5. Problems

1. A. Represent each sinusoid as a phasor. Write the phasors in polar form. Plot each phasor as an arrow in the complex plane.

   i. $5\cos\left(2\pi 60t + \frac{\pi}{3}\right)$

   ii. $0.002\sin\left(2\pi 10^7 t - 1.40\right)$

   iii. $150\cos\left(2\pi 1000t - 135°\right)$

   B. Plot each phasor as an arrow in the complex plane. Then write each phasor as a sinusoid in the form $A\cos(\omega_0 t + \theta)$. Let the frequency be $\omega_0 = 377$ r / s in each case.

   i. $4e^{j\frac{\pi}{3}}$

   ii. $5 + j7$

   iii. $35\angle(-45°)$

2. A. Represent each sinusoid as a phasor. Write the phasors in polar form. Plot each phasor as an arrow in the complex plane.

   i. $2\cos\left(2\pi 10^5 t - \frac{\pi}{3}\right)$ V

   ii. $6\sin\left(2\pi 60t - 2.7\right)$ mA

   iii. $150\cos\left(2\pi 10^8 t - 35°\right)$ μV

   B. Plot each phasor as an arrow in the complex plane. Then write each phasor as a sinusoid in the form $A\cos(\omega_0 t + \theta)$. Let the frequency be $\omega_0 = 2\pi 10^3$ r / s in each case.

   i. $5e^{-j\left(\frac{\pi}{6}\right)}$ A

   ii. $25 - j30$ A

   iii. $5\angle(65°)$ V

3. A. Represent each sinusoid as a phasor. Write the phasors in polar form. Plot each phasor as an arrow in the complex plane.

   i. $25\sin\left(2\pi 60t + \frac{\pi}{3}\right)$ mA

ii. $0.002\cos\left(2\pi 10^{7} t - 1.40\right)$ V

iii. $250\cos\left(2\pi 100t + 225°\right)$ V

B. Plot each phasor as an arrow in the complex plane. Then write each phasor as a sinusoid in the form $A\cos\left(\omega_0 t + \theta\right)$. Let the frequency be $\omega_0 = 2\pi 10^6$ r / s in each case.

i. $25e^{j\frac{2\pi}{3}}$ mV

ii. $-5 + j7$ V

iii. $155\angle(-35°)$ A

4. Use phasors to add the following waveforms. Express each result in the form $A\cos\left(\omega_0 t + \theta\right)$. In each case show the addition of the phasors on a graph of the complex plane.

A. $5\cos(377t) + 7\sin\left(377t + \frac{\pi}{3}\right)$

B. $-2\cos(1000t + 0.75) + 3\sin(1000t)$

C. $25\sin\left(100t - \frac{\pi}{4}\right) - 31\cos\left(100t + \frac{\pi}{3}\right)$

D. $0.010\cos\left(2\pi 10^{7} t - 3.23\right) + 0.025\cos\left(2\pi 10^{7} t + 3.23\right)$

E. $9\cos(1000t + 0.223) + 5\cos(1000t - 0.223) - 6\sin(1000t + 3.22)$

5. Use phasors to add the following waveforms. Express each result in the form $A\cos\left(\omega_0 t + \theta\right)$. In each case show the addition of the phasors on a graph of the complex plane.

A. $35\cos\left(2\pi 10^{6} t\right) + 42\sin\left(2\pi 10^{6} t\right)$

B. $6\cos(2\pi 60t) + 3\cos\left(2\pi 60t + \frac{\pi}{6}\right)$

C. $-5\cos(500t + 0.35) + 8\sin(500t)$

D. $13\sin\left(1000t - \frac{\pi}{4}\right) - 11\cos\left(1000t + \frac{\pi}{4}\right)$

E. $9\cos\left(10^{8} t + 0.113\right) - 4\cos\left(10^{8} t - 0.113\right) + 7\sin\left(10^{8} t - 2.15\right)$

6. Use phasors to add the following waveforms. Express each result in the form $A\cos\left(\omega_0 t + \theta\right)$. In each case show the addition of the phasors on a graph of the complex plane.

A. $0.03\sin\left(2\pi 60t - \frac{\pi}{3}\right) + 0.03\cos\left(2\pi 60t - \frac{\pi}{3}\right)$

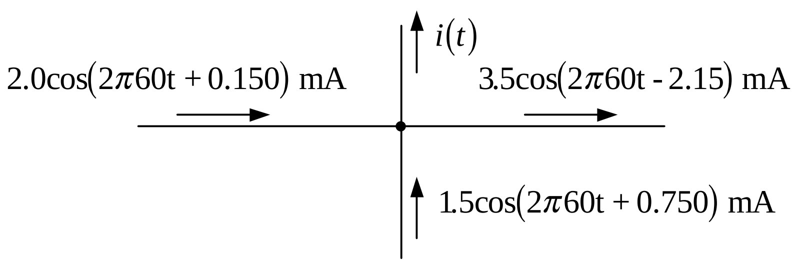
B. $5\cos(2\pi 1000t) - 8\sin(2\pi 1000t)$

C. $32\cos(2\pi 5\times 10^5 t) + 48\cos\left(2\pi 5\times 10^5 t + \frac{\pi}{3}\right)$

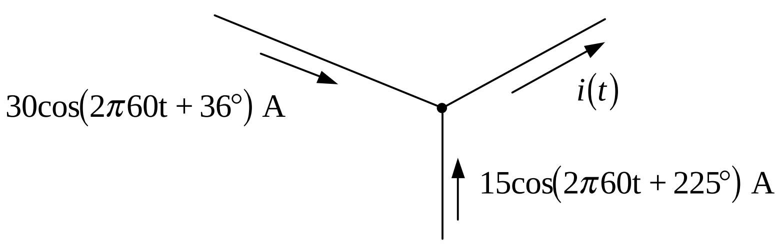
D. $2\times 10^{-3}\sin(377t + 40°) + 3\times 10^{-3}\sin(377t - 64°)$

E. $5\cos\left(2\pi 3.5\times 10^6 + \frac{\pi}{6}\right) + 5\sin\left(2\pi 3.5\times 10^6 + \frac{\pi}{6}\right) - 5\cos\left(2\pi 3.5\times 10^6 - \frac{\pi}{6}\right)$

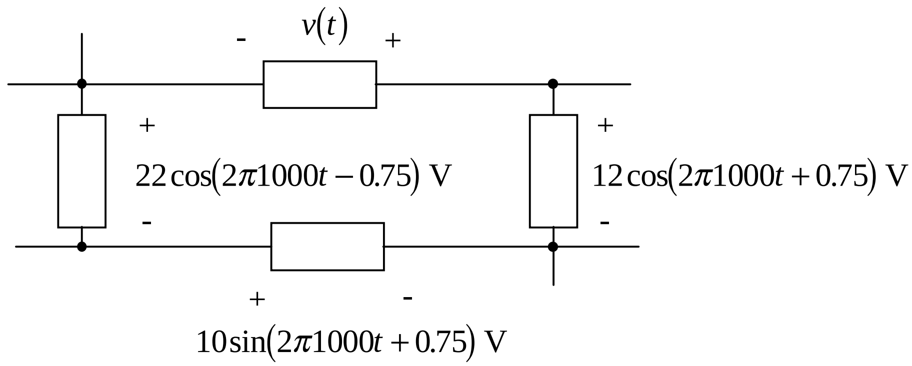
7. Use phasors to find the current $i(t)$ at the circuit node shown below. Express your answer as a time function. Draw a phasor diagram showing your calculation.



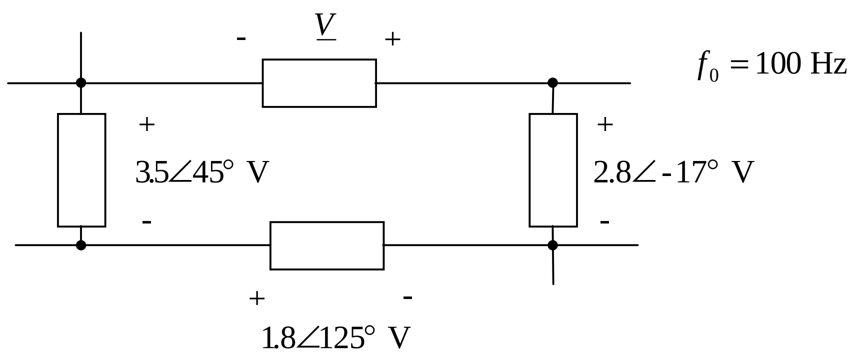
8. Use phasors to find the current $i(t)$ at the circuit node shown below. Express your answer as a time function. Draw a phasor diagram showing your calculation.



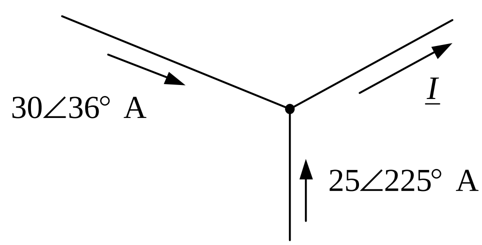
9. Use phasors to find the voltage $v(t)$ in the circuit mesh shown below. Express your answer as a time function. Draw a phasor diagram showing your calculation.



10. Use phasors to find the voltage $\underline{V}$ in the circuit mesh shown below. Then express your answer as a time function $v(t)$. Draw a phasor diagram showing your calculation.



11. Use phasors to find the current $\underline{I}$ at the circuit node shown below. Then express your answer as a time function $i(t)$. Draw a phasor diagram showing your calculation.



12. The variable $v_{out}(t)$ in a first-order circuit satisfies the differential equation

$$\frac{dv_{out}}{dt} + 400v_{out}(t) = 8000\cos(400t).$$

A. Replace the sinusoidal forcing function with a complex exponential.
B. Find a phasor particular solution $\underline{V_{out}}$.
C. Find the sinusoidal steady-state solution $v_{outSS}(t)$ of the original equation.

13. The variable $i_{out}(t)$ in a first-order circuit satisfies the differential equation

$$\frac{di_{out}}{dt}+\frac{1}{4.33\times10^{-3}}i_{out}(t)=\frac{2}{4.33\times10^{-3}}\sin(400t).$$

A. Replace the sinusoidal forcing function with a complex exponential.
B. Find a phasor particular solution $\underline{I_{out}}$.
C. Find the sinusoidal steady-state solution $i_{outSS}(t)$ of the original equation.

14. The variable $v_C(t)$ in a second-order circuit satisfies the differential equation

$$\frac{d^2v_C}{dt^2}+628\frac{dv_C}{dt}+3.16\times10^6 v_C(t)=126\times10^3\frac{di_{in}}{dt},$$

where $i_{in}(t)=20\cos(2\pi200t)$ mA.

A. Replace the sinusoidal forcing function with a complex exponential.
B. Find a phasor particular solution $\underline{V_C}$.
C. Find the sinusoidal steady-state solution $v_{Css}(t)$ of the original equation.

15. The variable $i_2(t)$ in a second-order circuit satisfies the differential equation

$$\frac{1}{125000}\frac{d^2i_2}{dt^2}+\frac{1}{200}\frac{di_2}{dt}+25i_2(t)=-8000\pi\sin(2\pi200t).$$

A. Replace the sinusoidal forcing function with a complex exponential.
B. Find a phasor particular solution $\underline{I_2}$.
C. Find the sinusoidal steady-state solution $i_{2ss}(t)$ of the original equation.

16. The variable $v_L(t)$ in a first-order circuit satisfies the differential equation

$$\frac{dv_L}{dt}+10v_L(t)=-60000\pi\sin(2\pi60t).$$

A. Replace the sinusoidal forcing function with a complex exponential.
B. Find a phasor particular solution $\underline{V_L}$.
C. Find the sinusoidal steady-state solution $v_{Lss}(t)$ of the original equation.

17. The variable $i_R(t)$ in a second-order circuit satisfies the differential equation

$$2.5\times10^{-3}\frac{d^2i_R}{dt^2}+250\frac{di_R}{dt}+\frac{1}{250\times10^{-12}}i_R(t)=-20\times10^7\sin\left(4\times10^7t+\frac{\pi}{3}\right).$$

A. Replace the sinusoidal forcing function with a complex exponential.

B. Find a phasor particular solution $\underline{I_R}$.

C. Find the sinusoidal steady-state solution $i_{Rss}(t)$ of the original equation.

18. The variable $v_o(t)$ in a first-order circuit satisfies the differential equation

$$0.002\times10^{-6}\frac{dv_o}{dt}+\frac{1}{500}v_o(t)=0.2\cos\left(2\pi10^3t-\frac{\pi}{6}\right).$$

A. Replace the sinusoidal forcing function with a complex exponential.

B. Find a phasor particular solution $\underline{V_o}$.

C. Find the sinusoidal steady-state solution $v_{oSS}(t)$ of the original equation.

19. Find the impedance of

A. A $0.01\ \mu\text{F}$ capacitor at 3.5 MHz.

B. A $0.01\ \mu\text{F}$ capacitor at 10.7 MHz.

C. A 25 mH inductor at 1000 Hz.

D. A 2.5 H inductor at 60 Hz.

E. A 250 pF capacitor at 10^6 rad/s.

20. Find the impedance of

A. A $25\ \mu\text{F}$ capacitor at 100 Hz.

B. A $250\ \mu\text{H}$ inductor at 350 MHz.

C. A 30 mH inductor at 2500 rad/s.

Find the admittance of

D. A $10\ \mu\text{F}$ capacitor at 1 kHz.

E. A 30 mH inductor at 2500 rad/s.

21. Find the impedance of

A. An $0.001\ \mu\text{F}$ capacitor at 850 MHz.

B. An $0.01\ \mu\text{F}$ capacitor at 450 rad/s.

C. A $10\ \mu\text{H}$ inductor at 850 MHz

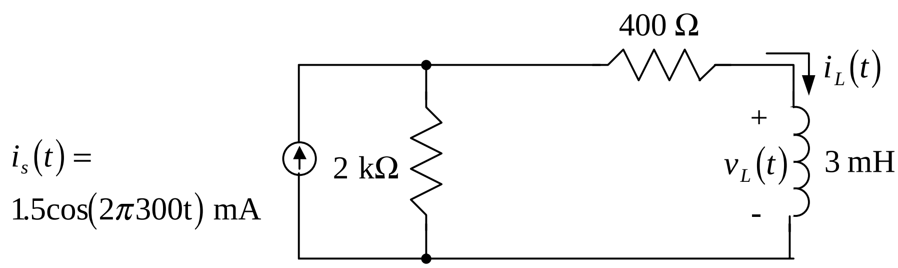
Find the admittance of

D. A 3.5 H inductor at 120 Hz

E. A $10\ \mu\text{F}$ capacitor at 377 rad/s.

22. The circuit shown below has been in operation for a long time.

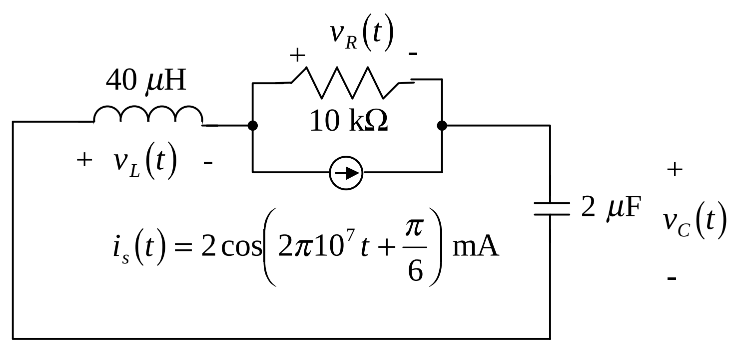
A. Transform the circuit to impedance and phasor form.



B. Find the phasors $\underline{V_L}$ and $\underline{I_L}$. Plot $\underline{I_s}$, $\underline{V_L}$, and $\underline{I_L}$ on a phasor diagram.

C. Find $v_L(t)$ and $i_L(t)$ in the sinusoidal steady state.

23. The circuit shown below has been in operation for a long time.

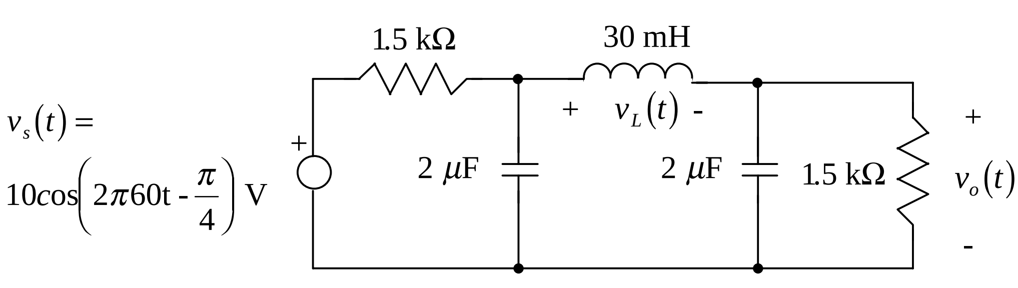


A. Transform the circuit to impedance and phasor form.

B. Find the phasors $\underline{V_L}$, $\underline{V_R}$, and $\underline{V_C}$. Plot these phasors on a phasor diagram.

C. Find $v_L(t)$, $v_R(t)$, and $v_C(t)$ in the sinusoidal steady state.

24. The circuit shown below has been in operation for a long time.

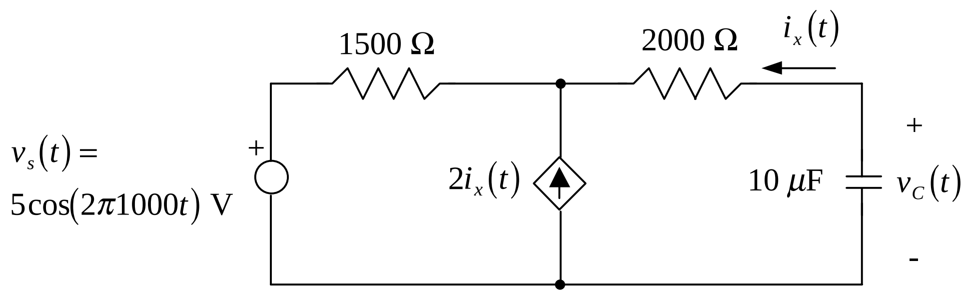


A. Transform the circuit to impedance and phasor form.

B. Find the phasors $\underline{V_L}$ and $\underline{V_o}$. Plot $\underline{V_s}$, $\underline{V_L}$, and $\underline{V_o}$ on a phasor diagram.

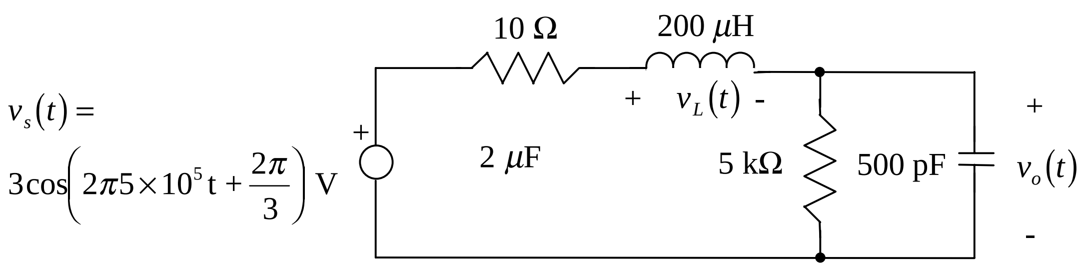
C. Find $v_L(t)$ and $v_o(t)$ in the sinusoidal steady state.

25. The circuit shown below has been in operation for a long time.



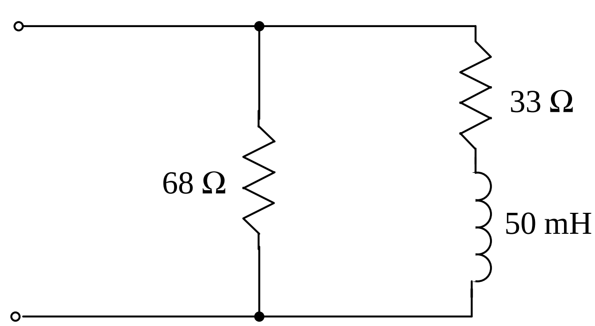
A. Transform the circuit to impedance and phasor form.
B. Find the phasors $\underline{I_x}$ and $\underline{V_C}$. Plot $\underline{V_s}$, $\underline{I_x}$, and $\underline{V_C}$ on a phasor diagram.
C. Find $i_x(t)$ and $v_C(t)$ in the sinusoidal steady state.

26. The circuit shown below has been in operation for a long time.

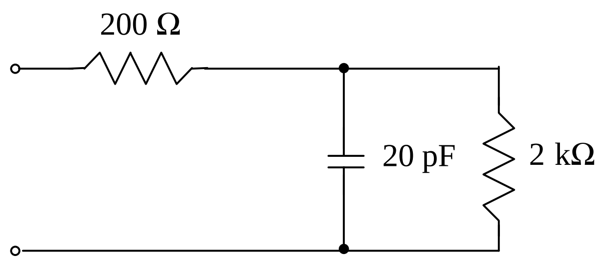


A. Transform the circuit to impedance and phasor form.
B. Find the phasor $\underline{V_C}$. Plot $\underline{V_s}$ and $\underline{V_C}$ on a phasor diagram.
C. Find $v_C(t)$ in the sinusoidal steady state.

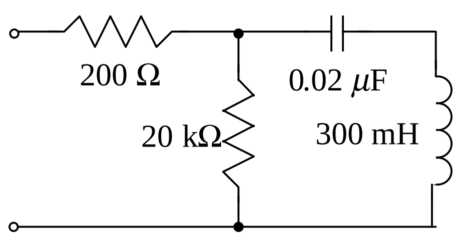
27. Find the equivalent impedance of the combination of elements shown below at a frequency of 200 Hz.



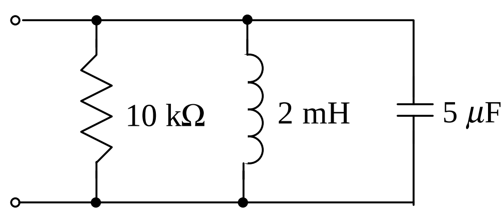
28. Find the equivalent impedance of the combination of elements shown below at a frequency of 40 MHz.



29. Find the equivalent impedance of the combination of elements shown below at a frequency of 12,000 rad/s.

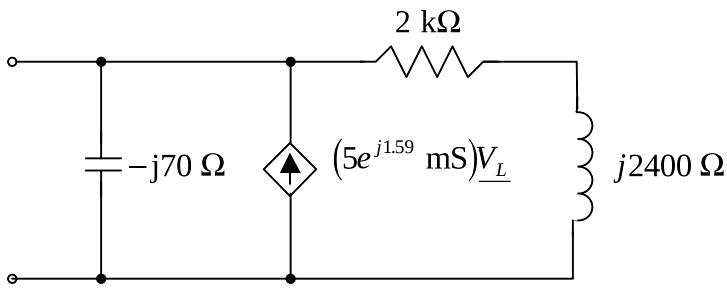


30. Find the equivalent impedance of the combination of elements shown below at a frequency of

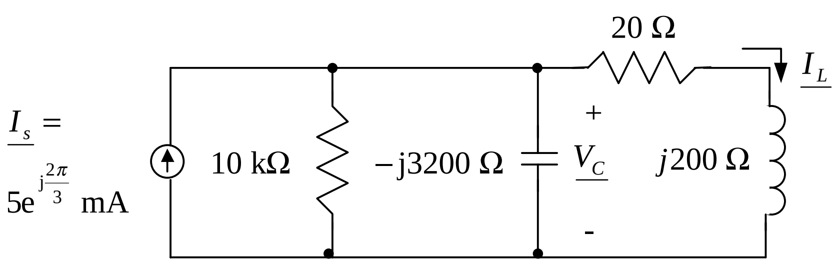


A. 9000 rad/s.
B. 10,000 rad/s.
C. 11,000 rad/s.

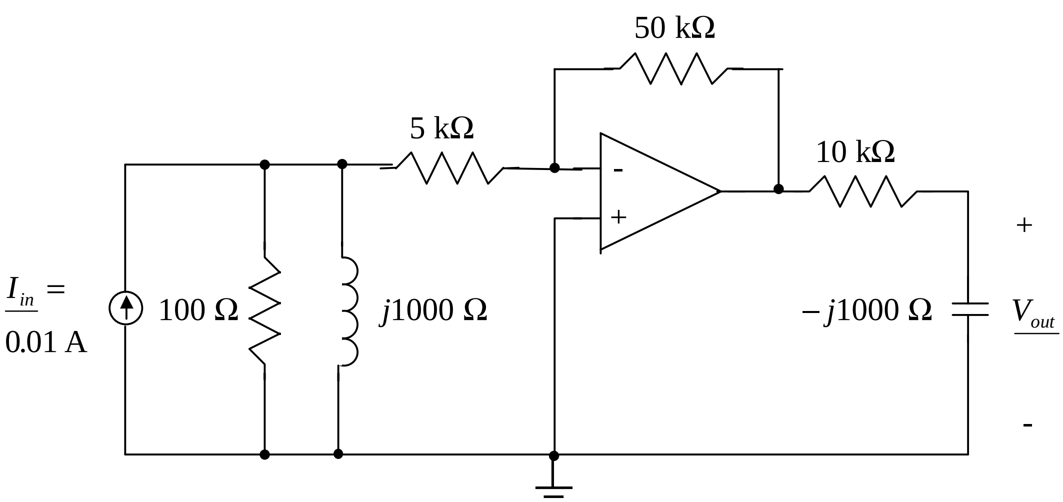
31. Find the equivalent impedance of the combination of elements shown below. (Owing to the dependent source this cannot be solved by a simple set of series and parallel combinations.)



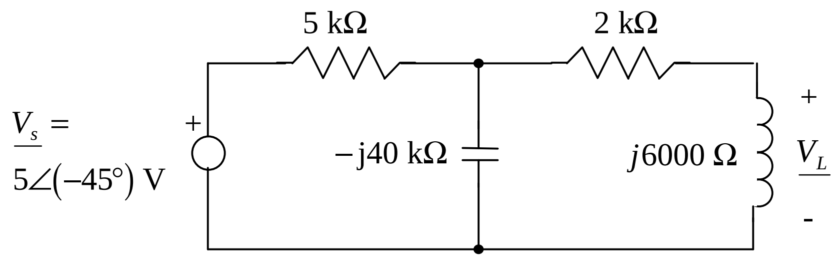
32. Find $\underline{V_C}$ and $\underline{I_L}$. Plot $\underline{I_s}$, $\underline{V_C}$, and $\underline{I_L}$ on a phasor diagram.



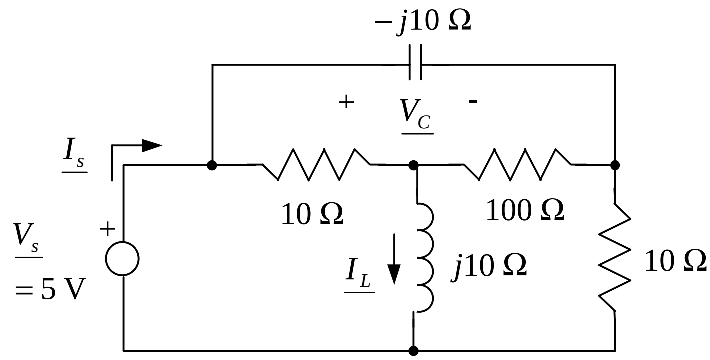
33. Find $\underline{V_{out}}$. Plot $\underline{I_{in}}$ and $\underline{V_{out}}$ on a phasor diagram.



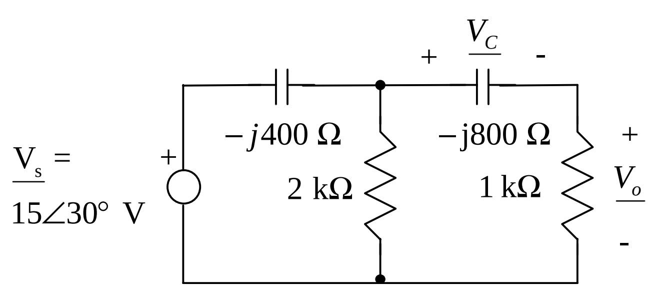
34. Find $\underline{V_L}$. Plot $\underline{V_s}$ and $\underline{V_L}$ on a phasor diagram.



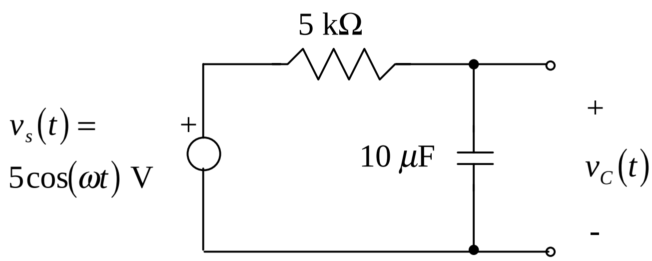
35. Find $\underline{I_s}$, $\underline{V_C}$, and $\underline{I_L}$. Plot $\underline{V_s}$ and $\underline{V_C}$ to scale on a phasor diagram. Plot $\underline{I_s}$ and $\underline{I_L}$ to scale on a second phasor diagram.



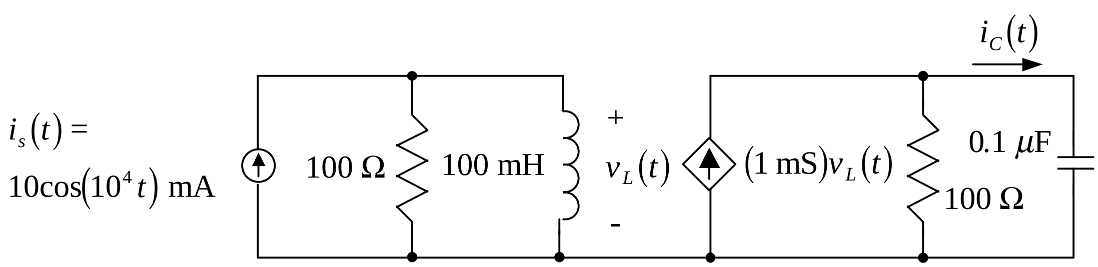
36. Find $\underline{V_C}$ and $\underline{V_o}$. Plot $\underline{V_s}$, $\underline{V_C}$ and $\underline{V_o}$ to scale on a phasor diagram.



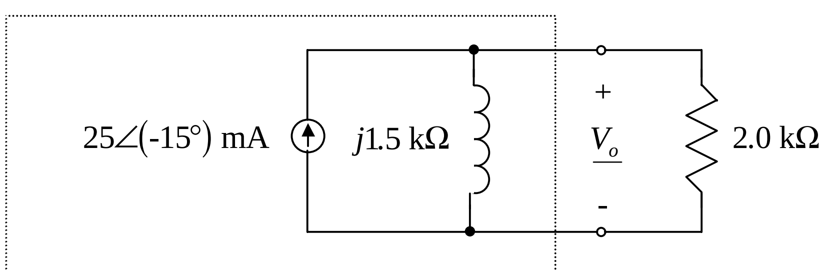
37. Use the voltage divider to find the phasor $\underline{V_C}$. Plot $\underline{V_C}$ to scale on a phasor diagram for $\omega = 1,\ 10,\ 100,\ \text{and } 500\ \text{r/s}$.



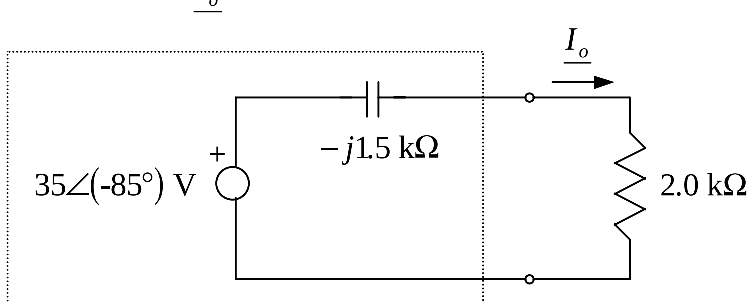
38. The circuit below is in the sinusoidal steady state. Use the current divider to find $\underline{I_C}$. Then find $i_C(t)$.



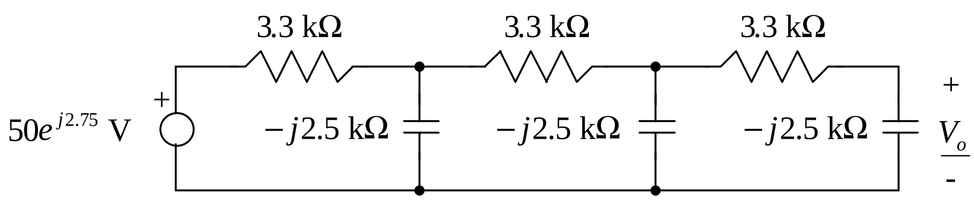
39. In the circuit shown below, replace the nonideal source by its Thevenin equivalent. Then use the voltage divider to find $\underline{V_o}$.



40. In the circuit shown below, replace the nonideal source by its Norton equivalent. Then use the current divider to find $\underline{I_o}$.

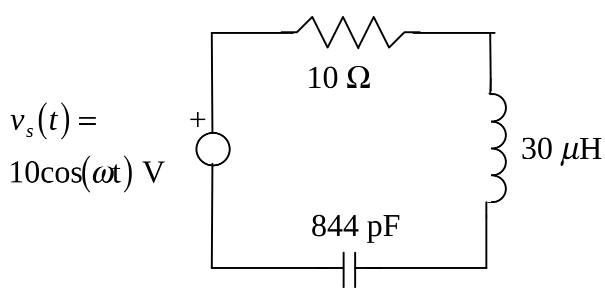


41. Use series and parallel combinations, voltage and current dividers, and source transformations as appropriate to find $\underline{V}_o$ in the circuit shown below.



42. The series circuit shown below is in the sinusoidal steady state.

A. i. Find the impedance seen by the voltage source. Express your answer as a function of the radian frequency ω.
   ii. Is there a resonant frequency? That is, is there a frequency at which the impedance is purely real? If so, find it.
   iii. Find the value of the impedance seen by the voltage source at the resonant frequency.



B. i. For the rest of this problem assume that the frequency of the voltage source is the resonant frequency of the circuit. Find the sinusoidal steady-state current and the voltage across the resistor, the inductor, and the capacitor. Use phasors to solve the problem, but express your answers as time functions.
   ii. Draw a phasor diagram to scale showing the resistor voltage, the inductor voltage, and the capacitor voltage.

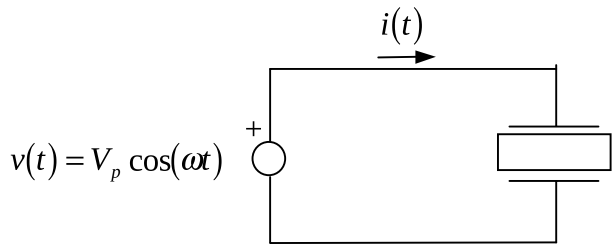
C. i. Find the energy $W_m(t)$ stored in the inductor. Find the energy $W_e(t)$ stored in the capacitor.
   ii. Find the total stored energy W_T.

D. i. Find the power $p_R(t)$ delivered to the resistor.
   ii. Find the energy W_R converted to heat in the resistor during a time interval equal to one radian of phase change of the input voltage at the resonant frequency.
   iii. Find the quality factor Q of the circuit.

43. In the diagram below, a device is made by sandwiching a piece of piezoelectric material (such as a quartz crystal) between two metal plates. Because the piezoelectric material deforms in response to an applied voltage, the constitutive relation between the device

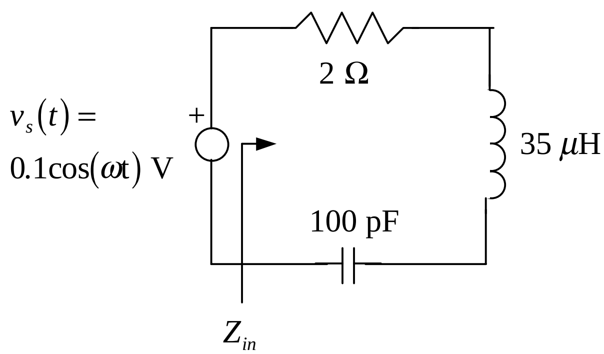
voltage and the device current depends on the way the crystal vibrates when a voltage is applied.

In the circuit shown the applied voltage comes from a sinusoidal function generator, and both the voltage and current can be monitored on an oscilloscope. The circuit is operating in the sinusoidal steady state. Deformation of the piezoelectric material is small, so that the circuit is linear.



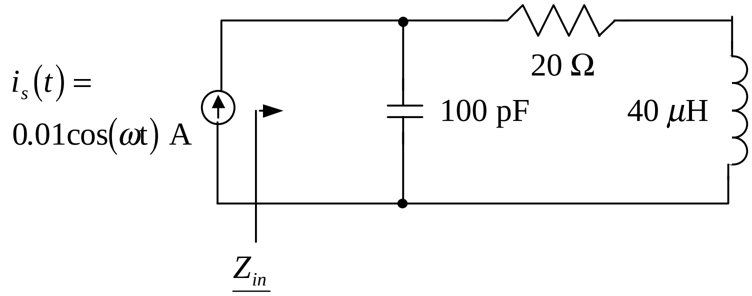
A. The frequency of the applied voltage is slowly varied by turning the knob on the function generator. How can you tell when the function generator is set to a *resonant frequency* of the circuit?
B. How can you define the *quality factor* Q of the resonant circuit?

44. The circuit shown below is in the sinusiodal steady state at radian frequency ω.



A. Find an expression for the input impedance $\underline{Z}_{in}$ in terms of ω.
B. Find the resonant frequency ω_0 of this circuit.
C. Find the total stored energy in the inductor and capacitor (together) at frequency ω_0.

45. The circuit shown below is in the sinusiodal steady state at radian frequency ω.



A. i. Find the impedance $\underline{Z}_{in}$ seen by the current source. Express your answer as a function of the radian frequency ω.

ii. Is there a resonant frequency? That is, is there a frequency at which the impedance is purely real? If so, find it. (You will want to use Maple for this.)

iii. Find the value of the impedance seen by the current source at the resonant frequency.

B. i. For the rest of this problem assume that the frequency of the current source is the resonant frequency of the circuit. Find the sinusoidal steady-state voltage across the capacitor, and the sinusoidal steady-state current through the resistor and the inductor. Use phasors to solve the problem, but express your answers as time functions.

ii. Draw a phasor diagram to scale showing the source current and the inductor current.

C. i. Find the energy $W_m(t)$ stored in the inductor. Find the energy $W_e(t)$ stored in the capacitor.

ii. Find the total stored energy $W_T(t)$. It will not be constant, but it will be very nearly constant compared with $W_m(t)$ or $W_e(t)$. Find the average value W_T of $W_T(t)$.

D. i. Find the power $p_R(t)$ delivered to the resistor.

ii. Find the energy W_R converted to heat in the resistor during a time interval equal to one radian of phase change of the input voltage at the resonant frequency.

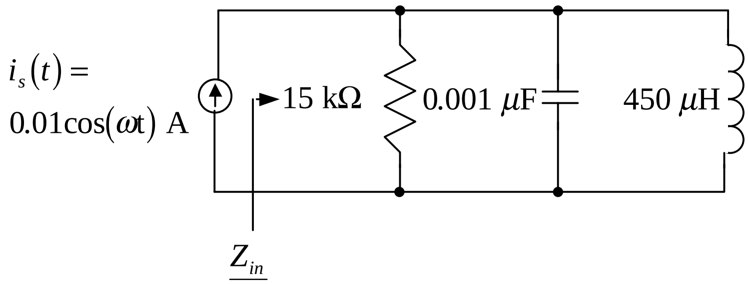
iii. Find the quality factor Q of the circuit.

46. The parallel circuit shown below is in the sinusoidal steady state.

A. i. Find the impedance $\underline{Z}_{in}$ seen by the current source. Express your answer as a function of the radian frequency ω.

ii. Is there a resonant frequency? That is, is there a frequency at which the impedance is purely real? If so, find it.

iii. Find the value of the impedance seen by the current source at the resonant frequency.



B. i. For the rest of this problem assume that the frequency of the current source is the resonant frequency of the circuit. Find the sinusoidal steady-state voltage and the current through the resistor, the inductor, and the capacitor. Use phasors to solve the problem, but express your answers as time functions.

ii. Draw a phasor diagram to scale showing the resistor current, the inductor current, and the capacitor current.

C. i. Find the energy $W_m(t)$ stored in the inductor. Find the energy $W_e(t)$ stored in the capacitor.

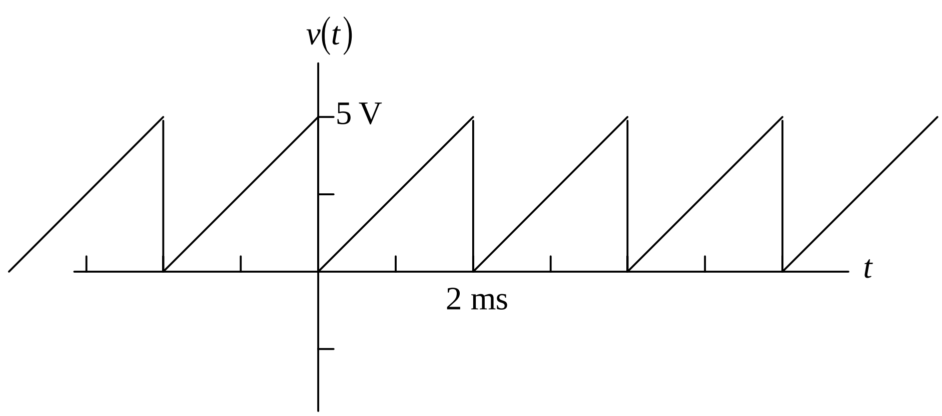
ii. Find the total stored energy W_T.

D. i. Find the power $p_R(t)$ delivered to the resistor.

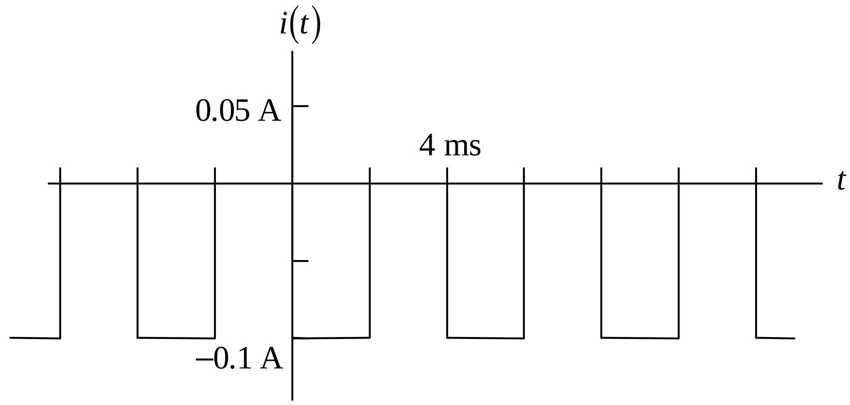
ii. Find the energy W_R converted to heat in the resistor during a time interval equal to one radian of phase change of the input voltage at the resonant frequency.

iii. Find the quality factor Q of the circuit.

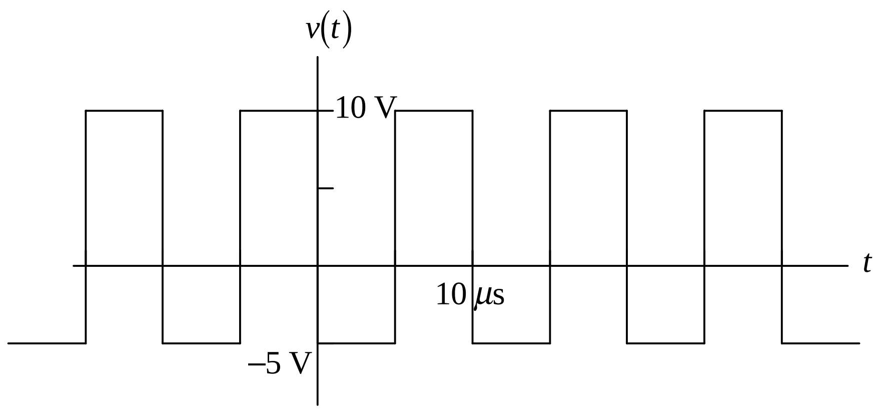
47. Find the effective (RMS) value of the voltage waveform shown below.



48. Find the effective (RMS) value of the current waveform shown below.



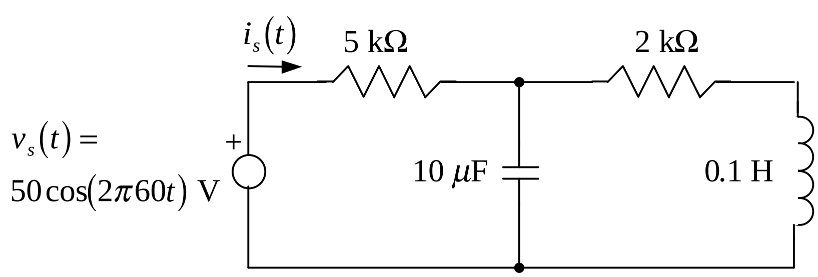
49. Find the effective (RMS) value of the voltage waveform shown below.



50. Find the effective (RMS) value of the current $i(t)$ given by

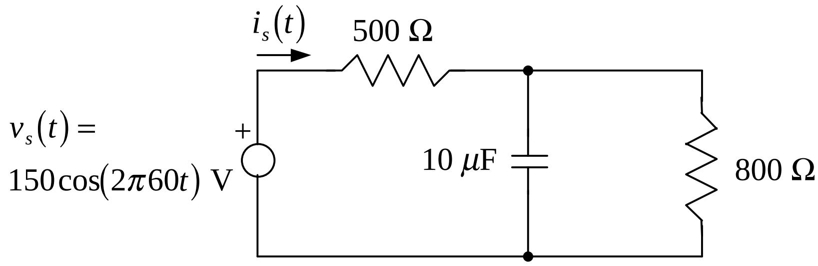
$$i(t) = 20\cos(2\pi 60t) + 15\cos(2\pi 180t)\ \text{A}.$$

51. The circuit shown below is in the sinusoidal steady state.



A. Find the source current $i_s(t)$.
B. Find the instantaneous power $p_s(t)$ provided by the voltage source.
C. Find the effective source voltage and effective source current.
D. Find the average power provided by the voltage source.
E. Find the power factor. Identify the power factor as “leading” or “lagging.”

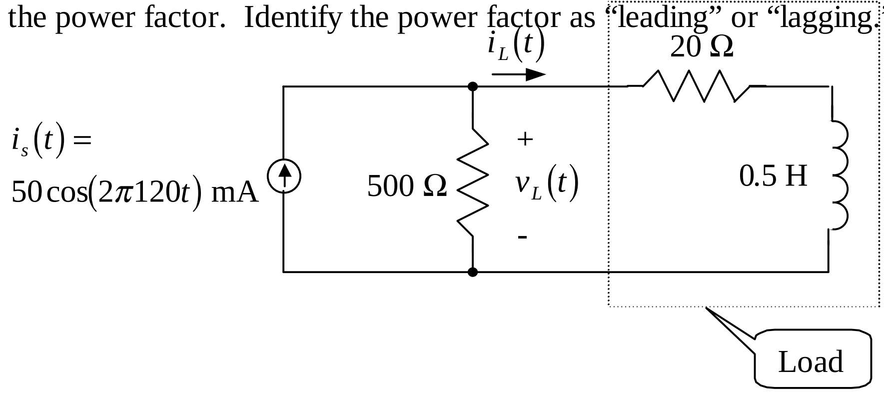
52. The circuit shown below is in the sinusoidal steady state.



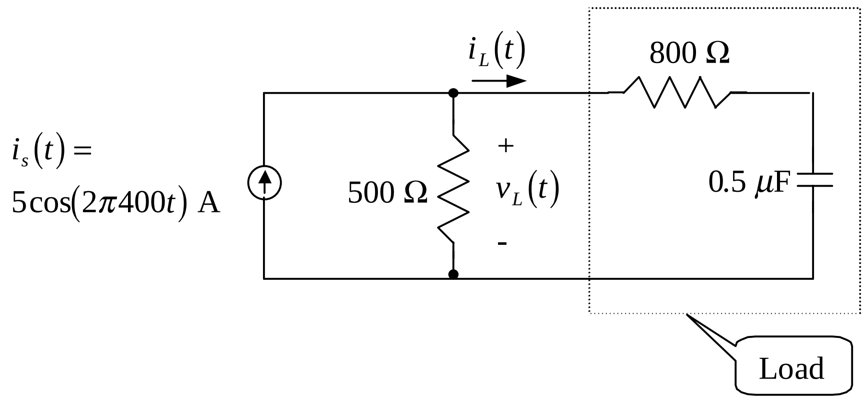
A. Find the source current $i_s(t)$.
B. Find the instantaneous power $p_s(t)$ provided by the voltage source.
C. Find the effective source voltage and effective source current.
D. Find the average power provided by the voltage source.
E. Find the power factor. Identify the power factor as "leading" or "lagging."

53. The circuit shown below is in the sinusoidal steady state.

A. Find the load voltage $v_L(t)$ and the load current $i_L(t)$.
B. Find the instantaneous power $p_L(t)$ delivered to the load. Plot your answer vs. time, showing several cycles.
C. Find the effective load voltage and effective load current.
D. Find the apparent power and the average power delivered to the load.
E. Find the power factor. Identify the power factor as "leading" or "lagging."

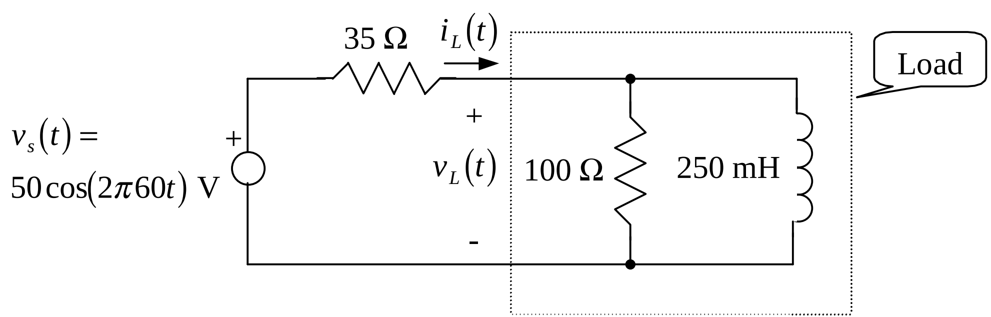


54. The circuit shown below is in the sinusoidal steady state.



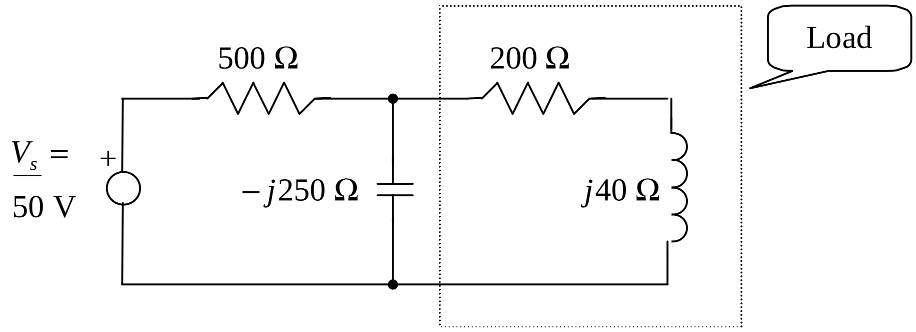
A. Find the load voltage $v_L(t)$ and the load current $i_L(t)$.
B. Find the instantaneous power $p_L(t)$ delivered to the load. Plot your answer vs. time, showing several cycles.
C. Find the effective load voltage and effective load current.
D. Find the apparent power and the average power delivered to the load.
E. Find the power factor. Identify the power factor as "leading" or "lagging."

55. The circuit shown below is in the sinusoidal steady state.



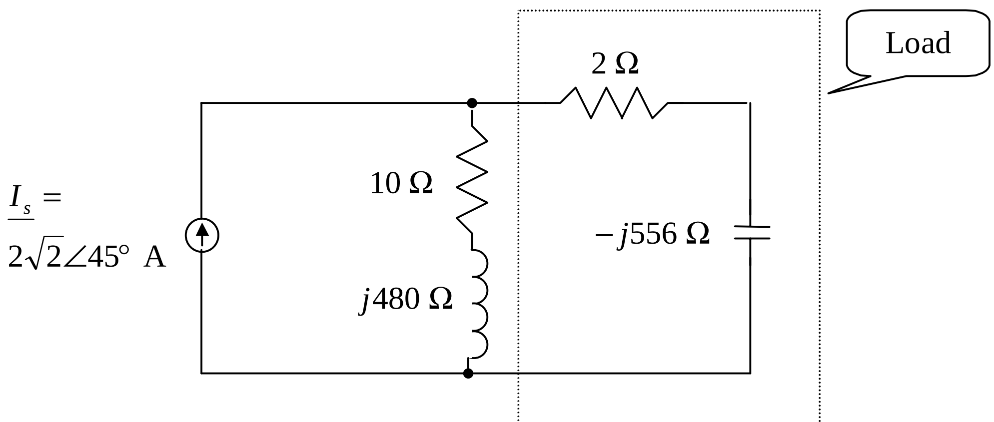
A. Find the load voltage $v_L(t)$ and the load current $i_L(t)$.
B. Find the instantaneous power $p_L(t)$ delivered to the load. Plot your answer vs. time, showing several cycles.
C. Find the effective load voltage and effective load current.
D. Find the apparent power and the average power delivered to the load.
E. Find the power factor. Identify the power factor as "leading" or "lagging."

56. In the circuit shown below,



A. Determine the complex power delivered to the load.
B. Determine the average power and the reactive power delivered to the load.
C. Find the load power factor.
D. Draw the power triangle for the load.

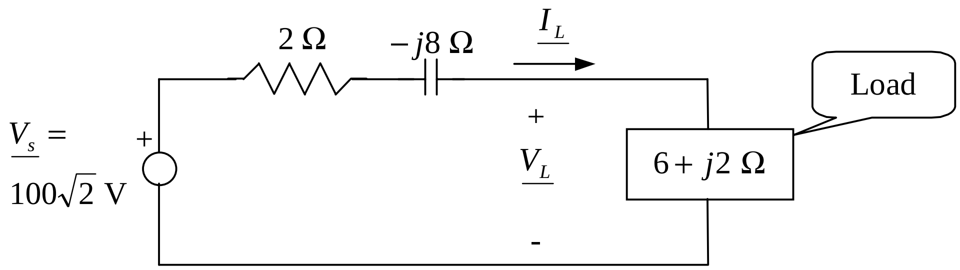
57. In the circuit shown below,



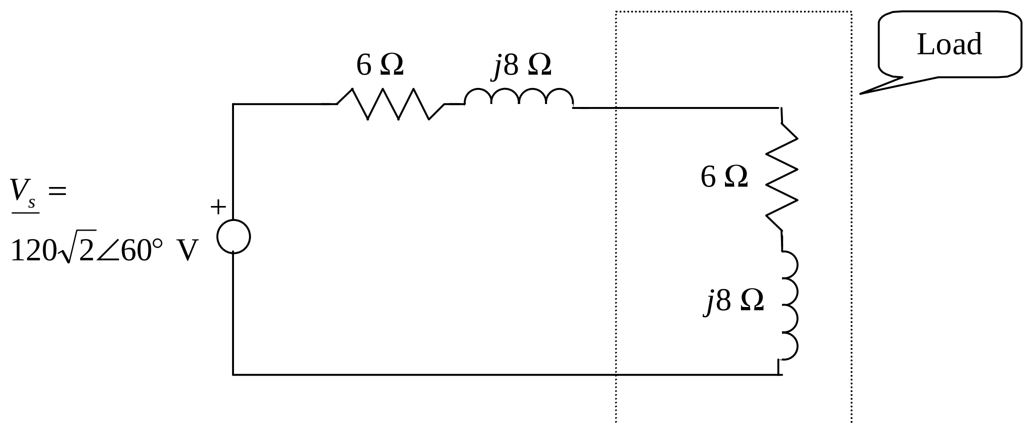
A. Determine the complex power delivered to the load.
B. Determine the average power and the reactive power delivered to the load.
C. Find the load power factor. Identify whether it is “leading” or “lagging.”
D. Draw the power triangle for the load.

58. In the circuit shown below,

A. Find the load voltage $\underline{V_L}$ and the load current $\underline{I_L}$.
B. Find the complex power $\underline{S_s}$ generated by the source and the complex power $\underline{S_L}$ delivered to the load.
C. Find the average power generated by the source, the average power delivered to the load, and the average power delivered to the 2 Ω resistor. Verify that average power is conserved.
D. Find the reactive power generated by the source and the reactive power delivered to the load. How much reactive power is delivered to the capacitor?

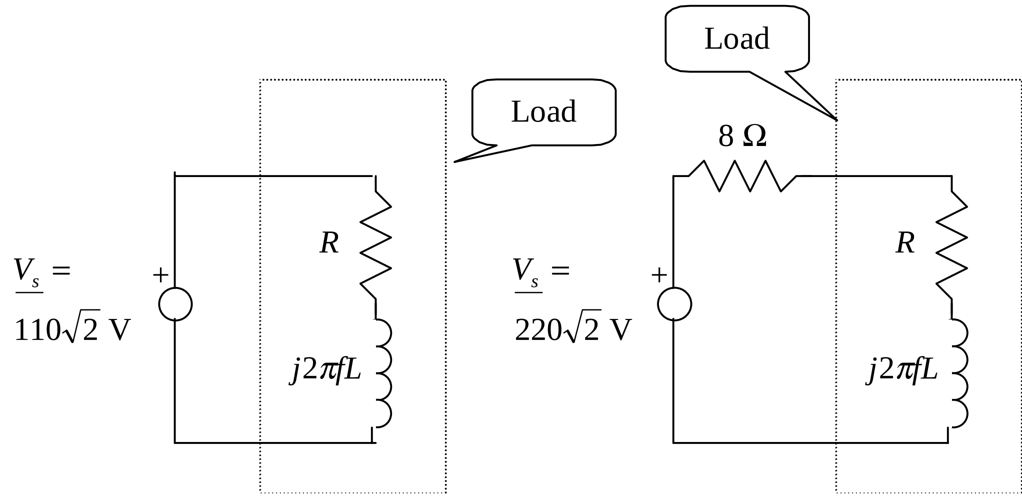


59. In the circuit shown below,



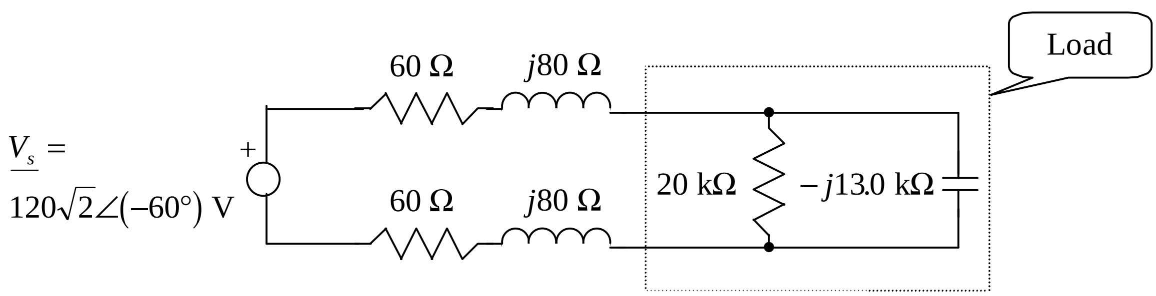
A. Find the complex power generated by the source.
B. Find the complex power delivered to the load.
C. Find the load power factor. Identify it as "leading" or "lagging."
D. Draw the power triangle for the load. Find the real power and the reactive power delivered to the load.

60. In the two circuits shown below the sources have a frequency of 60 Hz. The load is the same in both circuits. Suppose in Circuit A the average power delivered to the load is 300 W. In Circuit B the source voltage is doubled, a resistor is added, and the average power delivered to the load is (still) 300 W. Find the load resistance R and the load inductance L.



Circuit A Circuit B

61. In the circuit shown below,



A. Find the complex power generated by the source.
B. Find the complex power delivered to the load.
C. Find the load power factor. Identify it as “leading” or “lagging.”
D. Draw the power triangle for the load. Find the apparent power, the real power, and the reactive power delivered to the load.