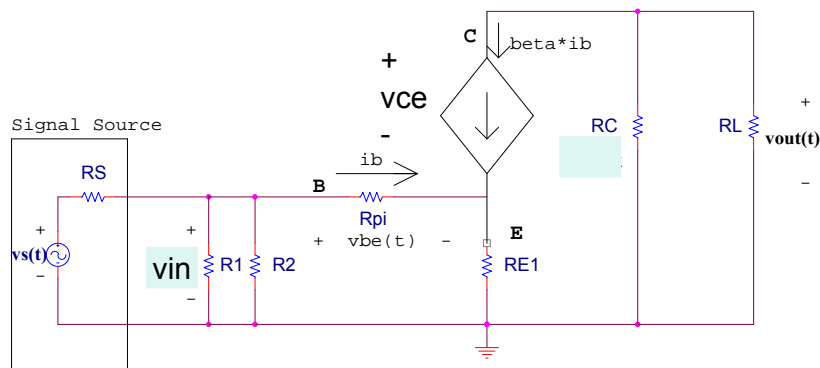
$$\Rightarrow I_c = -1 / (R_C + R_E) V_{ce} + V_{cc} / (R_C + R_E)$$

Thus I_c vs. V_{ce} DC load line has I_c intercept of $I_c = V_{cc} / (R_C + R_E)$,
a slope of $-1 / (R_C + R_E)$, and a V_{ce} intercept of $V_{ce} = V_{cc}$

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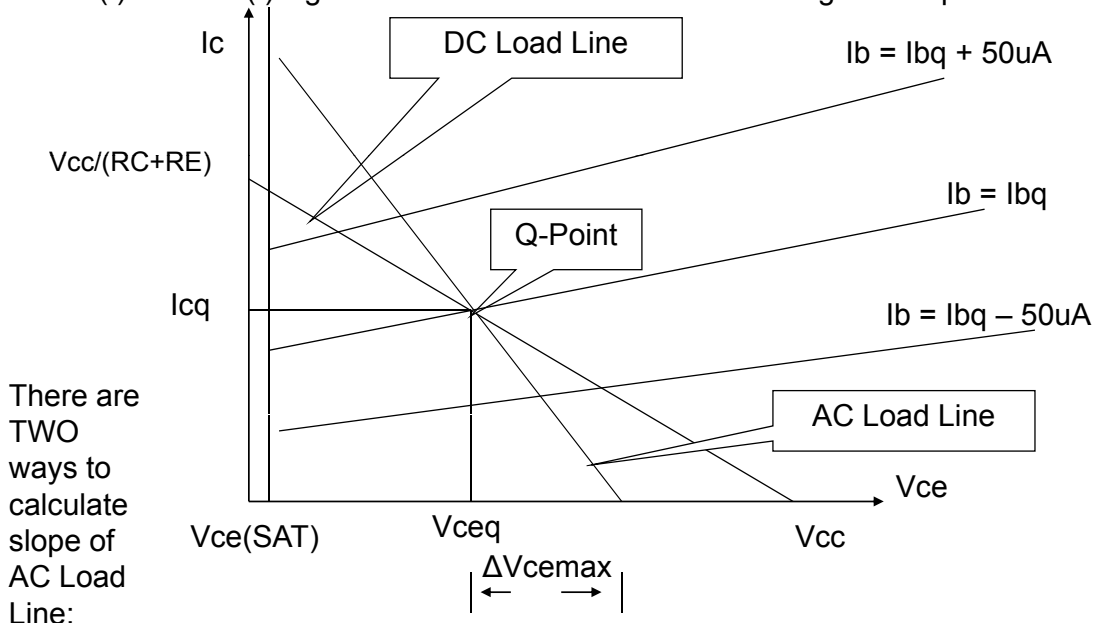
In the AC model of this circuit, where V_{cc} has been set to zero, and capacitors C_C and C_E have become short circuits, a KVL equation around the collector loop reveals (Assuming : $i_e \approx i_c$)

$$0 = i_c(t) \cdot (R_C \parallel R_L) + v_{ce}(t) + R_{E1} \cdot i_c(t)$$

$$i_c = -v_{ce} / (R_{E1} + (R_C \parallel R_L))$$

Thus the slope of the AC load line is $1 / (R_{E1} + (R_C \parallel R_L))$, which is considerably steeper than slope of the DC load line, $-1 / (R_E + R_C)$

The AC and DC load lines must both pass through the DC bias Q-point, since the total $i_c(t)$ and $v_{ce}(t)$ signals are the sum of the AC and DC signal components.



There are TWO ways to calculate slope of AC Load Line:

$$\text{Slope AC Load Line} = -1 / ((R_C \parallel R_L) + R_{E1}) = -I_{cq} / \Delta V_{cemax}$$

We may solve for $\Delta V_{c\text{emax}}$. For our fully-bypassed example ($R_{E1} = 0$),

$$\begin{aligned}\Delta V_{c\text{emax}} &= I_{cQ} * ((R_C // R_L) + R_{E1}) \\ &= (1 \text{ mA}) * (2\text{k} // 4\text{k}) = 1.33 \text{ V}\end{aligned}$$

Thus the $V_{ce}(t)$ total (dc + ac component) output voltage signal may swing **above** the Q-point value " V_{ceQ} " by $\Delta V_{c\text{emax}} = 1.33 \text{ V}$ before the BJT enters cutoff and stops amplifying. Likewise, V_{ceQ} may swing **below** the V_{ceQ} by $(V_{ceQ} - V_{ce(\text{SAT})})$ volts = $2 - 0.2 = 1.8 \text{ V}$ before it hits saturation and stops amplifying. Because the output voltage is typically thought to swing symmetrically (sinusoidally) about the Q-point, just as much above the Q-pt as below it, we must take the smaller of these two distances. Because

$$\Delta V_{c\text{emax}} = 1.33 \text{ V} < (V_{ceQ} - V_{ce(\text{SAT})}) = 1.8 \text{ V}$$

We take our Maximum Symmetrical $V_{ce}(t)$ Output Voltage Swing to be

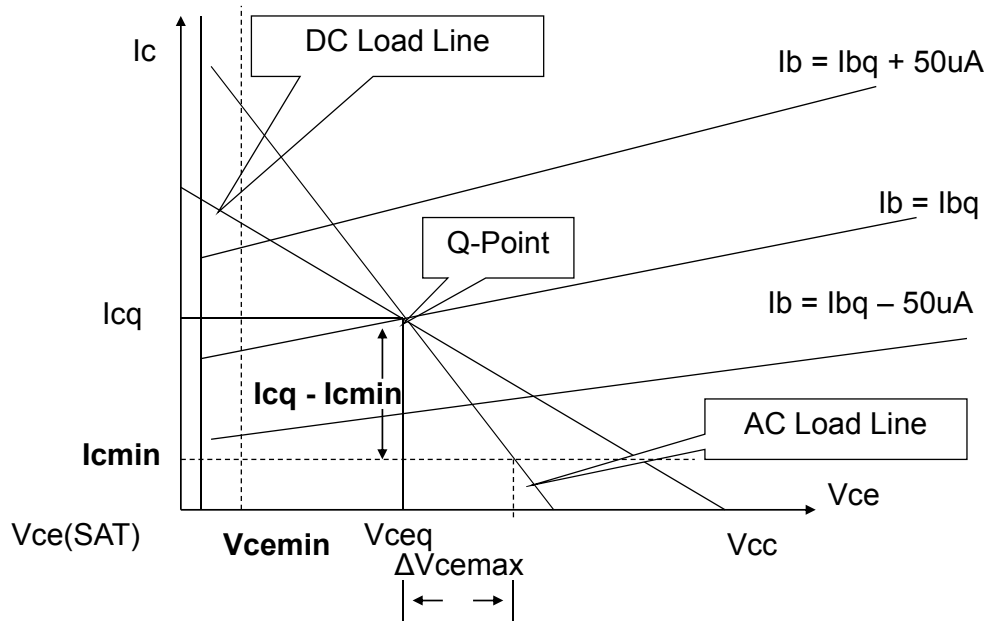
$$\text{Max. Symm Vce Swing} = 2 * (\Delta V_{c\text{emax}}) = 2(1.33) = 2.66 \text{ V, peak-to-peak.}$$

If the above inequality were reversed, as it sometimes is, then

$$\text{Max. Symm Vce Swing} = 2 * ((V_{ceQ} - V_{ce(\text{SAT})})) \quad (\text{in V, peak-to-peak})$$

Note: Take the smaller distance and double it – since "a chain is only as strong as its weakest link"

Sometimes a "margin for error" is allowed for, by not allowing V_{ce} to rise so far as to allow I_c to hit zero, but rather requiring I_c to remain above a certain specified minimum value " $I_{c\text{min}}$ " that is slightly above 0. This is especially desirable because the BJT's β decreases markedly as I_c comes close to 0, and thus this guarantees a more linear amplifying range. Likewise, V_{ce} may not be allowed to fall so far as to reach saturation, where $V_{ce} = V_{ce(\text{SAT})}$. Instead, V_{ce} might be restricted to lie above a specified minimum V_{ce} value " $V_{ce\text{min}}$ " that is slightly above $V_{ce(\text{SAT})}$. The modified procedure is outlined on the next slide:



$$\text{Slope AC Load Line} = -1/((RC // RL)+RE1) = -(Icq - Icmin)/ \Delta Vcemax$$

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Solve for $\Delta Vcemax$ = distance Vce can swing ABOVE Vceq

Calculate $(Vceq - Vcemin)$ = distance Vce can swing BELOW Vceq

Take whichever distance is SMALLER, double it, and that predicts the Maximum Symmetrical Vce Swing

Example 1 (Consider the Fully Bypassed Case => $RE1 = 0$)

Let us assume that the following constraints have been specified:

$$Vcemin = 0.4 \text{ V and } Icmin = 0.1 \text{ mA}$$

Then the distance Vce may swing above Vceq is

$$-1/((RC // RL)+RE1) = -(Icq - Icmin)/ \Delta Vcemax \Rightarrow \Delta Vcemax = 1.2 \text{ V}$$

The distance Vce may swing below Vceq is

$$Vceq - Vcemin = 2 - 0.4 = 1.6 \text{ V}$$

Because 1.2 V < 1.6 V, the Max Symm Swing is 2(1.2) = 2.4 V peak-peak

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Example 2 Again consider the Fully Bypassed Case => $RE1 = 0$; but this time, assume RL has been removed; $RL = \infty$

Let us assume that same constraints as in Example 1:

$$V_{cemin} = 0.4 \text{ V and } I_{cmin} = 0.1 \text{ mA}$$

Then the distance V_{ce} may swing above V_{ceq} is

$$-1/(RC+RE1) = -(I_{cq} - I_{cmin})/\Delta V_{cemax} \Rightarrow \Delta V_{cemax} = 1.8 \text{ V}$$

The distance V_{ce} may swing below V_{ceq} is

$$V_{cq} - V_{cemin} = 2 - 0.4 = 1.6 \text{ V}$$

Because $1.6 \text{ V} < 1.8 \text{ V}$, the Max Symm Swing is $2(1.6) = 3.2 \text{ V}$ peak-peak.

Note that in the first example the Max Symm Swing was limited by the amount V_{ce} could RISE above V_{ceq} , while in the second example, the Max Symm Swing was limited by the amount V_{ce} could FALL below V_{ceq} !

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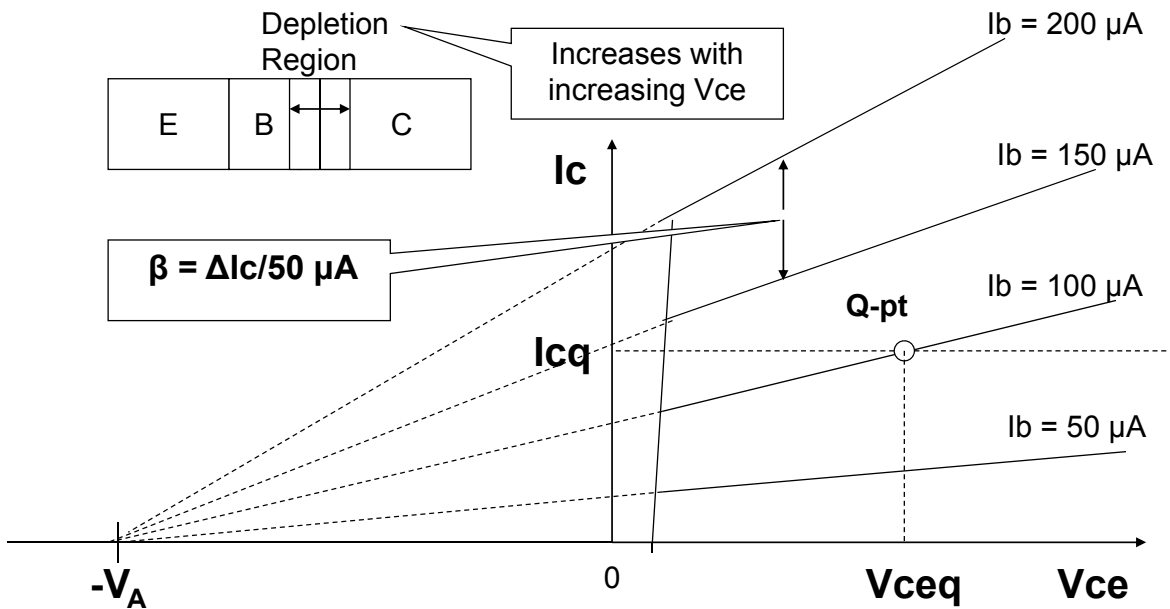
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Including the effects of Base-Width Modulation (also called the “Early Effect”, V_A) in the AC Model of the NPN BJT:
The BJT’s output resistance
“ r_o ” parameter

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Note $\beta = \Delta I_c / \Delta I_b$ increases with increasing V_{ce} because as V_{ce} increases the width of the reverse-biased depletion region increases, making the effective width of the base region thinner. The thinner the base region, the closer α is to 1.0, so $\beta = \alpha / (1 - \alpha)$ becomes larger.

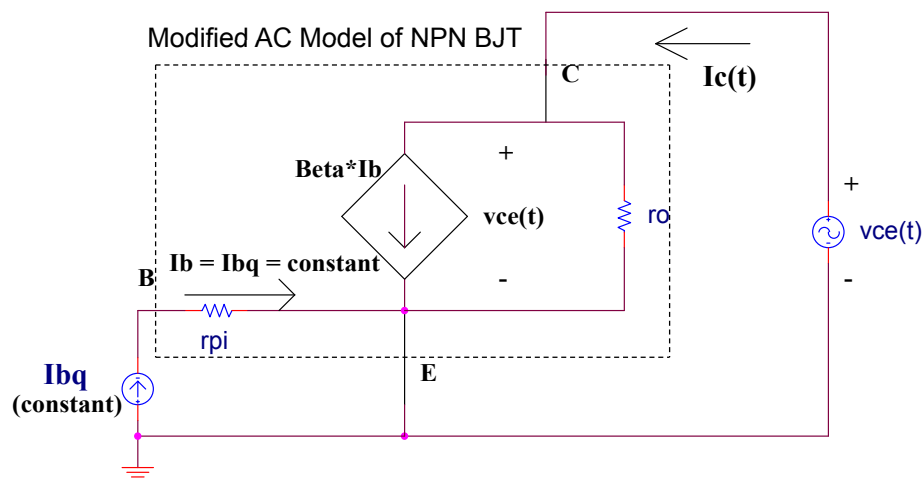
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Assuming $I_b(t) = I_{bq} = \text{constant}$, and thus the controlled current source “pumps” constant current $= \beta * I_{bq}$. Then as $V_{ce}(t)$ is varied, an I_c vs. V_{ce} curve is obtained that passes through the Q-point, whose (slight) upward slope may be accounted for in the AC model of the BJT by placing a high value of resistance (r_o) across the controlled-current source in the BJT’s AC model, as shown below:

$$I_c = \beta * I_{bq} + V_{ce}(t) / r_o$$



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Because in the forward-active region,

$$I_c = \beta \cdot I_{bq} + v_{ce(t)}/r_o$$

It should be apparent that the forward-active portion of the I_c vs. V_{ce} curve should be linear (of the form $y = mx + b$), and have a slope = $1/r_o$.

Note from the previous slide that another way of calculating this slope is in terms of the Early voltage parameter, V_A :

$$\text{Slope} = 1/r_o = I_{cq}/(V_A + V_{ceq})$$

Furthermore, because $50 \text{ V} < V_A < 200 \text{ V}$, V_{ceq} is typically on the order of several volts, we may drop V_{ceq} from the following expression:

$$1/r_o = I_{cq} / V_A \Rightarrow$$

$$r_o = V_A / I_{cq}$$

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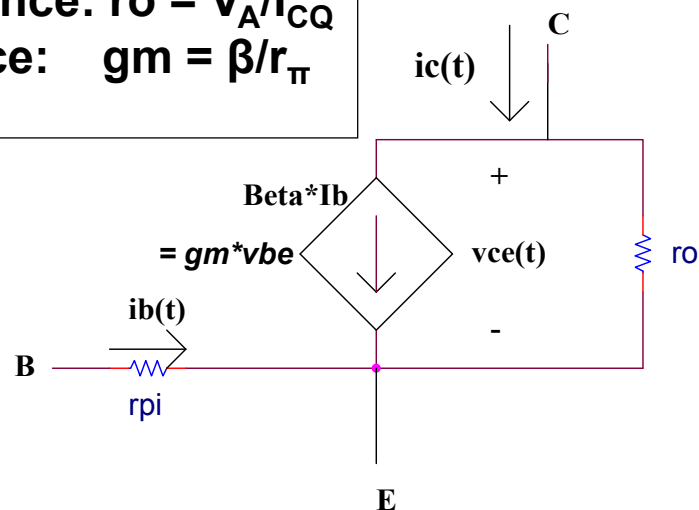
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Improved Forward-Active NPN BJT AC Small-Signal “Hybrid π ” Model

AC base resistance $r_{\pi} = \eta V_T / I_{BQ}$

AC output resistance: $r_o = V_A / I_{CQ}$

Transconductance: $g_m = \beta / r_{\pi}$

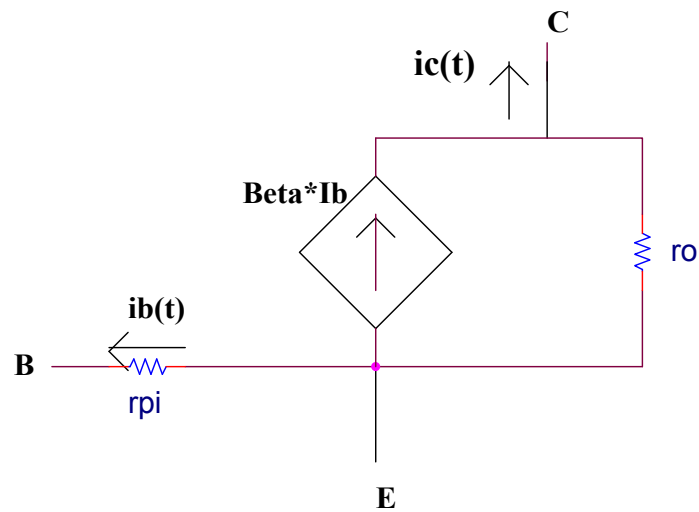


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AC “Hybrid- π ” Model of Forward-Active PNP BJT



Note that the AC Models of the NPN and PNP BJT are actually equivalent, as we can see by reversing all current reference directions.

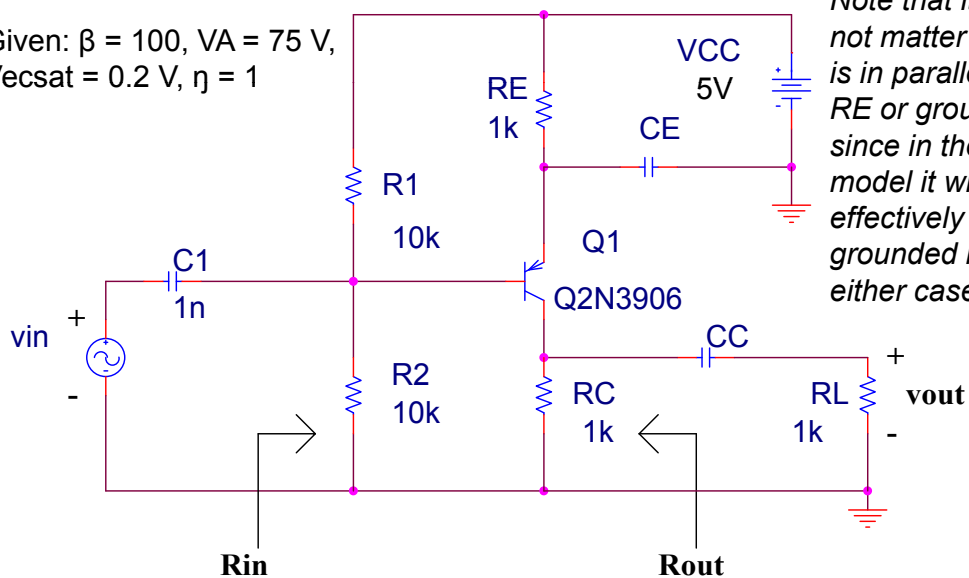
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Example: PNP BJT CE Amplifier Analysis

Given: $\beta = 100$, $V_A = 75$ V,
 $V_{\text{ecsat}} = 0.2$ V, $\eta = 1$



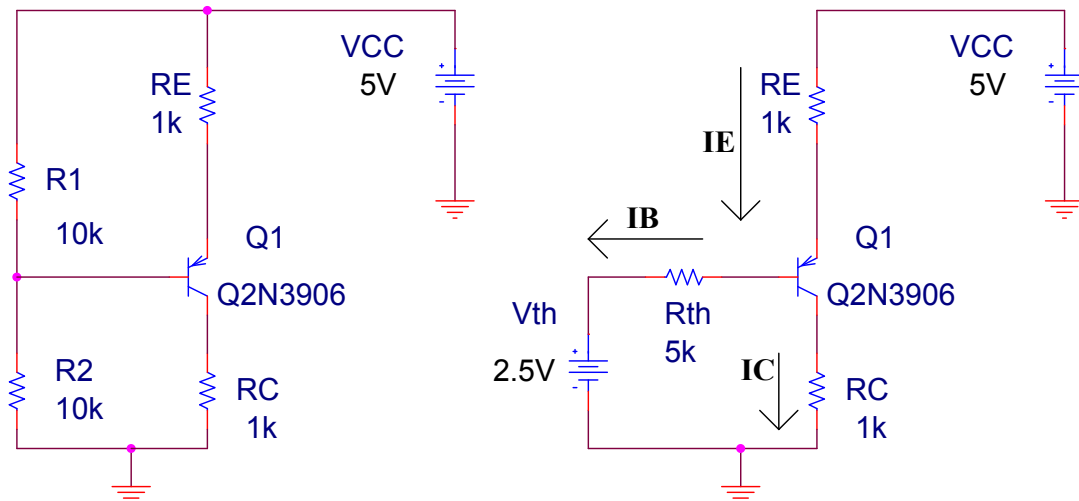
Note that it does not matter if CE is in parallel with RE or grounded, since in the AC model it will be effectively grounded in either case.

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DC Model before and after source Theveninization



To find I_B , write a KVL equation Around Base Loop. Remember this is PNP BJT!

$$V_{cc} = (\beta + 1)I_B \cdot R_E + V_{E_{on}} + R_{th} \cdot I_B + V_{th}$$

$$\Rightarrow 5 = 101 \cdot I_B \cdot 1k + 0.7 + 5k \cdot I_B + 2.5 \quad \Rightarrow \quad \underline{I_B = 16.98 \mu A}$$

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$$V_{EC} = (V_{cc} - (\beta + 1)I_B \cdot R_E) - \beta \cdot I_B \cdot R_C$$

$$= (5 - 101 \cdot 16.98 \mu A \cdot 1k) - 100 \cdot 16.98 \mu A \cdot 1k$$

$$= 1.587 V > V_{ECsat} \Rightarrow \text{BJT Fwd Active, as assumed}$$

$$I_C = I_B \cdot \beta = 16.98 \mu A \cdot 100 = 1.698 mA$$

Q-point of the PNP BJT is ($V_{EC} = 1.587 V$, $I_C = 1.698 mA$)

Now that the DC analysis is complete, we can calculate the small-signal ac (hybrid- π) model parameters

$$r_{\pi} = \eta V_T / I_{BQ} = (1 \cdot 26 \text{ mV}) / 16.98 \mu A = \underline{1.53 k\Omega}$$

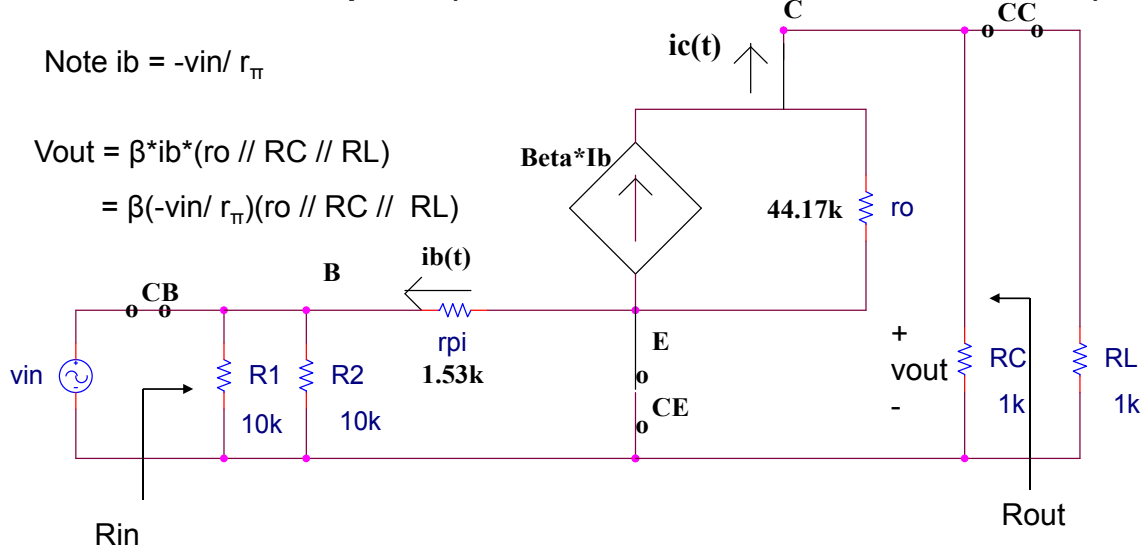
$$r_o = V_A / I_{CQ} = 75 V / 1.698 \mu A = \underline{44.17 k\Omega}$$

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AC model of PNP CE Amplifier ($V_{cc} \rightarrow 0$; $C_B, C_C, \& C_E \rightarrow$ short circuits)



$$R_{out} = R_C \parallel r_o = 1 \text{ k}\Omega \parallel 44.17 \text{ k}\Omega = \underline{977.9 \Omega}$$

$$R_{in} = R_1 \parallel R_2 \parallel r_{\pi} = 10 \text{ k}\Omega \parallel 10 \text{ k}\Omega \parallel 1.53 \text{ k}\Omega = \underline{1.17 \text{ k}\Omega}$$

$$A_{vo} = -\beta(R_C \parallel r_o) / r_{\pi} = -100(1 \text{ k}\Omega \parallel 44.17 \text{ k}\Omega) / 1.53 \text{ k}\Omega = \underline{-63.9}$$

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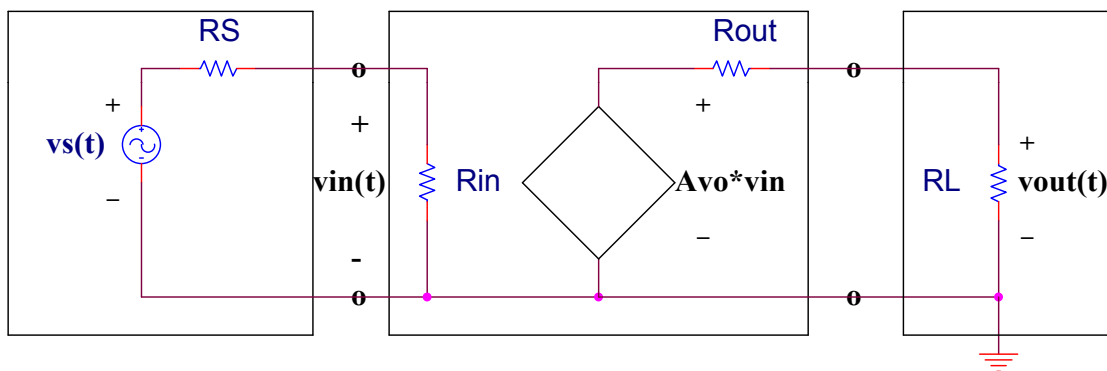
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Source or Transducer

General Voltage Amp Model

Load



In this case, $R_S = 0$,

so $A_{vt} = A_v = v_{out} / v_{in} = A_{vo} \cdot (R_L / (R_{out} + R_L))$

$$A_v = -63.9 \cdot (1 \text{ k} / (1 \text{ k} + 977.9 \Omega)) = \underline{-32.3}$$

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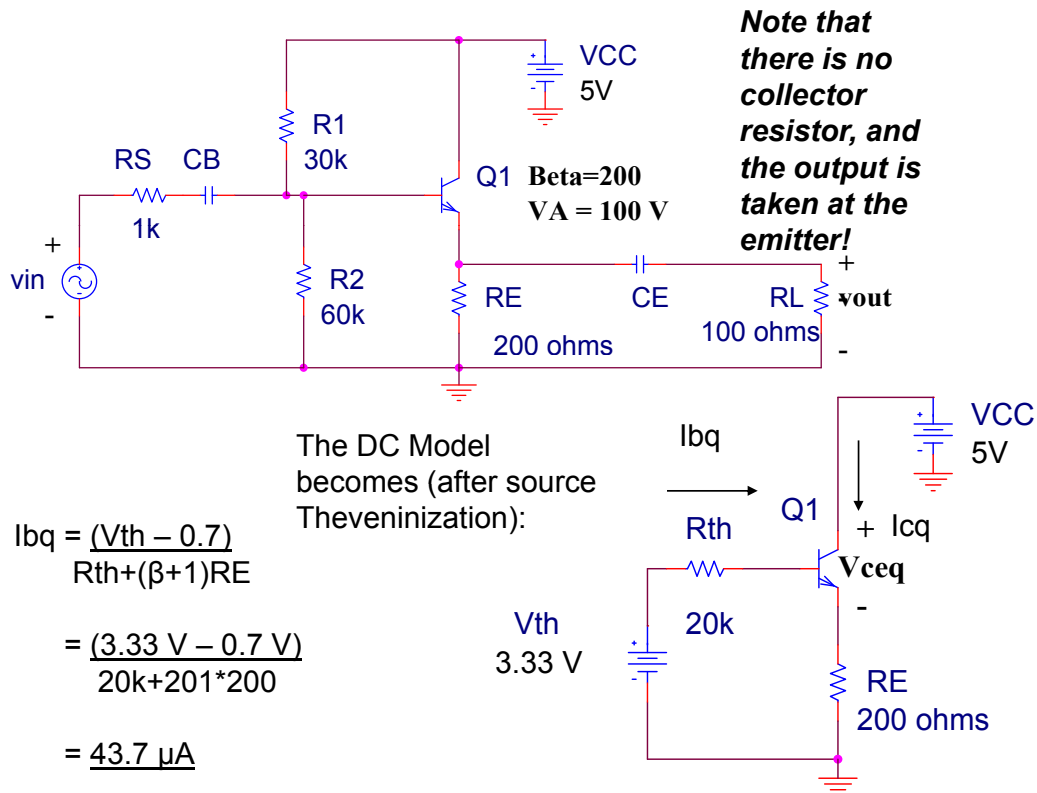
Common-Collector (also called Emitter Follower) BJT Amplifier

Same 3-resistor biasing network and DC model as before, but now the output is taken across the (unbypassed) emitter resistor, so the AC model changes **dramatically**.

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$$V_{ceq} = V_{cc} - (\beta + 1)I_{bq}R_E = \underline{3.24 \text{ V}}$$

> $V_{ce}(\text{SAT}) \Rightarrow$ Our assumption of forward-active mode is valid

$$I_{cq} = \beta I_b = 200 * (43.7 \mu\text{A}) = \underline{8.74 \text{ mA}}$$

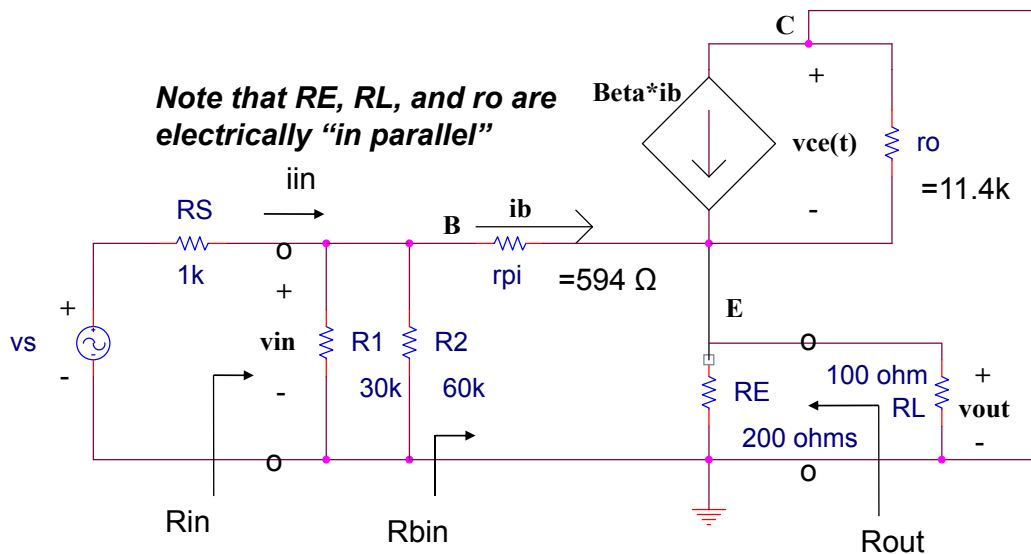
Now that the dc analysis is complete, we can find r_{π} and r_o

$$r_{\pi} = n * V_T / I_{bq} = (1)(26 \text{ mV}) / 43.7 \mu\text{A} = \underline{594.4 \Omega}$$
 Assume IC BJT ($n=1$)

$$r_o = V_A / I_{cq} = 100 \text{ V} / 8.74 \text{ mA} = \underline{11.44 \text{ k}\Omega}$$

Now it is time to construct the AC model of the CC (Emitter Follower) amplifier:

AC Model of CC (Emitter Follower) Amplifier



Find Avo (Remove RL)

Remember to REMOVE RL

When finding Avo!

KVL around base loop =>

$$v_{in} = i_b \cdot r_{\pi} + (\beta + 1) \cdot i_b \cdot \left(\frac{R_E \cdot r_o}{R_E + r_o} \right)$$

$$\Rightarrow i_b = \frac{v_{in}}{r_{\pi} + (\beta + 1) \cdot \frac{R_E \cdot r_o}{R_E + r_o}}$$

KVL around emitter loop =>

$$v_{out} = (\beta + 1) \cdot i_b \cdot \frac{R_E \cdot r_o}{R_E + r_o}$$

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$$v_{out} = (\beta + 1) \cdot \left[\frac{v_{in}}{r_{\pi} + (\beta + 1) \cdot \frac{R_E \cdot r_o}{R_E + r_o}} \right] \cdot \frac{R_E \cdot r_o}{R_E + r_o}$$

Substituting i_b equation into v_{out} yields:

$$A_{vo} = \frac{v_{out}}{v_{in}} = \frac{(\beta + 1) \cdot \frac{R_E \cdot r_o}{R_E + r_o}}{r_{\pi} + (\beta + 1) \cdot \frac{R_E \cdot r_o}{R_E + r_o}} = (\beta + 1)(R_E // r_o) / (r_{\pi} + (\beta + 1)(R_E // r_o))$$

Note that A_{vo} can never be > 1 , and A_{vo} is approximately = 1 if

$$(\beta + 1)(R_E // r_o) \gg r_{\pi}$$

In our example

$$A_{vo} := \frac{(\beta + 1) \cdot \frac{R_E \cdot r_o}{R_E + r_o}}{r_{\pi} + (\beta + 1) \cdot \frac{R_E \cdot r_o}{R_E + r_o}} \quad A_{vo} = 0.985$$

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The input resistance R_{bin} may be found by writing a KVL loop around the base loop with R_L re-inserted in the circuit.

$$v_{in} = r_{\pi} \cdot i_b + (\beta + 1) \cdot i_b \cdot \left(\frac{1}{\frac{1}{R_E} + \frac{1}{r_o} + \frac{1}{R_L}} \right)$$

$$R_{bin} = \frac{v_{in}}{i_b} = r_{\pi} + (\beta + 1) \cdot \left(\frac{1}{\frac{1}{R_E} + \frac{1}{r_o} + \frac{1}{R_L}} \right)$$

Note that, unlike the CE amplifier, R_{bin} for the CC amplifier depends upon the value of load resistance, R_L .

In our example, $R_{bin} := 13.92 \cdot k\Omega$

$$R_{in} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{bin}}} \quad R_{in} := 8.208 \cdot k\Omega$$

Note that, compared to R_E and R_L , R_{in} is a fairly large input resistance. It can be made larger if R_1 and R_2 are made larger, though this increases the dependence of the Q-point on β .

The input resistance R_{bin} "with R_L removed" will be needed later:

$$v_{in} = r_{\pi} \cdot i_b + (\beta + 1) \cdot i_b \cdot \left(\frac{1}{\frac{1}{R_E} + \frac{1}{r_o}} \right)$$

$$R_{bin_no_RL} = \frac{v_{in}}{i_b} = r_{\pi} + (\beta + 1) \cdot \left(\frac{1}{\frac{1}{R_E} + \frac{1}{r_o}} \right)$$

Note that, unlike the CE amplifier, R_{bin} for the CC amplifier depends upon the value of load resistance, R_L .

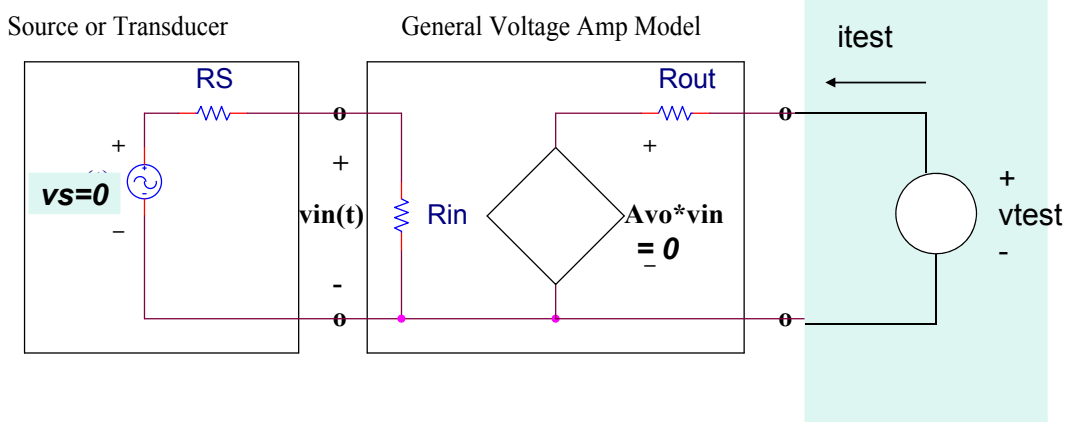
In our example, $R_{bin_no_RL} := 40.1 \cdot k\Omega$

$$R_{in_no_RL} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{bin_no_RL}}}$$

$$R_{in_no_RL} = 1.334 \times 10^4 \Omega$$

R_{in} with R_L removed is used in the general voltage amplifier model of the CC amplifier

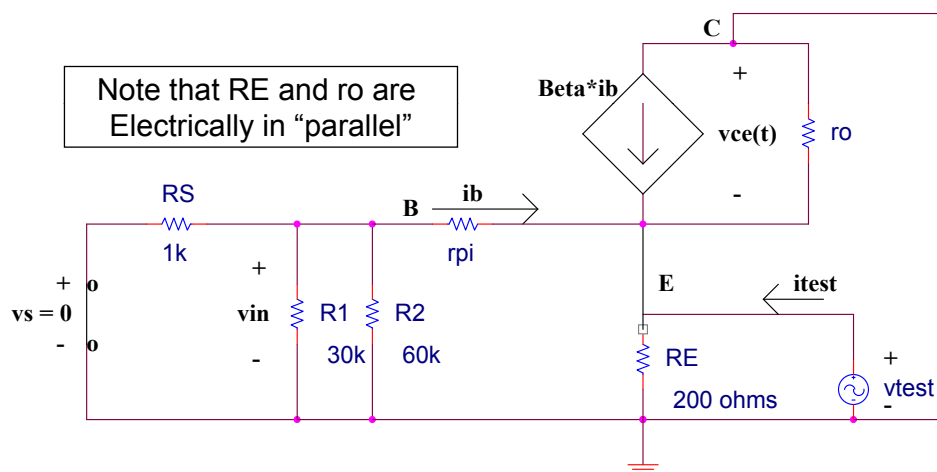
Finding R_{out} for the CC amplifier is *more complicated* than it was for the CE amplifier case, where we found $R_{out} = R_C$. With a small ac test voltage source connected across the output terminals of the CC amplifier, which is modeled below using the “General Voltage Amplifier Model”:



It should be clear that

$$R_{out} = (v_{test}/i_{test})_{v_s = 0}$$

Applying this ac test voltage source to the AC model of the CC BJT circuit (***making sure to set $v_s = 0$***) yields:



KCL at emitter node: $-i_b - \beta i_b - i_{test} + v_{test}/R_E + v_{test}/r_o = 0$

Also, note that i_b may be calculated in terms of v_{test} :

$$i_b = -v_{test} / (r_{pi} + R_1 // R_2 // R_S)$$

Substituting this expression for i_b into the KCL equation yields

$$(\beta+1)v_{test} / (r_{pi} + R1 // R2 // RS) - i_{test} + v_{test}/R_E + v_{test}/r_o = 0$$

$$\Rightarrow v_{test}\{(\beta+1)/ (r_{pi} + R1 // R2 // RS) + 1/R_E + 1/r_o\} = i_{test}$$

$R_{out} = v_{test} / i_{test}$

$$= 1/\{1/[(r_{pi} + R1 // R2 // RS) / (\beta+1) + 1/R_E + 1/r_o]\}$$

$$= \{R_E // r_o\} // \{(r_{pi} + R1 // R2 // RS) / (\beta+1)\}$$

$$R_{out} := \frac{1}{\frac{1}{\frac{1}{R_E} + \frac{1}{r_o}} + \frac{1}{\left[\frac{1}{\left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{RS}\right)}\right]^{+\beta+1}}}$$

In our example, with $R_S = 1k$,
 $R_{out} = 196.6 // [1.55k/(\beta+1)]$
 $R_{out} = 7.41 \Omega$

R_o relatively low; &
 depends upon R_S

Note how the "reflection rule" is used in reverse. The equiv resistance in the base circuit is divided by $(\beta+1)$

CC Amplifier Summary

$$A_{vo} = \frac{(\beta+1)(R_E // r_o)}{r_{\pi} + (\beta+1)(R_E // r_o)}$$

Note: A_{vo} is slightly less than 1.0

$$R_{bin} = r_{\pi} + (\beta+1)(R_E // r_o // R_L)$$

Note: R_{in} is relatively high. It depends upon R_L , so the general voltage amplifier model is NOT independent of output termination as it is for the CE amplifier.

$$R_{bin_no_RL} = r_{\pi} + (\beta+1)(R_E // r_o)$$

$$R_{in} = (R_{bin} // R1 // R2)$$

$$R_{out} = (R_E // r_o) // (r_{\pi} + R1 // R2 // RS) / (\beta+1)$$

Note: R_{out} is relatively low. It depends upon R_S , so the general voltage amplifier model is NOT independent of input termination, as it is for the CE amplifier

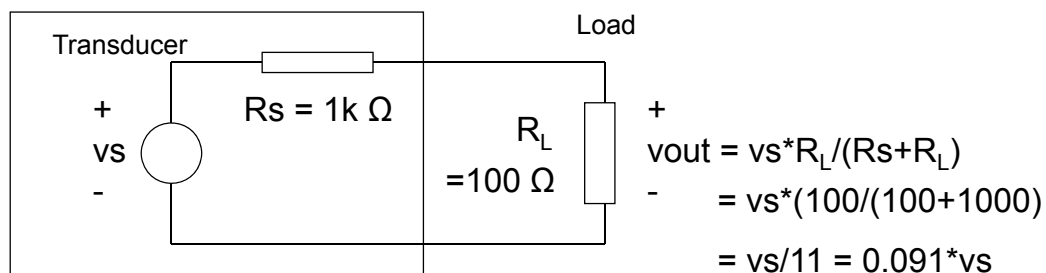
But what good is the CC
(Emitter Follower) Amplifier,
since it has a voltage gain
that is slightly less than unity?
What advantage does this
amplifier have over a wire that
connects input to output?

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ECE250 (KEH)

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Consider a situation where a transducer
with open-circuit voltage v_s and output
resistance $R_s = 1\text{k}\Omega$ that must deliver a
signal across a $R_L = 100\ \Omega$ load



Only 9.1% of the available transducer voltage (v_s) is delivered to the load!

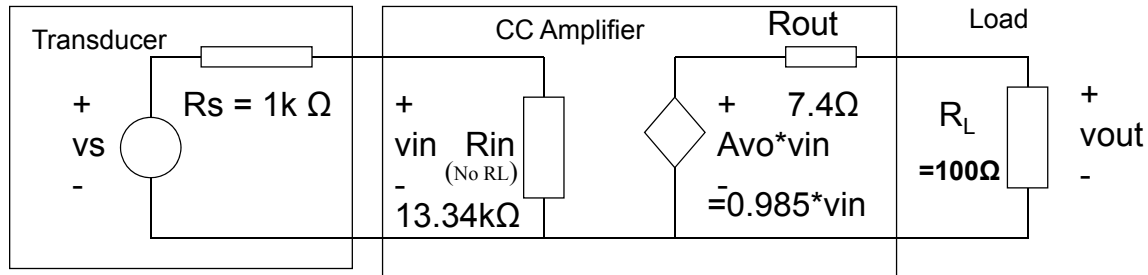
We say that the transducer's output has been severely "loaded" by R_L

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This “loading” problem may be solved by placing the CC (Emitter Follower) amplifier that we have just designed between the transducer and the load:



$$\begin{aligned} \text{Now } v_{out} &= v_s \cdot R_{in_no_RL} / (R_{in_no_RL} + R_s) \cdot A_{vo} \cdot R_L / (R_{out} + R_L) \\ &= v_s \cdot (13.34 / (1 + 13.34)) \cdot 0.985 \cdot (100 / (7.4 + 100)) = 0.853 \cdot v_s \end{aligned}$$

Now 85.3% of the available transducer voltage is delivered to the load!

Note that there is a very subtle difference in applying the general voltage amplifier model for the CC Amplifier.

This difference is due to the fact that in a CC amplifier, R_{in} depends upon R_L .

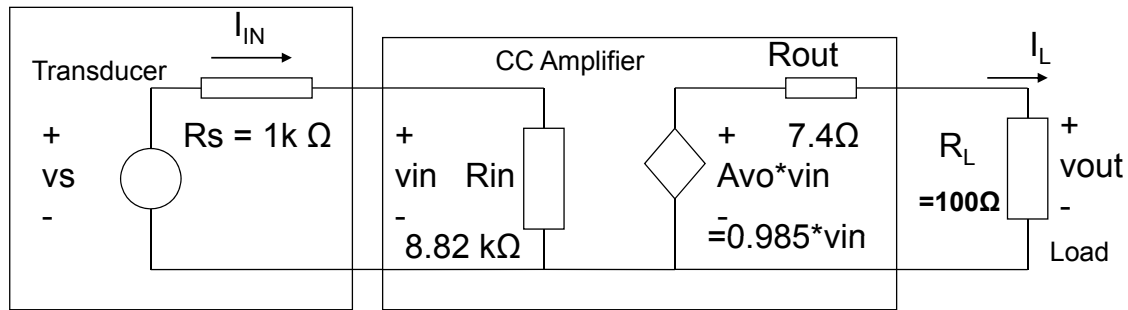
When finding the loaded voltage gain (A_v) using the general voltage amplifier model, we must use the value of R_{in} with R_L removed (“ $R_{in_no_RL}$ ”).

This is because we are calculating the strength of the “open-circuit” voltage source “ $A_{vo} \cdot v_{in}$ ”. Because it must represent the open circuit voltage at that point in the circuit, it must be calculated assuming that no load has yet been placed across the output terminals:

$$A_{vo} \cdot v_{in} = A_{vo} \cdot v_s \cdot (R_{in_no_RL} / (R_{in_no_RL} + R_s))$$

Thus, the overall loaded voltage gain is calculated as

$$A_v = R_{in_no_RL} / (R_{in_no_RL} + R_s) \cdot A_{vo} \cdot R_L / (R_{out} + R_L)$$



While the CC amplifier exhibits a voltage gain slightly less than 1, it exhibits a *much higher* current gain and power gain.

Let us define **current gain** as

$$A_I = I_L / I_{IN}$$

$$= (v_{out} / R_L) / (v_s / (R_s + R_{in})) = (v_{out} / v_s) * (R_s + R_{in}) / R_L$$

$$= A_V * (R_s + R_{in}) / R_L$$

When calculating A_I , the actual loaded value of R_{in} is used!

Note A_I is expressed in terms of A_V (not A_{VO})

Likewise let us define **power gain** as

$$A_P = P_{out} / P_{in} = (v_{out} * I_L) / (v_s * I_{IN}) = (v_{out} / v_s) * (I_L / I_{IN}) = A_V * A_I$$

In this example,

$$A_V = v_{out} / v_s$$

$$= R_{in_no_RL} / (R_{in_no_RL} + R_s) * A_{VO} * R_L / (R_{out} + R_L)$$

$$= (13.34 / (13.34 + 1)) * 0.985 * (100 / (7.4 + 100)) = 0.853$$

$$A_I = A_V * (R_s + R_{in}) / R_L$$

$$= 0.853 * ((1k\Omega + 8.2k\Omega) / 100\Omega) = 78.5$$

$$A_P = A_V * A_I = 0.853 * 78.5 = 67$$

Note that a CC amplifier may have a voltage gain slightly less than one, but it has a current gain and a power gain that is usually much greater than one!

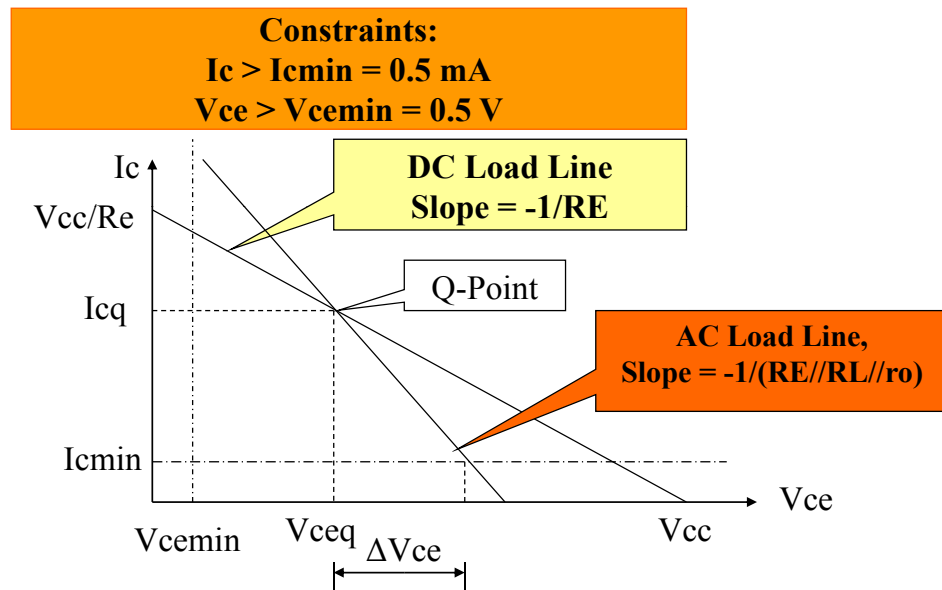
Finding Maximum Symmetrical Vce Output Swing for CC Amplifier

(Use our present example)

12/14/2009

ECE250 (KEH)

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$$\begin{aligned} \text{Slope AC load line} &= -1/(RE // RL // ro) = -(I_{cq} - I_{cmin}) / \Delta V_{ce} \\ &= -1/(200 // 100 // 11.4k) = -(8.74 \text{ mA} - 0.5 \text{ mA}) / \Delta V_{ce} \end{aligned}$$

$$\Rightarrow \underline{\Delta V_{ce} = 0.546 \text{ V}}$$

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Thus the amount that V_{ce} may RISE above V_{ceq} is given by

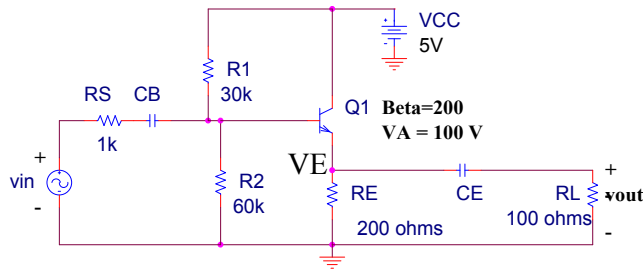
$$\Delta V_{ce} = 0.546 \text{ V}$$

The amount that v_{ce} may FALL below V_{ceq} is given by

$$V_{ceq} - V_{cem\text{in}} = 3.24 \text{ V} - 0.5 \text{ V} = \underline{2.74 \text{ V}}$$

Since $0.549 \text{ V} < 2.74 \text{ V}$, the amount that V_{ce} RISES is the limiting factor.

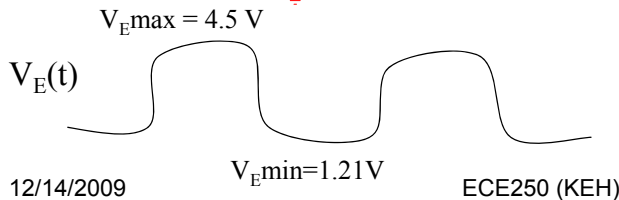
Max Symmetrical VCE swing = $2(0.546 \text{ V}) = \underline{1.09 \text{ V, peak-peak}}$



$V_E(t)$ distortion points:

$$V_{E\text{max}} = V_{cc} - V_{cem\text{in}} = 5 \text{ V} - 0.5 \text{ V} = \underline{4.5 \text{ V}}$$

$$V_{E\text{min}} = V_{cc} - V_{cem\text{ax}} = V_{cc} - (V_{ceq} + \Delta V_{ce}) = 5 \text{ V} - (3.24 \text{ V} + 0.546 \text{ V}) = \underline{1.21 \text{ V}}$$



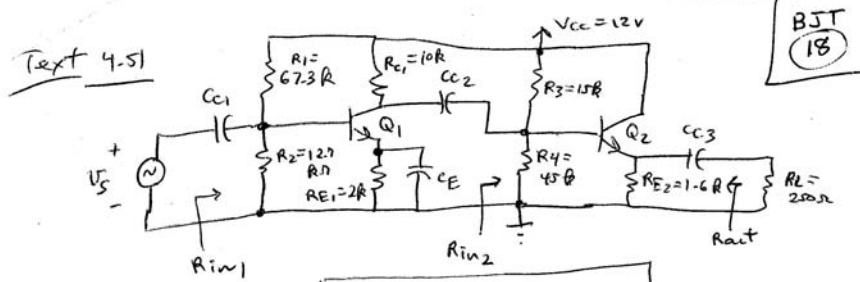
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$V_{E\text{min}}=1.21\text{V}$

ECE250 (KEH)

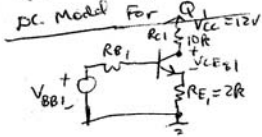
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Cascading two amplifier stages using the general voltage amplifier model



Both BJTs have $\beta_F = 120, V_A = \infty$

(a) Find g_m, r_{π}, r_o for both BJTs $V_{CC} = +12V$



$$V_{BB1} = V_{CC} \left(\frac{R_2}{R_1 + R_2} \right) = 1.905V$$

$$R_{B1} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = 10.68 k\Omega$$

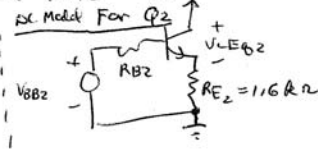
KVL around b-e loop \Rightarrow

$$I_{B1Q} = \frac{V_{BB1} - 0.7V}{R_{B1} + (\beta_F + 1)R_{E1}} = 4.77 \mu A$$

$$I_{C1Q} = \beta_F I_{B1Q} = 0.572 mA$$

$$V_{CE1Q} = V_{CC} - R_{C1} I_{C1Q} - (\beta_F + 1) I_{B1Q} R_{E1}$$

$$= 5.123V > V_{CEsat} \Rightarrow \text{Fwd Active!}$$



$$V_{BB2} = V_{CC} \left(\frac{R_4}{R_3 + R_4} \right) = 9V$$

$$R_{B2} = R_3 \parallel R_4 = \frac{R_3 R_4}{R_3 + R_4} = 11.25 k\Omega$$

$$I_{B2Q} = \frac{V_{BB2} - 0.7V}{R_{B2} + (\beta_F + 1)R_{E2}} = 40.52 \mu A$$

$$I_{C2Q} = \beta_F I_{B2Q} = 4.862 mA$$

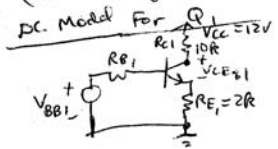
$$V_{CE2Q} = V_{CC} - (\beta_F + 1) I_{B2Q} R_{E2} = 4.155V$$

$$> V_{CEsat} \Rightarrow \text{Fwd Active!}$$

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(a) Find g_m, r_{π}, r_o for both BJTs $V_{CC} = +12V$



$$V_{BB1} = V_{CC} \left(\frac{R_2}{R_1 + R_2} \right) = 1.905V$$

$$R_{B1} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = 10.68 k\Omega$$

KVL around b-e loop \Rightarrow

$$I_{B1Q} = \frac{V_{BB1} - 0.7V}{R_{B1} + (\beta_F + 1)R_{E1}} = 4.77 \mu A$$

$$I_{C1Q} = \beta_F I_{B1Q} = 0.572 mA$$

$$V_{CE1Q} = V_{CC} - R_{C1} I_{C1Q} - (\beta_F + 1) I_{B1Q} R_{E1}$$

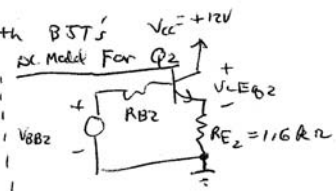
$$= 5.123V > V_{CEsat} \Rightarrow \text{Fwd Active!}$$

For AC Model,

$$r_{\pi 1} = \frac{nVT}{I_{B1Q}} = \frac{1(26mV)}{4.77 \mu A} = 5.45 k\Omega$$

$$r_{o1} = \frac{V_A}{I_{C1Q}} = \frac{\infty}{I_{C1Q}} = \infty$$

$$g_{m1} = \frac{i_{c1}}{v_{be1}} = \frac{i_{c1}}{r_{\pi 1} i_{b1}} = \frac{\beta_F}{r_{\pi 1}} = 0.022 \frac{A}{V}$$



$$V_{BB2} = V_{CC} \left(\frac{R_4}{R_3 + R_4} \right) = 9V$$

$$R_{B2} = R_3 \parallel R_4 = \frac{R_3 R_4}{R_3 + R_4} = 11.25 k\Omega$$

$$I_{B2Q} = \frac{V_{BB2} - 0.7V}{R_{B2} + (\beta_F + 1)R_{E2}} = 40.52 \mu A$$

$$I_{C2Q} = \beta_F I_{B2Q} = 4.862 mA$$

$$V_{CE2Q} = V_{CC} - (\beta_F + 1) I_{B2Q} R_{E2} = 4.155V$$

$$> V_{CEsat} \Rightarrow \text{Fwd Active!}$$

For AC Model

$$r_{\pi 2} = \frac{nVT}{I_{B2Q}} = \frac{1(26mV)}{40.52 \mu A} = 641.7 \Omega$$

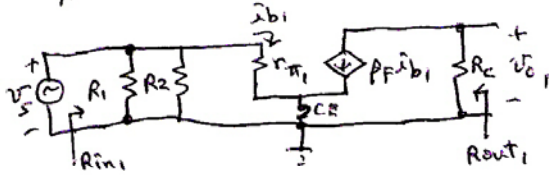
$$r_{o2} = \frac{V_A}{I_{C2Q}} = \frac{\infty}{I_{C2Q}} = \infty$$

$$g_{m2} = \frac{\beta_F}{r_{\pi 2}} = 0.1875 \frac{A}{V}$$

1:

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AC Model for Q₁ (CE) stage

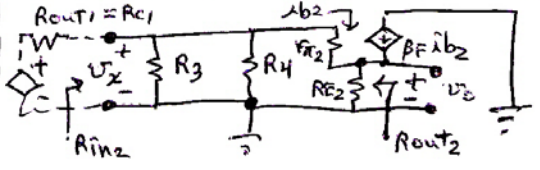


$$R_{in1} = R_1 // R_2 // r_{\pi 1} = 3.61 k\Omega$$

$$A_{Vc1} = \frac{-\beta F R_C}{r_{\pi 1}} = -220.1$$

$$R_{out1} = R_C = 10 k\Omega$$

AC Model for Q₂ (CC) stage

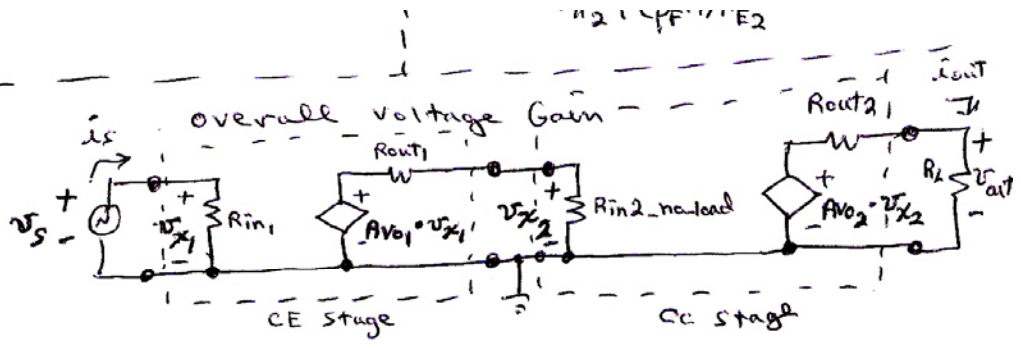
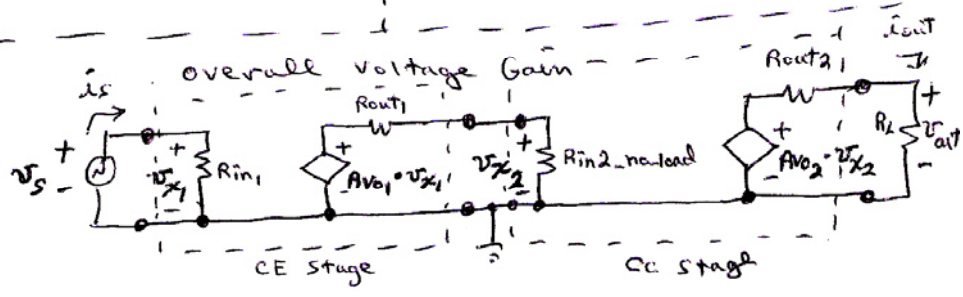


$$R_{out1} = R_{C1}$$

$$R_{in2_no_load} = R_3 // R_4 // [r_{\pi 2} + (\beta F + 1) R_{E2}] = 10.63 k\Omega$$

$$R_{out2} = R_{E2} // \left(\frac{r_{\pi 2} + (R_3 // R_4 // R_{out1})}{\beta F + 1} \right) = 47.6 \Omega$$

$$A_{V_o2} = \frac{(\beta F + 1) R_{E2}}{r_{\pi 2} + (\beta F + 1) R_{E2}} = 0.997$$



$$A_V = \frac{v_{out}}{v_s} = A_{Vc1} \cdot \frac{R_{in2_no_load}}{R_{in2_no_load} + R_{out1}} \cdot A_{Vc2} \cdot \frac{R_L}{R_L + R_{out2}}$$

$$A_V = -94.9$$

$$A_I = \frac{v_{out} / R_L}{v_s / (R_{in1})} = A_V \cdot \frac{R_{in1}}{R_L} = -1370$$

$$A_P = \frac{P_{out}}{P_{in}} = \frac{v_{out} \cdot i_{out}}{v_s \cdot i_s} = A_V \cdot A_I = 1.3 \times 10^5$$

THE END OF BJT Coverage!

(MOSFETs come next)