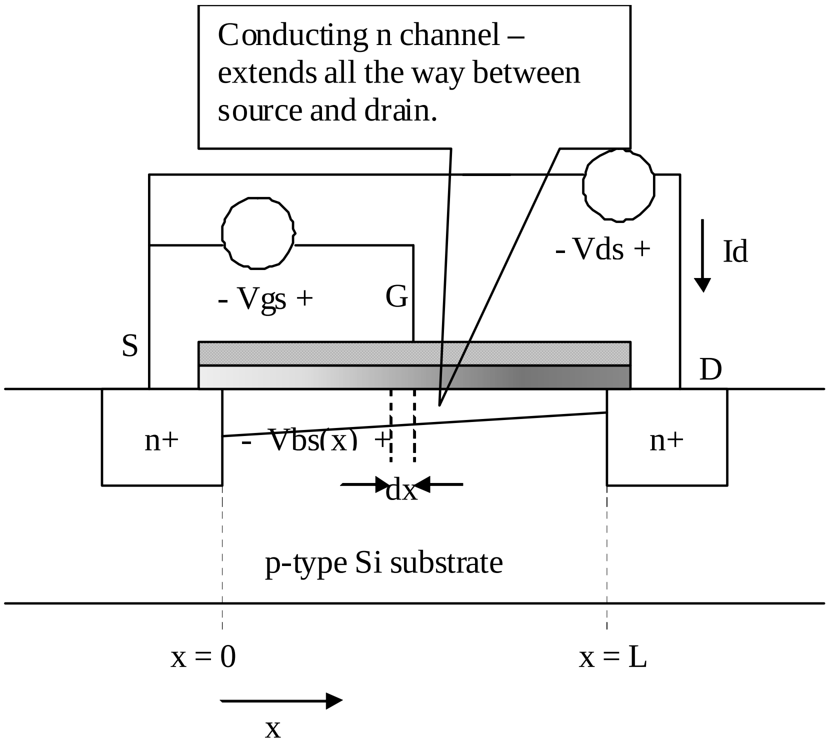
Derivation of NMOSFET Drain Current Equations

Part 1. Ohmic (Triode) Mode NMOSFET (Vgs - Vds > Vtn) N channel not pinched off

We start by deriving an expression for the free electron charge per unit length, or the charge density "q(x)" that is in the n-channel "inversion layer" fully formed on the surface of the p-type substrate under the gate in a turned-on, ohmic NMOSFET as a function of distance "x" from the source toward the drain. The ohmic mode NMOSFET is depicted in Figure 1 below.

Figure 1. Ohmic Mode NMOSFET (Vgs - Vds > Vtn)



Recall that the capacitance of any parallel-plate capacitor is $\varepsilon A/d$. Thus the gate-to-substrate capacitance of a MOSFET must be given by $\frac{\varepsilon_{ox}}{t_{ox}}\cdot(W\cdot L) = C_{ox}\cdot(W\cdot L)$ where ε_{ox} is the permittivity of the gate silicon dioxide layer = $3.9\varepsilon o$= $3.9(8.854\text{x}10^{-12})$ F/m, tox is the thickness of the gate oxide layer, W is the width of the source and drain diffusions (into the paper), and L is the length of the MOSFET, or the distance between the source and drain diffusions. Thus the gate-to-substrate capacitance per unit of distance between the source and the drain diffusions must be given by

$$W\cdot C_{ox} = W\cdot\frac{\varepsilon_{ox}}{tox} \qquad (1)$$

Let Vbs(x) be the voltage difference between the surface of the p-type silicon substrate under the gate and the source diffusion. Note that this voltage difference is a function of the distance "x" from source to drain. At the source end of the channel (x = 0), Vbs(0) = 0 (the voltage between the channel and the source at the source end of the channel must be 0.) Likewise, over at the drain end of the channel (x = L), Vbs(L) = Vds (the voltage between the channel and the source at the drain end of the channel must be Vds).

Recall that for a capacitor C = Q/V, and thus Q = C*V, and because no mobile charge can exist for Vgs < Vtn, the free electronic charge per unit length at the source end (x = 0) must be given by

$$q(0) = W \cdot C_{ox} \cdot (Vgs - Vtn) \qquad (2)$$

The above equation implies that as Vgs is increased above 0, the inversion layer at the source end just begins to form as Vgs reaches the threshold voltage of Vtn volts. This is the potential difference that must exist between the surface of the p-type silicon subtrate and the body of the substrate that is required in order to create an electric field that is strong enough to begin repelling holes near the surface of the p-type substrate. However at the point where Vgs = Vtn, no free electrons are yet stored in the newly formed n-channel. The amount that Vgs rises *above* Vtn is the voltage that "charges" the capacitance between the gate and the surface of the substrate, and thereby results in free electrons being stored in the channel.

As we move a distance "x" away from the source (toward the drain), recall that the substrate surface voltage with respect to the source increases from 0 toward Vds, and so the voltage existing between the gate and the surface of the substrate decreases from Vgs to Vgs - Vbs(x). Therefore at a distance x from the source, the free electron charge density in the channel at position "x" between source and drain must be given by

$$q(x) = W \cdot C_{ox} \cdot [(Vgs - Vbs(x)) - Vtn] \qquad (3)$$

Now we derive an expression for the drain-to-source current, Id in the NMOSFET.

Consider a specific "constant x" cross-section, x = xo located somewhere between the drain and the source diffusions. Also, note that q(x)dx is the amount of electronic charge contained in a small surface strip (of length W and width dx) located at position x = xo. Let us imagine that this small strip of charge crosses the x = xo cross section in differential time "dt". Then the drain current, Id, is simply the differential amount of charge q(x)dx that crosses in differential time "dt" divided by "dt". But dx/dt is the drift velocity, u(x). This reasoning is shown in Equation (4)

$$Id = \frac{q(x) \cdot dx}{dt} = q(x) \cdot \frac{dx}{dt} = q(x) \cdot u(x) \qquad (4)$$

It is interesting to note that moving toward the drain, as x increases, q(x) decreases and thus u(x) must increase, since the product of q(x) and u(x) at any value of x must equal a constant, Id.

Recall that the electronic drift velocity is directly proportional to the applied electric field E field (which in this case is set up by the applied terminal voltage Vds, and is directed between drain and source). Note the minus sign in Equation (5), since electrons drift *against* the applied E field.

$$u = -\mu_n \cdot E \qquad (5)$$

Also, recall that voltage is the work required to move a 1-Coulomb test charge against the force exerted by an electric field. Recalling that F = QE, The work done in moving *against* the electric field along the x direction from drain (x = 0) to a position x = x_o is:

$$Vbs = \int_0^{x_0} F\,dx = -1\text{Coulomb} \cdot \int_0^{x_0} E\,dx = -\int_0^{x_0} E\,dx$$

The minus sign is because work is done moving against the electric field.

The differential quantity dVbs is $dVbs = -Edx$

And thus we have $$E = \frac{-dVbs}{dx} \qquad (6)$$

Thus from Eqns (5) and (6) we may write $$u = -\mu_n \cdot E = \mu_n \cdot \frac{d}{dx} Vbs \qquad (7)$$

And therefore from Eqn (4)

$$Id = q(x) \cdot u(x) = W \cdot C_{ox} \cdot [(Vgs - Vbs) - Vtn] \cdot \left(\mu_n \cdot \frac{d}{dx} Vbs\right) \qquad (8)$$

Multiplying through by dx and integrating over the length of the channel from x = 0 to x = L yields the ohmic mode equation for the MOSFET:

$$\int_0^L Id\, dx = \int_{Vbs(0)=0}^{Vbs(L)=Vds} W \cdot C_{ox} \cdot [(Vgs - Vbs) - Vtn] \cdot (\mu_n)\, dVbs \qquad (9)$$

$$Id \cdot L = \frac{-1}{2} \cdot W \cdot C_{ox} \cdot \mu_n \cdot Vds \cdot (Vds - 2 \cdot Vgs + 2 \cdot Vtn)$$

$$Id = \mu_n \cdot C_{ox} \cdot \frac{W}{2 \cdot L} \cdot \left[2 \cdot (Vgs - Vtn) \cdot Vds - Vds^2\right] \quad \text{for} \quad Vgs - Vds > Vtn \qquad (10)$$

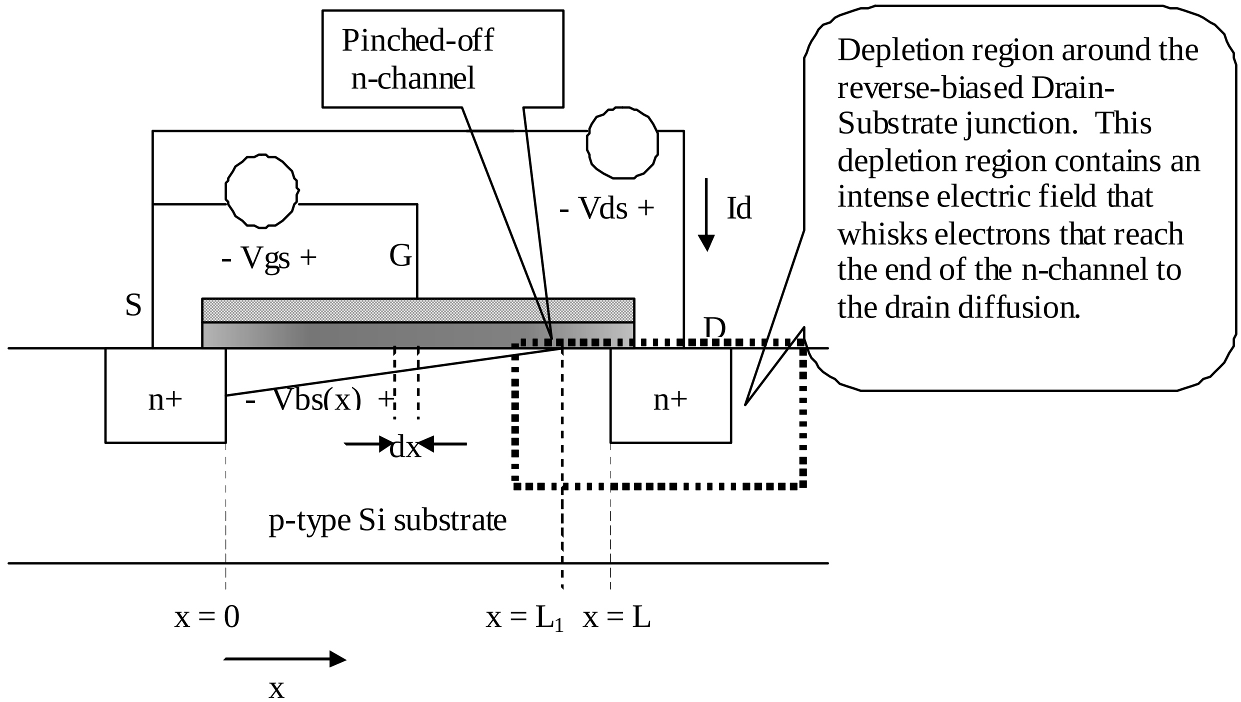
(Ohmic Mode)

Note that this derivation assumes the channel is not pinched off, since we are assuming the channel is fully formed all the way up to the drain diffusion, since we are integrating from x = 0 all the way up to x = L. In order for the channel to be formed all the way up to the drain, we require that Vds be low enough that Vgs - Vds > Vtn, which is the way we check to see if an NMOSFET is operating in the ohmic region.

Part 2. Saturation Mode NMOSFET (Vgs-Vds < Vtn) Channel is PInched Off

When Vds is raised to a point where Vgs-Vds < Vtn, the channel is NOT formed all the way to the drain, but rather is pinched off somewhere to the left of the drain at x = L_1 < L. This situation is depicted in Figure 2. Since the conducting n-channel is no longer completely formed, it might seem that Id will now go to 0, but in reality, the drain-to-substrate junction (n to p) forms a reverse-biased diode junction (since in saturation mode, Vds >> Vsubstrate), and thus a depletion region with an intense E field surrounds the drain region. Thus, electrons reaching the end of the pinched off channel will experience the strong E field of this depletion region, and be whisked across to the drain diffusion in much the same way that current is collected in a forward-active BJT.

Figure 2. Saturation Mode NMOSFET (Vgs - Vds < Vtn)



Equation (8) now only applies from x = 0 up to the pinch-off point, $x = L_1$, where L_1 is < L, since it was derived assuming a conducting n-channel. Note that the substrate surface voltage with respect to the source at the pinchoff point $x = L_1$ is $Vbs(L_1)$ = (Vgs-Vtn), since we must require that the gate-to-substrate surface voltage at the pinchoff point be Vgs - (Vgs-Vtn) = Vtn. This is in sharp contrast to the ohmic case considered above, where Vbs(L) = Vds. Therefore, we must multiply Equation (8) by dx and integrate only from x = 0 up to $x = L_1$

$$\int_0^L \mathrm{Id}\,dx = \int_{Vbs(0)=0}^{Vbs(L_1)=Vgs-Vtn} W\cdot C_{ox}\cdot[(Vgs - Vbs) - Vtn]\cdot(\mu_n)\,dVbs \qquad (11)$$

$$L\cdot Id = \frac{1}{2}\cdot W\cdot C_{ox}\cdot\mu_n\cdot Vgs^2 - W\cdot C_{ox}\cdot\mu_n\cdot Vgs\cdot Vtn + \frac{1}{2}\cdot W\cdot C_{ox}\cdot\mu_n \qquad (12)$$

$$Id = C_{ox}\cdot\mu_n\cdot\frac{W}{2\cdot L}\cdot(Vgs - Vtn)^2 \quad \text{for} \quad Vgs - Vds < Vtn \qquad (13)$$

(Saturation Mode)