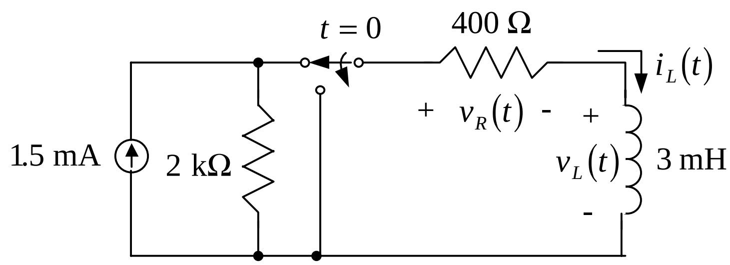
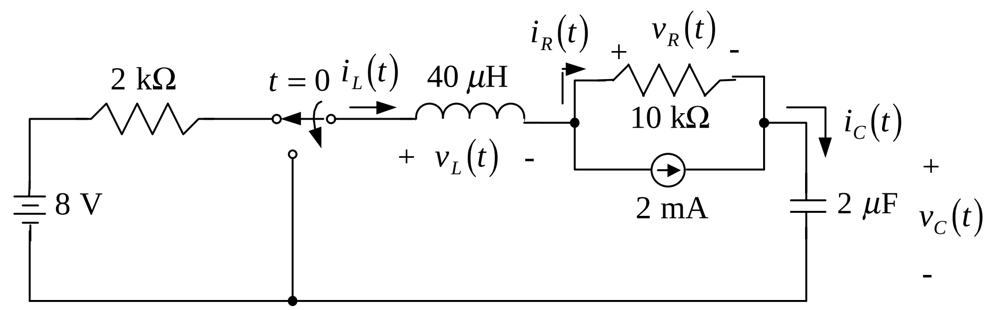
4. Problems

1. In the circuit shown below the switch has been in the position shown for a long time. At $t = 0$ the switch is thrown to the lower position.



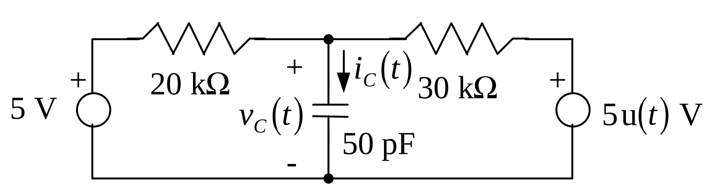
   A. Find $v_R(t)$, $v_L(t)$, and $i_L(t)$ at $t = 0^-$.
   B. Find $v_R(t)$, $v_L(t)$, and $i_L(t)$ at $t = 0^+$.
   C. Find the energy stored in the inductor at $t = 0$.

2. In the circuit shown below the switch has been in the position shown for a long time. At $t = 0$ the switch is thrown to the lower position.



   A. Identify the state variables.
   B. Find $v_L(t)$, $i_L(t)$, $v_R(t)$, $i_R(t)$, $v_C(t)$, and $i_C(t)$ at $t = 0^-$.
   C. Find $v_L(t)$, $i_L(t)$, $v_R(t)$, $i_R(t)$, $v_C(t)$, and $i_C(t)$ at $t = 0^+$.

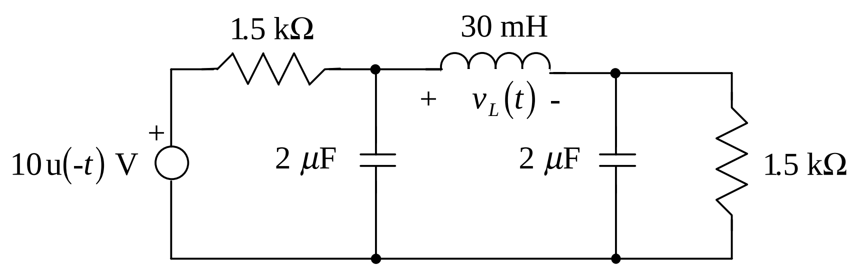
3. In the circuit shown below,



   A. Identify the state variable(s).

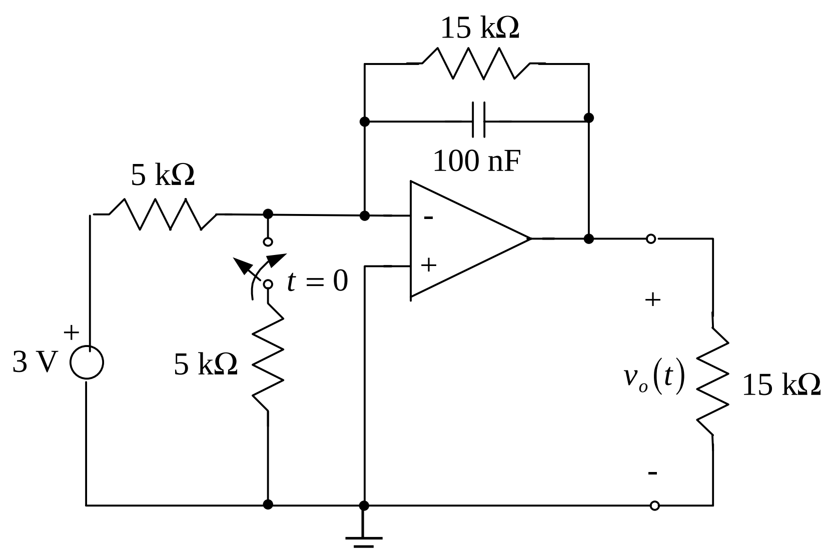
B. Find $v_C(t)$ and $i_C(t)$ at $t = 0^-$.
C. Find $v_C(t)$ and $i_C(t)$ at $t = 0^+$.

4. In the circuit shown below,



A. Identify the state variables.
B. What is the order of the circuit?
C. Find values of the state variables at $t = 0$.
D. Find $v_L(0^+)$.

5. The circuit shown below uses an ideal op-amp. The switch has been open for a long time before $t = 0$, and at $t = 0$ it suddenly closes.

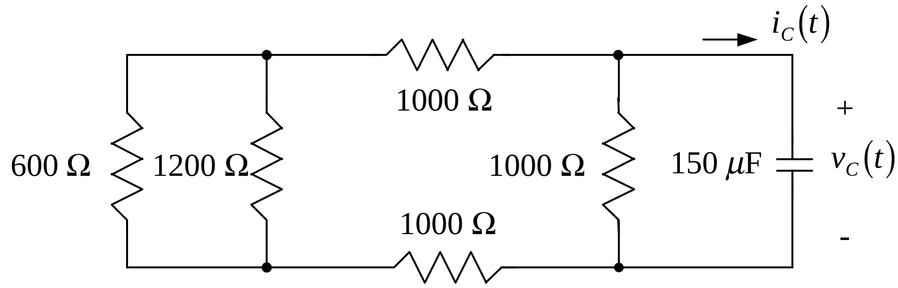


A. Identify the state variable(s).
B. Find the value(s) of the state variable(s) at $t = 0$.
C. Find $v_o(0^-)$ and $v_o(0^+)$.

6. In the circuit shown below $v_C(0) = 80$ V.

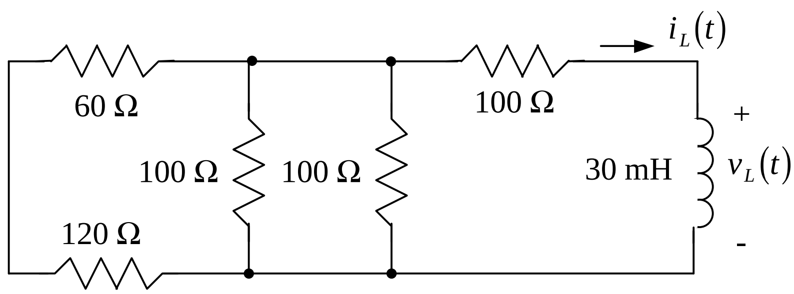
A. Find $v_C(t)$ for $t \geq 0$.

B. Find the time constant τ.
C. Plot $v_C(t)$ for $t \geq 0$ to scale by hand. Include about five time constants of elapsed time in your plot.

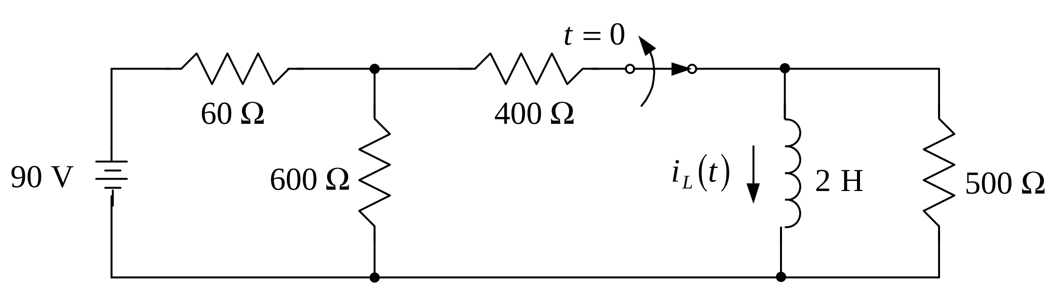


7. In the circuit shown below $i_L(0) = 20$ mA.

A. Find $i_L(t)$ for $t \geq 0$.
B. Find the time constant τ.
C. Plot $i_L(t)$ for $t \geq 0$ to scale by hand. Include about five time constants of elapsed time in your plot.

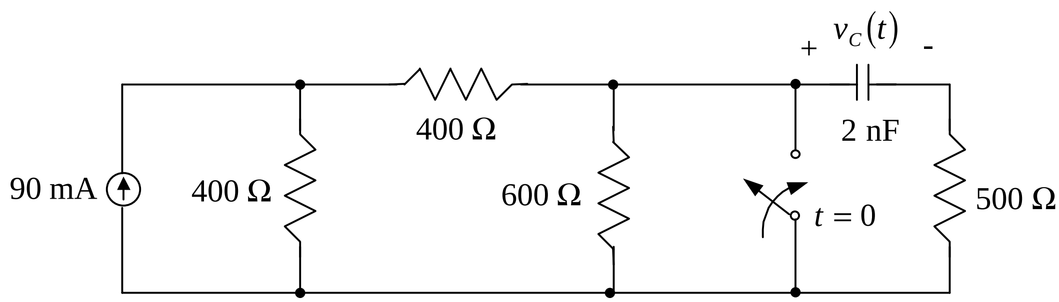


8. In the circuit below the switch has been closed for a long time before it opens at $t = 0$.



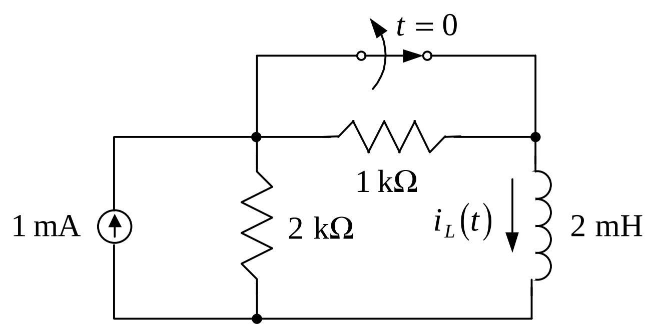
A. Find $i_L(0)$.
B. Find $i_L(t)$ for $t \geq 0$.
C. Find the time constant τ.
D. Plot $i_L(t)$ to scale by hand. Include the time interval $-5\tau \leq t \leq 5\tau$ in your plot.

9. In the circuit below the switch has been open for a long time before it closes at $t = 0$.



A. Find $v_C(0)$.
B. Find $v_C(t)$ for $t \geq 0$.
C. Find the time constant τ.
D. Plot $v_C(t)$ to scale by hand. Include the time interval $-5\tau \leq t \leq 5\tau$ in your plot.

10. In the circuit shown below the switch has been closed for a long time before it suddenly opens at $t = 0$.

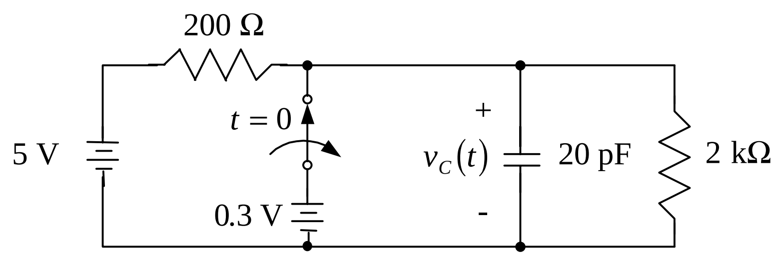


A. Find $i_L(0)$.
B. Find $i_L(t)$ for $t \geq 0$.
C. Write $i_L(t)$ as the sum of a *transient* part and a *steady-state* part.
D. Find the time constant τ of the circuit for $t > 0$.
E. After the switch is opened, how long does it take for the circuit to return to steady state? Use a conventional approximation.

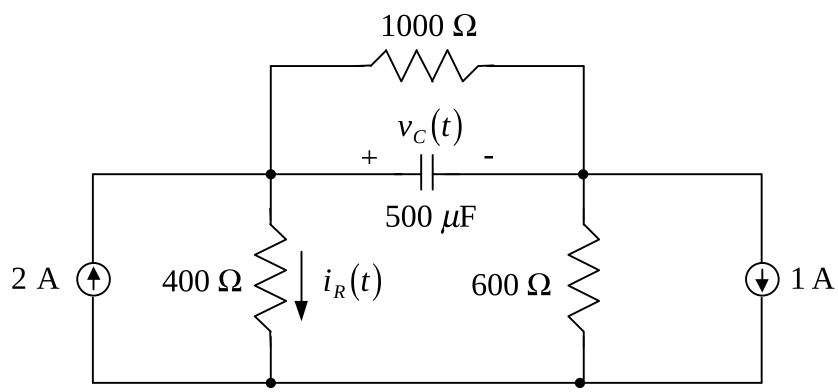
11. In the circuit shown below the switch has been closed for a long time, when it suddenly opens at $t = 0$.

A. Find $v_C(0)$.
B. Find the "final" capacitor voltage $v_C(\infty)$.
C. Find $v_C(t)$ for $t \geq 0$.

D. Determine the time constant τ of the circuit for $t > 0$.

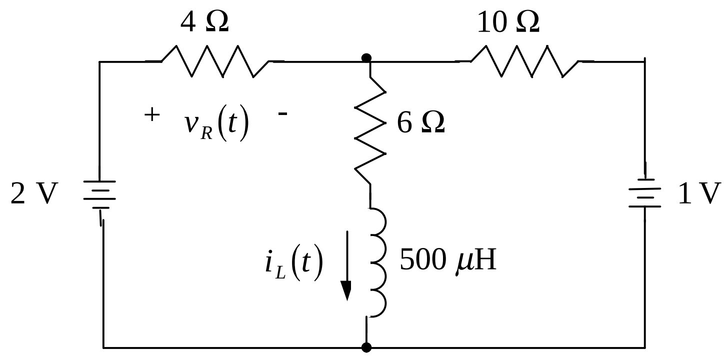


12. In the circuit shown below $v_C(0) = 15$ V.

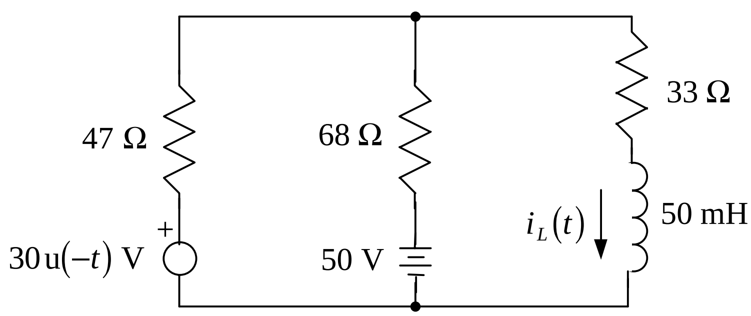


A. Find $v_C(t)$ for $t \geq 0$.
B. Find the time constant τ.
C. Find the initial value $i_R(0^+)$.
D. Find the final value $i_R(\infty)$.
E. Find $i_R(t)$ for $t > 0$.
F. Plot your answers to A and E.

13. In the circuit shown below $i_L(0) = 150$ mA.
A. Find $i_L(t)$ for $t \geq 0$.
B. Find the time constant τ.
C. Find the initial value $v_R(0^+)$.
D. Find the final value $v_R(\infty)$.
E. Find $v_R(t)$ for $t > 0$.
F. Plot your answers to A and E.

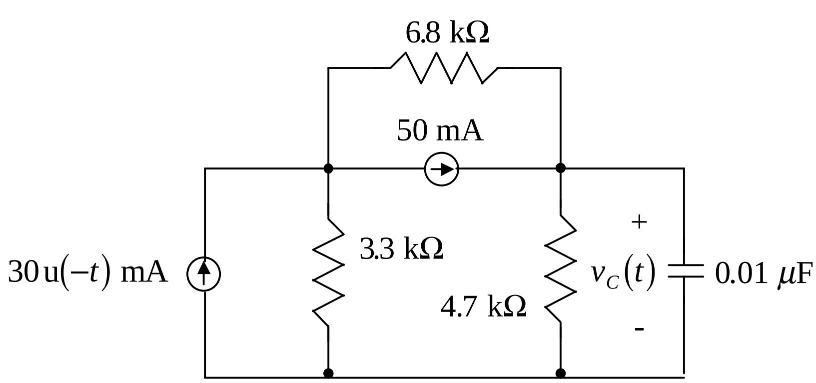


14. The circuit shown below contains one constant source and one source that turns *off* at $t = 0$.



A. Find $i_L(0)$.
B. Find $i_L(t)$ for $t \geq 0$.
C. Write $i_L(t)$ as the sum of a *transient* response and a *steady-state* response.
D. Write $i_L(t)$ as the sum of a *zero-state* response and a *zero-input* response.
E. Find the energy $W_m(t)$ stored in the inductor for $t \geq 0$. Plot $W_m(t)$ for $t = 0$ to $t = 5\tau$, where τ is the time constant.

15. The circuit shown below contains one constant source and one source that turns *off* at $t = 0$.

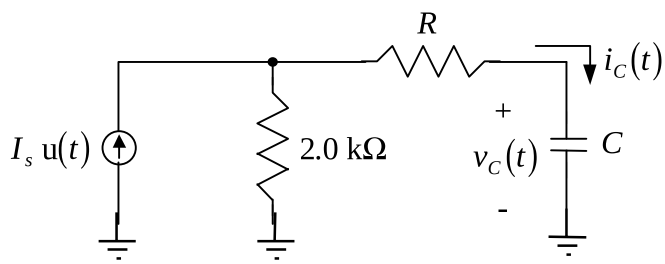


A. Find $v_C(0)$.
B. Find $v_C(t)$ for $t \geq 0$.
C. Write $v_C(t)$ as the sum of a *transient* response and a *steady-state* response.
D. Write $v_C(t)$ as the sum of a *zero-state* response and a *zero-input* response.
E. Find the energy $W_e(t)$ stored in the capacitor for $t \geq 0$. Plot $W_e(t)$ for $t = 0$ to $t = 5\tau$, where τ is the time constant.

16. In the circuit shown below,

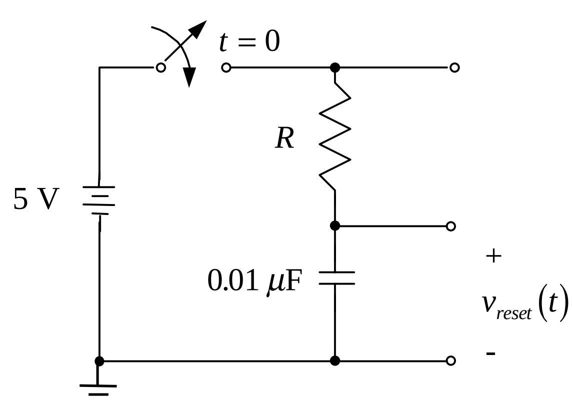
$$v_C(t) = 24 - 14e^{-\left(\frac{t}{7\times10^{-6}}\right)} \text{ V}, \ t \geq 0$$

$$i_C(t) = 2e^{-\left(\frac{t}{7\times10^{-6}}\right)} \text{ mA}, \ t > 0.$$



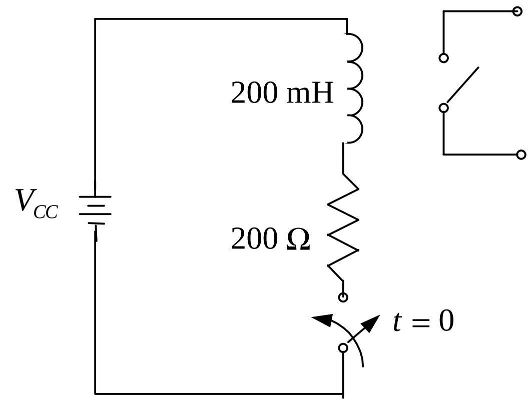
Find I_s, R, and C.

17. Power-On Reset.
In the circuit shown below the switch closes at $t = 0$, providing power to a microprocessor that is not shown. It is necessary that the microprocessors “reset” voltage be kept near zero for a short interval of time after the power is turned on, so that the microprocessor will begin operation in a known state. Assume that $v_{reset}(0) = 0$.



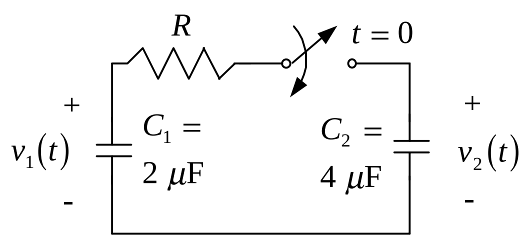
A. Find R so that $v_{reset}(10^{-3}\text{ s}) = 0.8\text{ V}$.
B. If it is required that $v_{reset}(t) \le 0.8\text{ V}$ for $t \le 1\text{ ms}$, is your answer to A a maximum value or a minimum value for R?

18. The circuit shown below is a model for a transistor-operated relay. The switch (representing the transistor) closes at $t = 0$. The relay closes when the inductor current reaches 50 mA.



Find V_{CC} so that the relay will close at $t = 2\text{ ms}$.

19. The Two-Capacitor Problem (revisited).
In the circuit shown below $v_1(0) = 3\text{ V}$ and $v_2(0) = 5\text{ V}$. The switch closes at $t = 0$.

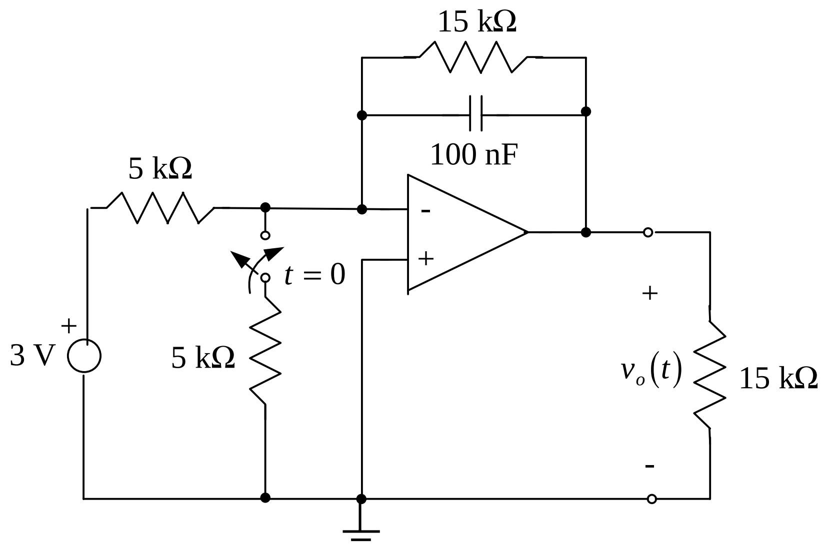


A. Find the charge $q_1(0)$ and $q_2(0)$ initially stored on (the top plate of) each capacitor. How much total charge is this?
B. Find the energy $W_{e1}(0)$ and $W_{e2}(0)$ initially stored in each capacitor. How much energy is this?
C. Find $v_1(t)$, $v_2(t)$, and $i(t)$ for $t \ge 0$. Your answers will depend on the value of R. Find the final values $v_1(\infty)$ and $v_2(\infty)$. Do these values depend on the value of R?
D. Find the charge $q_1(\infty)$ and $q_2(\infty)$. How much total charge is this? Compare with A; is charge conserved?
E. Find the energy $W_{e1}(\infty)$ and $W_{e2}(\infty)$. How much total energy is this?

F. Find the total energy converted to heat in the resistor. Does your answer depend on the value of R? Compare your answer to B with your answers to E and F; is energy conserved?

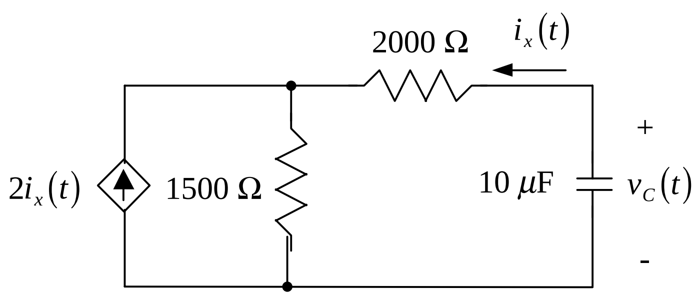
G. Plot $i(t)$ vs. time for $R = 1000\ \Omega$, $R = 10\ \Omega$, and $R = 0.1\ \Omega$. What is happening as $R \to 0$?

20. This problem continues problem 5 above. The circuit shown below uses an ideal op-amp. The switch has been open for a long time before $t = 0$, and at $t = 0$ it suddenly closes.



Find $v_o(t)$ for $t \geq 0$.

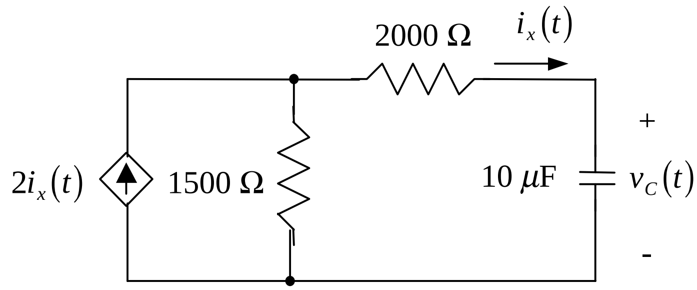
21. In the circuit shown below $v_C(0) = 5$ V.



A. Find $v_C(t)$ and $i_x(t)$ for $t > 0$.

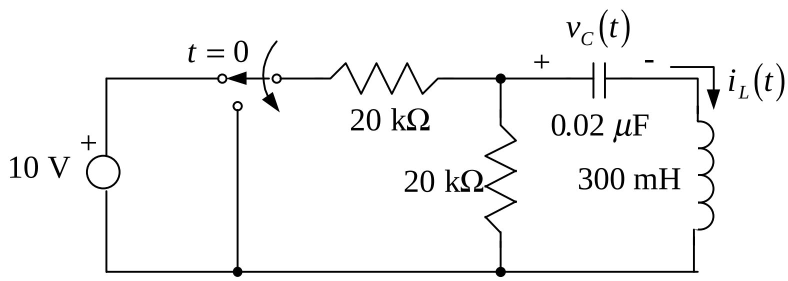
B. Plot $v_C(t)$ and $i_x(t)$ for $t > 0$. Show at least five time constants of elapsed time.

22. In the circuit shown below $v_C(0) = 5$ V.



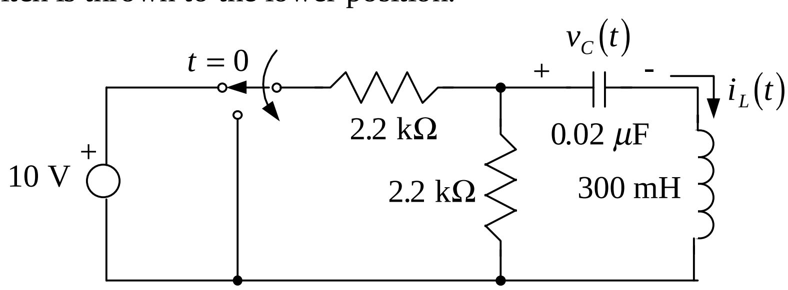
A. Find $v_C(t)$ and $i_x(t)$ for $t > 0$.
B. Plot $v_C(t)$ and $i_x(t)$ for $t > 0$. Show at least five time constants of elapsed time.

23. In the circuit shown below the switch has been in the position shown for a long time. At $t = 0$ the switch is thrown to the lower position.



A. Find $v_C(t)$, for $t \geq 0$. Plot your answer vs. time.
B. Is the circuit overdamped, critically damped, or underdamped?
C. Find the eigenvalues (natural frequencies) of the circuit. If the circuit is overdamped, find the two time constants. If the circuit is underdamped, find the damped ringing frequency and the damping coefficient.

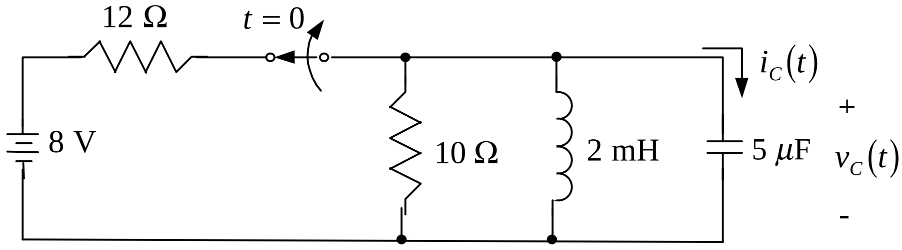
24. In the circuit shown below the switch has been in the position shown for a long time. At $t = 0$ the switch is thrown to the lower position.



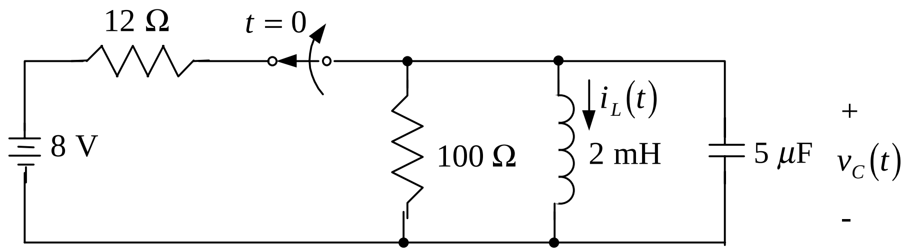
A. Find $i_L(t)$, for $t \geq 0$. Plot your answer vs. time.
B. Is the circuit overdamped, critically damped, or underdamped?
C. Find the eigenvalues (natural frequencies) of the circuit. If the circuit is overdamped, find the two time constants. If the circuit is underdamped, find the damped ringing frequency and the damping coefficient. Don't forget the units.

25. In the circuit shown below the switch has been in the position shown for a long time. At $t = 0$ the switch is suddenly opened.

    A. Find $v_C(t)$ and $i_C(t)$ for $t \geq 0$. Plot your answers vs. time.
    B. Is the circuit overdamped, critically damped, or underdamped?
    C. Find the eigenvalue(s) (natural frequencies). Don't forget units!

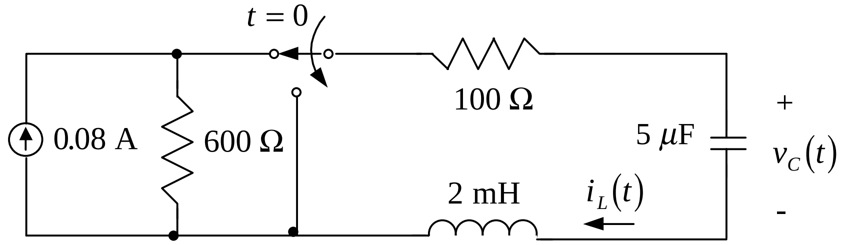


26. In the circuit shown below the switch has been in the position shown for a long time. At $t = 0$ the switch is suddenly opened.



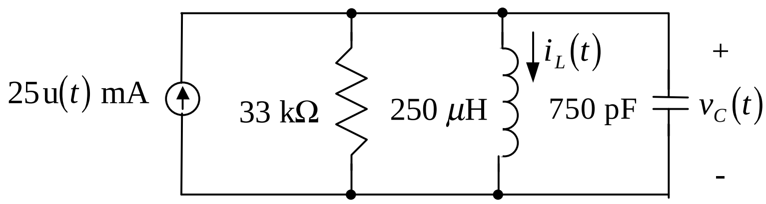
    A. Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$. Plot your answers vs. time.
    B. Is the circuit overdamped, critically damped, or underdamped?
    C. Find the eigenvalue(s) (natural frequencies). Don't forget units!
    D. Plot the energy stored in the capacitor and the energy stored in the inductor as functions of time for $t = 0$ to $t = 5\text{ ms}$.
    E. Find the total energy stored in the inductor and the capacitor together at $t = 0$.
    F. Find the total energy converted to heat in the $100\ \Omega$ resistor from $t = 0$ to $t \to \infty$.

27. In the circuit shown below the switch has been in the position shown for a long time. At $t = 0$ the switch is suddenly pushed to the lower position.



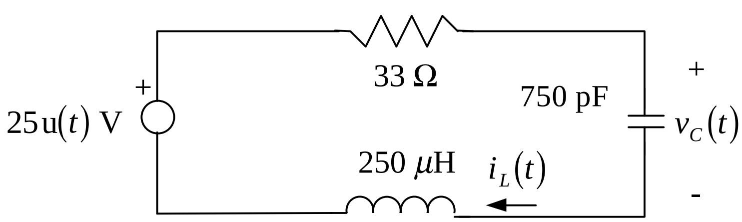
A. Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$. Plot your answers vs. time.
B. Is the circuit overdamped, critically damped, or underdamped?
C. Find the eigenvalue(s) (natural frequencies). Don't forget units!
D. Plot the energy stored in the capacitor and the energy stored in the inductor as functions of time for $t = 0$ to $t = 5\text{ ms}$.
E. Find the total energy stored in the inductor and the capacitor together at $t = 0$.
F. Find the total energy converted to heat in the $100\ \Omega$ resistor from $t = 0$ to $t \to \infty$.

28. In the circuit shown below,



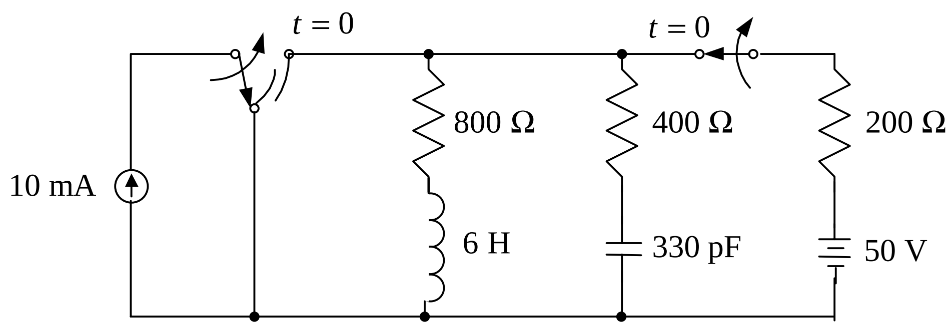
A. Find $i_L(t)$ for $t \geq 0$.
B. Find the eigenvalues (natural frequencies) of the circuit. Don't forget the units!
C. Find the damping coefficient and the damped ringing frequency.
D. Find the damping ratio and the undamped ringing frequency.
E. Find the transient part and the steady-state part of $i_L(t)$.

29. In the circuit shown below,



A. Find $v_C(t)$ for $t \geq 0$.
B. Find the eigenvalues (natural frequencies) of the circuit. Don't forget the units!
C. Find the damping coefficient and the damped ringing frequency.
D. Find the damping ratio and the undamped ringing frequency.
E. Find the transient part and the steady-state part of $v_C(t)$.

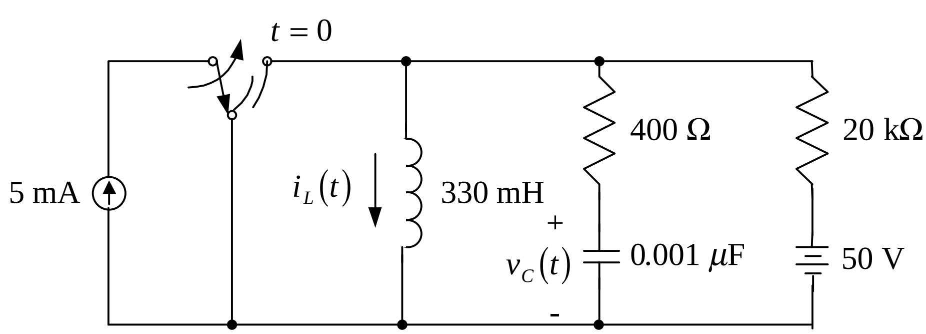
30. The switches in the circuit shown below have been in the positions shown for a long time, when they are simultaneously thrown at $t = 0$.



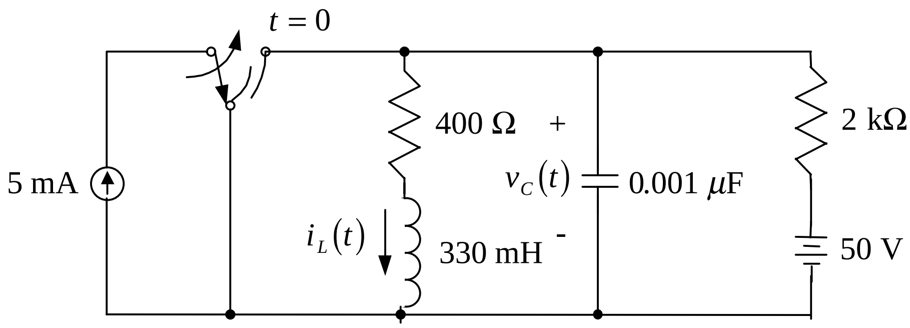
A. Find $i_L(t)$ for $t \geq 0$.
B. Is the circuit underdamped, critically damped, or overdamped?
C. Find the eigenvalues (natural frequencies) of the circuit. If the circuit is underdamped, find the damping coefficient and the damped ringing frequency. If the circuit is overdamped, find the two time constants.
D. Find the transient part and the steady-state part of $i_L(t)$.
E. Using a conventional approximation, how long does it take after $t = 0$ for steady-state to be reestablished?

31. In the circuit shown below the switch has been in the lower position for a long time, when it is suddenly moved to the upper position at $t = 0$.

A. Find $i_L(t)$ and $v_C(t)$ for $t \geq 0$.
B. Is the circuit overdamped, critically damped, or underdamped?
C. Find the eigenvalues (natural frequencies) of the circuit. If the circuit is overdamped, find the two time constants. If the circuit is underdamped, find the damping coefficient and the damped ringing frequency.
D. Write $i_L(t)$ as the sum of a transient part and a steady-state part. Indicate which is which.
E. Plot $i_L(t)$ and the energy $W_m(t)$ stored in the inductor as functions of time for $t \geq 0$.

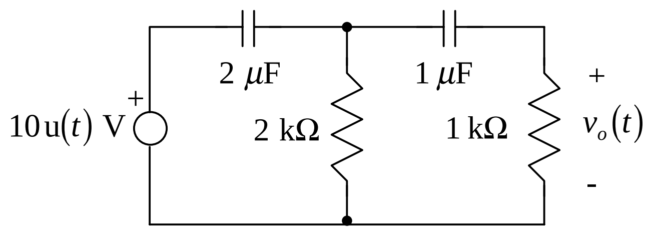


32. In the circuit shown below the switch has been in the lower position for a long time, when it is suddenly moved to the upper position at $t = 0$.



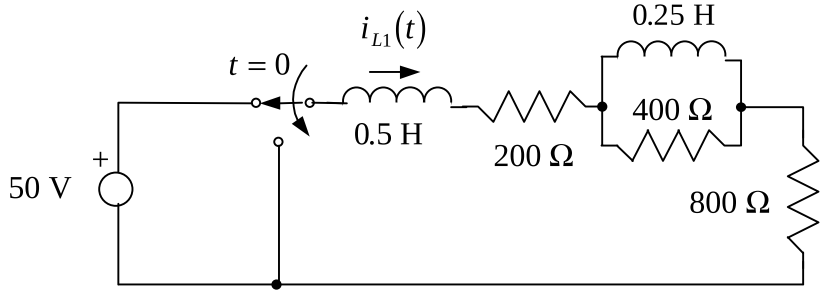
A. Find $i_L(t)$ and $v_C(t)$ for $t \geq 0$.
B. Is the circuit overdamped, critically damped, or underdamped?
C. Find the eigenvalues (natural frequencies) of the circuit. If the circuit is overdamped, find the two time constants. If the circuit is underdamped, find the damping coefficient and the damped ringing frequency.
D. Write $v_C(t)$ as the sum of a transient part and a steady-state part. Indicate which is which.
E. Plot $v_C(t)$ and the energy $W_e(t)$ stored in the capacitor as functions of time for $t \geq 0$.

33. In the circuit shown below,



A. Find $v_o(t)$ for $t \geq 0$.
B. Find each of the time constants.
C. Find the transient part and the steady-state part of $v_o(t)$.
D. Plot $v_o(t)$ and the two exponential components of the transient part together on a single set of axes.

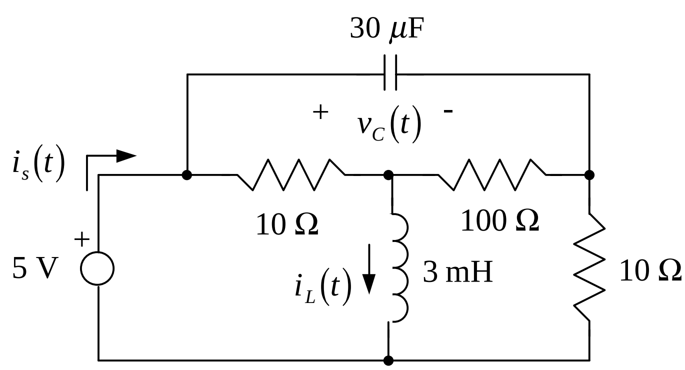
34. In the circuit shown below, the switch has been in the position shown for a long time. At $t = 0$ the switch is pushed down, disconnecting the voltage source.



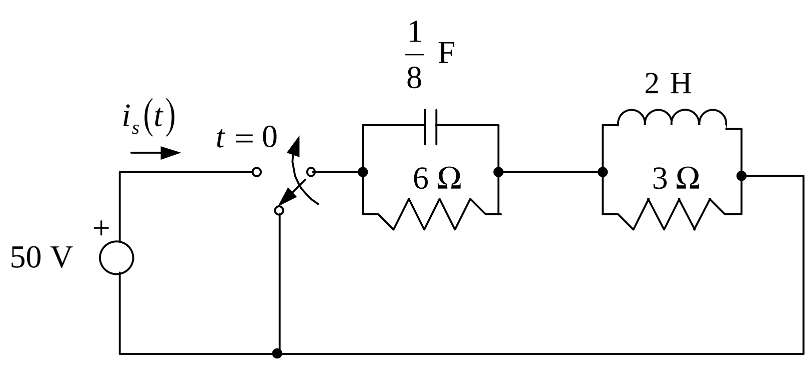
A. Find $i_{L1}(t)$ for $t \geq 0$.
B. Find each of the time constants.
C. Find the transient part and the steady-state part of $i_{L1}(t)$.
D. Plot $i_{L1}(t)$ and the two exponential components of the transient part together on a single set of axes.

35. The voltage source in the circuit shown below is constant. At $t = 0$ the inductor current is 1 mA and the capacitor voltage is zero.

A. Find the source current $i_s(t)$ for $t > 0$.
B. Is the circuit overdamped, underdamped, or critically damped?
C. If the circuit is overdamped, find the two time constants. If the circuit is underdamped, find the damping coefficient and the damped ringing frequency.
D. Plot $i_s(t)$ as a function of time.



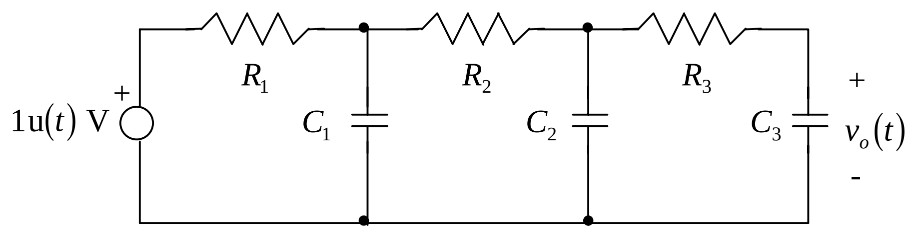
36. The switch in the circuit shown below has been in the position shown for a long time. At $t = 0$ the switch is suddenly moved to the upper position.



A. Find the source current $i_s(t)$ for $t > 0$.

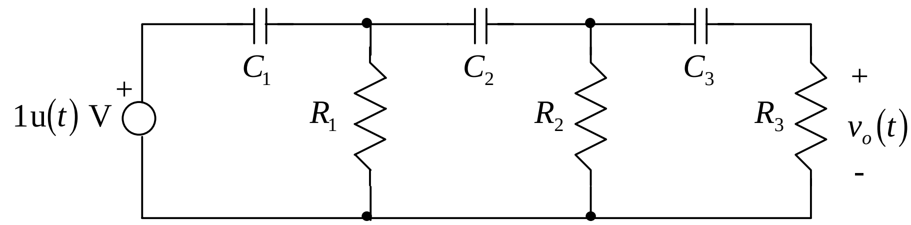
B. Plot $i_s(t)$ vs. time.
C. Is the circuit overdamped, critically damped, or underdamped?
D. Find the eigenvalues (natural frequencies) of the circuit.
E. Find the transient and steady-state components of $i_s(t)$.
F. Using a conventional approximation, how long does it take for steady-state to be re-established after the switch is thrown?

37. In the circuit shown below,



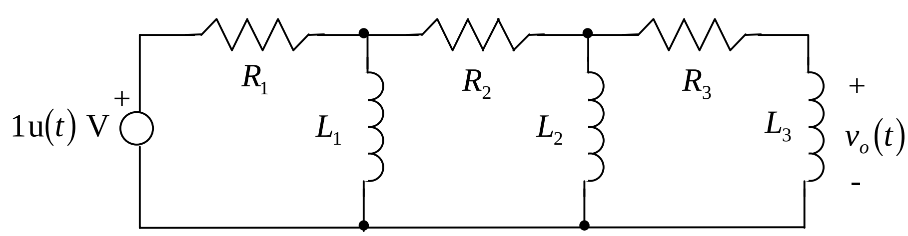
A. What is the order of the circuit? (How many energy storage elements does it have besides the voltage source?)
B. Identify and label the *state variables* for the circuit.
C. Suppose you wrote circuit equations for the circuit, and then eliminated the variables until you had a single equation in a single state variable. What order differential equation would you have?
D. Write a generic differential equation of the order you found in part C.
E. Write the characteristic equation of your differential equation.
F. How many eigenvalues (natural frequencies) does the circuit have?
G. A second-order circuit can have eigenvalues that are real and distinct, real and repeated, or that are complex conjugates of each other. List all of the possibilities for the circuit shown.
H. A second-order circuit can have a response that has the form $Ae^{s_1 t} + Be^{s_2 t}$, $Ae^{st} + Bte^{st}$, or $2Ke^{-\sigma t}\cos(\omega_d t + \theta)$, depending on the form of the eigenvalues. List all of the possible responses for the circuit shown.
I. Suppose $R_1 = R_2 = R_3 = 1000\ \Omega$ and $C_1 = C_2 = C_3 = 200\text{ pF}$. Let the inital capacitor voltages all be zero. Find and plot the voltage $v_o(t)$ as a function of time.
J. Find the numerical values of the eigenvalues of the circuit.
K. Find $v_o(0^+)$ and $v_o(\infty)$. How long does it take for the voltage to complete 99% of the change from $v_o(0^+)$ to $v_o(\infty)$?

38. In the circuit shown below,



A. What is the order of the circuit? (How many energy storage elements does it have besides the voltage source?)
B. Identify and label the *state variables* for the circuit.
C. Suppose you wrote circuit equations for the circuit, and then eliminated the variables until you had a single equation in a single state variable. What order differential equation would you have?
D. Write a generic differential equation of the order you found in part C.
E. Write the characteristic equation of your differential equation.
F. How many eigenvalues (natural frequencies) does the circuit have?
G. A second-order circuit can have eigenvalues that are real and distinct, real and repeated, or that are complex conjugates of each other. List all of the possibilities for the circuit shown.
H. A second-order circuit can have a response that has the form $Ae^{s_1 t} + Be^{s_2 t}$, $Ae^{st} + Bte^{st}$, or $2Ke^{-\sigma t}\cos(\omega_d t + \theta)$, depending on the form of the eigenvalues. List all of the possible responses for the circuit shown.
I. Suppose $R_1 = R_2 = R_3 = 1000\ \Omega$ and $C_1 = C_2 = C_3 = 200\ \text{pF}$. Let the inital capacitor voltages all be zero. Find and plot the voltage $v_o(t)$ as a function of time.
J. Find the numerical values of the eigenvalues of the circuit.
K. Find $v_o(0^+)$ and $v_o(\infty)$. How long does it take for the voltage to complete 99% of the change from $v_o(0^+)$ to $v_o(\infty)$?

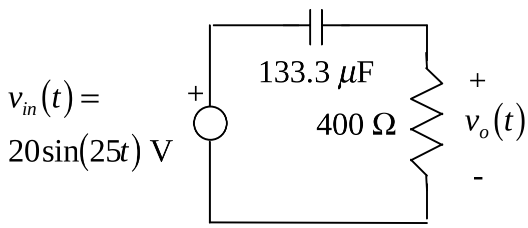
39. In the circuit shown below,



A. What is the order of the circuit? (How many energy storage elements does it have besides the voltage source?)
B. Identify and label the *state variables* for the circuit.

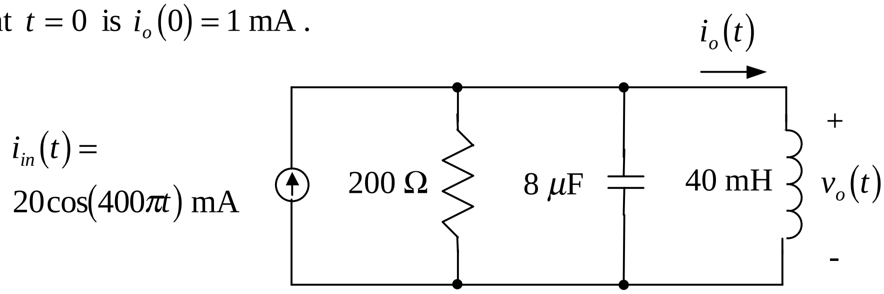
C. Suppose you wrote circuit equations for the circuit, and then eliminated the variables until you had a single equation in a single state variable. What order differential equation would you have?
D. Write a generic differential equation of the order you found in part C.
E. Write the characteristic equation of your differential equation.
F. How many eigenvalues (natural frequencies) does the circuit have?
G. A second-order circuit can have eigenvalues that are real and distinct, real and repeated, or that are complex conjugates of each other. List all of the possibilities for the circuit shown.
H. A second-order circuit can have a response that has the form $Ae^{s_1 t} + Be^{s_2 t}$, $Ae^{st} + Bte^{st}$, or $2Ke^{-\sigma t}\cos(\omega_d t + \theta)$, depending on the form of the eigenvalues. List all of the possible responses for the circuit shown.
I. Suppose $R_1 = R_2 = R_3 = 200\ \Omega$ and $L_1 = L_2 = L_3 = 50\text{ mH}$. Let the inital inductor currents all be zero. Find and plot the voltage $v_o(t)$ as a function of time.
J. Find the numerical values of the eigenvalues of the circuit.
K. Find $v_o(0^+)$ and $v_o(\infty)$. How long does it take for the voltage to complete 99% of the change from $v_o(0^+)$ to $v_o(\infty)$?

40. In the circuit shown below the capacitor voltage is zero at $t = 0$.



A. Write the circuit equations. Then combine them into a single differential equation containing one variable, $v_0(t)$.
B. Find $v_0(t)$ for $t \geq 0$. Plot $v_0(t)$ as a function of time for $t = 0$ to $t = 1\text{ s}$.
C. Identify the transient part and the sinusoidal steady-state part of your answer.
D. Write $v_0(t)$ as the sum of a zero-input response and a zero-state response.

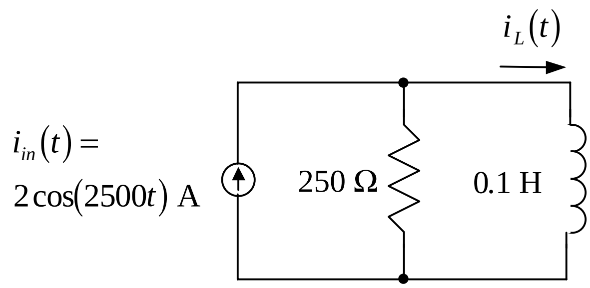
41. In the circuit shown, the capacitor voltage at $t = 0$ is $v_o(0) = -10\text{ V}$, and the inductor current at $t = 0$ is $i_o(0) = 1\text{ mA}$.



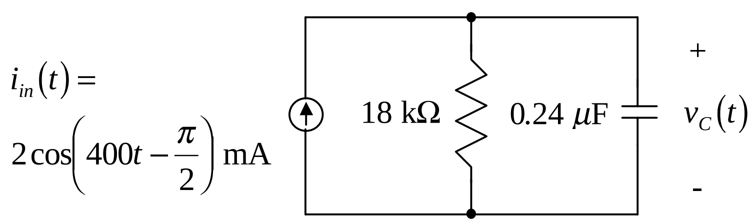
A. Using the Big Gun find $v_0(t)$ for $t \ge 0$.
B. Plot $v_0(t)$ vs. t for $t = 0$ to $t = 50$ ms.
C. Identify the transient part and the sinusoidal steady-state part of your answer.

42. In the circuit shown, the inductor current at $t = 0$ is $i_L(0) = 0$.

A. Find $i_L(t)$ for $t \ge 0$.
B. Plot $i_L(t)$ vs. t for $t = 0$ to $t = 2$ ms.
C. Identify the transient part and the sinusoidal steady-state part of your answer.

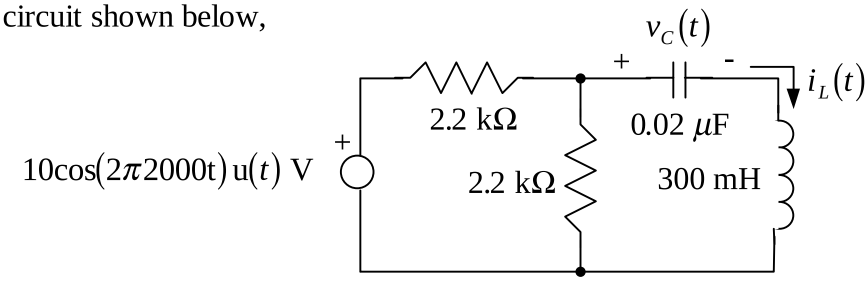


43. In the circuit shown below, the capacitor voltage at $t = 0$ is $v_C(0) = -10$ V.



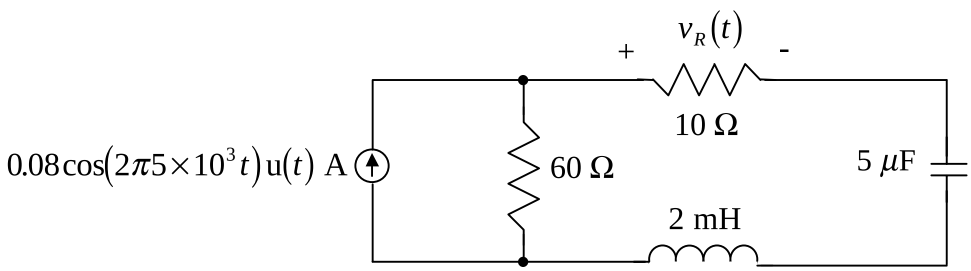
A. Find $v_C(t)$ for $t \ge 0$.
B. Plot $v_C(t)$ vs. t for $t = 0$ to $t = 100$ ms.
C. Identify the transient part and the sinusoidal steady-state part of your answer.

44. For the circuit shown below,



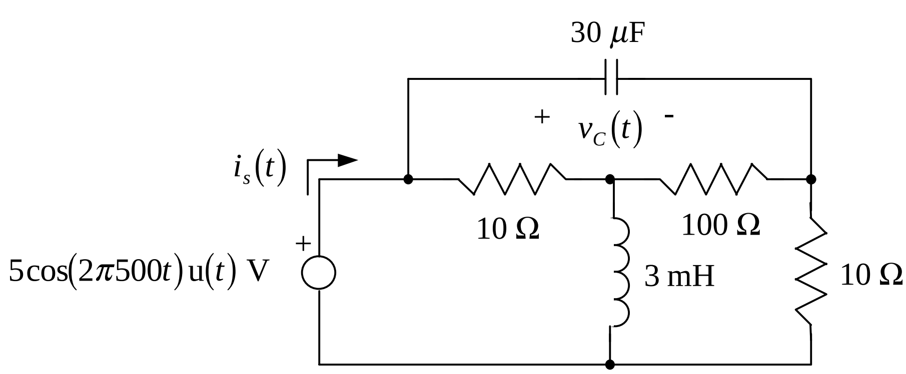
A. Find $i_L(t)$ for $t \geq 0$.
B. Identify the transient part and the sinusoidal steady-state part of $i_L(t)$.
C. Plot $i_L(t)$ vs. t. Be sure your plot shows both transient and steady-state behavior.
D. Plot on separate graphs the transient part and the sinusoidal steady-state part of $i_L(t)$ for $t \geq 0$.

45. For the circuit shown below,



A. Find $v_R(t)$ for $t \geq 0$.
B. Identify the transient part and the sinusoidal steady-state part of $v_R(t)$.
C. Plot $v_R(t)$ vs. t. Be sure your plot shows both transient and steady-state behavior.
D. Plot on separate graphs the transient part and the sinusoidal steady-state part of $v_R(t)$ for $t \geq 0$.

46. For the circuit shown below,



A. Find $v_C(t)$ and $i_s(t)$ for $t \geq 0$.

B. Identify the transient part and the sinusoidal steady-state part of $v_C(t)$. Repeat for $i_s(t)$.
C. Plot $v_C(t)$ and $i_s(t)$ vs. t on a single set of axes. Be sure your plot shows both transient and steady-state behavior.
D. Plot on separate graphs the transient part and the sinusoidal steady-state part of $v_C(t)$ for $t \geq 0$.