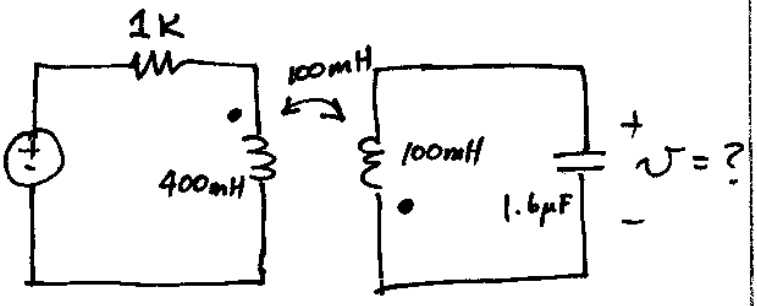
EC200

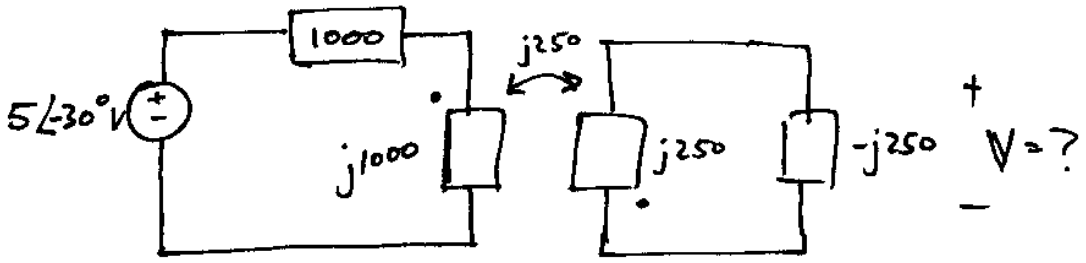
Example 1

$5\cos(2500t - 30°)$ V

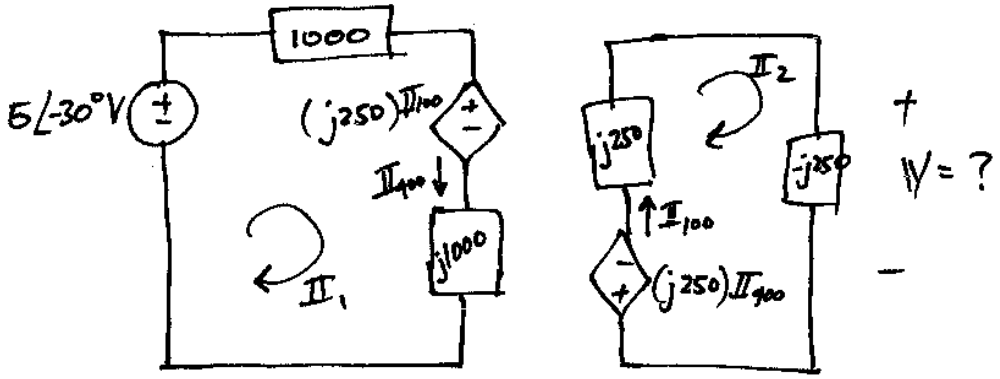


* Convert to phasor circuit:

400 mH @ $\omega = 2{,}500$ rad/s → $j1000\,\Omega$
100 mH " → $j250\,\Omega$
1.6 μF " → $-j250\,\Omega$



* Insert circuit model for coupled coils:



* Use mesh analysis:

Mesh 1: $-5\angle -30° + 1000\,\mathbb{I}_1 + j250\,\mathbb{I}_{100} + j1000\,\mathbb{I}_1 = 0$

Mesh 2: $j250\,\mathbb{I}_{400} + j250\,\mathbb{I}_2 - j250\,\mathbb{I}_2 = 0$

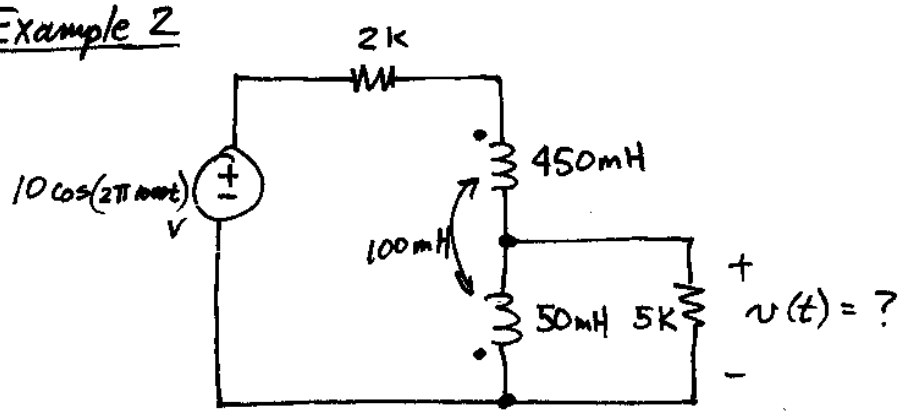
Control currents: $\mathbb{I}_{400} = \mathbb{I}_1$ & $\mathbb{I}_{100} = \mathbb{I}_2$

* Solve system:

$\mathbb{I}_1 = 0$ & $\mathbb{I}_2 = 20\angle -120°$ mA

* Solve for V: $\mathbb{V} = -j250\,\mathbb{I}_2 = 5\angle -210°$ V ⇒ $v(t) = 5\cos(2500t - 210°)$

Example 2

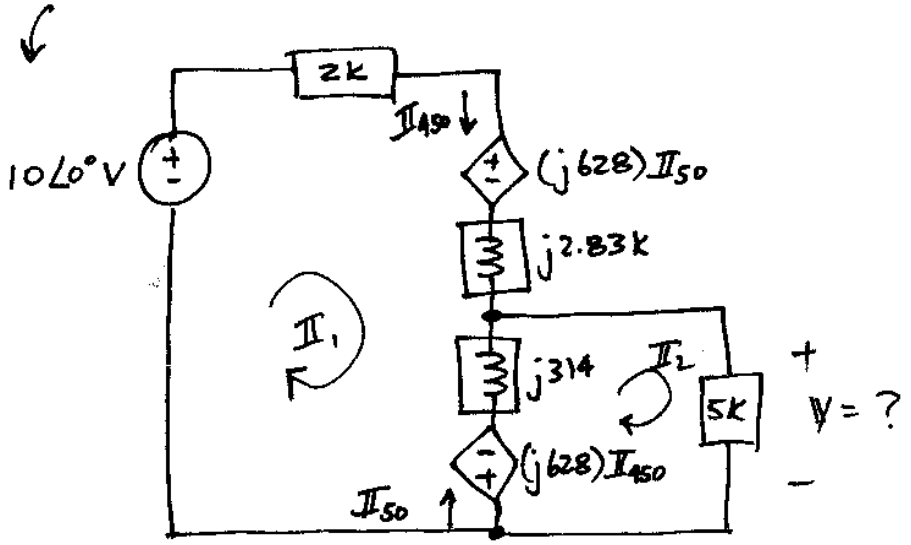


* Convert to phasor circuit:

450mH @ $\omega = 2\pi/1000$ rad/s → $j2.83\,k$

100mH @ " → $j628\,\Omega$

50mH @ " → $j314\,\Omega$



* Use mesh analysis (usually the better choice, since the dependent sources introduce more nodes)

Mesh 1: $-10 + 2000I_1 + j628I_{50} + (j2.83k)I_1 + j314(I_1 - I_2) - (j628)I_{450} = 0$

Mesh 2: $j628(I_2 - I_1) + j314(I_2 - I_1) + 5000I_2 = 0$

Control currents: $I_{450} = I_1$ & $I_{50} = I_2 - I_1$

* Solve system for I_1 & I_2, then $V = 5000I_2$. Convert V to $v(t)$. (see next page)

Example 2, cont'd:

Equations from mesh current analysis, ~~including the initial conditions~~:

```
> restart:
  Digits := 4:
  alias(I=I, j=sqrt(-1)):
  eqs := {
  -10 + 2000*I1 + j*628*I50 + j*2.83E3*I1 + j*314*(I1-I2) -
  j*628*I450,
  j*628*(I2-I1) + j*314*(I2-I1) + 5000*I2,
  I450 = I1,
  I50 = I2 - I1
  };
```

$$eqs := \{I450 = I1,\ -10 + 2000\,I1 + 628\,j\,I50 + 2830.\,j\,I1 + 314\,j\,(I1 - I2) - 628\,j\,I450,\ I50 = I2 - I1,\ 942\,j\,(I2 - I1) + 5000\,I2\}$$

Solve the system of equations:

```
> soln := solve(eqs);
```

$$soln := \{I50 = -.002074 + .002964\,j,\ I450 = .002633 - .002573\,j,\ I2 = .0005583 + .0003908\,j,\ I1 = .002633 - .002573\,j\}$$

```
> assign(soln);
```

Display the voltage of interest:

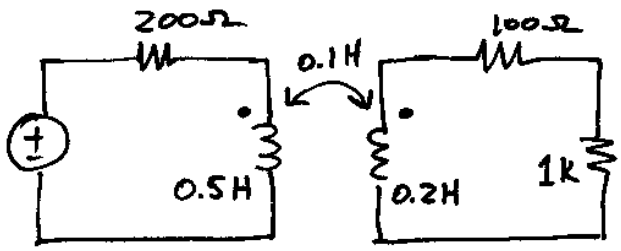
```
> V := 5000*I2;
```

$$V := 2.792 + 1.954\,j$$

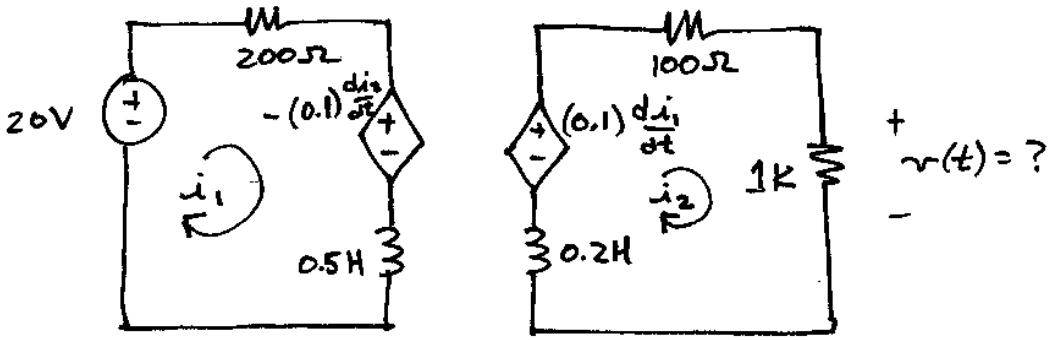
$$= 3.40 \angle 35^\circ \text{ V}$$

$$\Rightarrow \boxed{v(t) = 3.40 \cos(2\pi \cdot 1000\,t + 35^\circ)\ \text{V}}$$

Example 3



* Circuit for $t \geq 0$ (initial conditions are all zero):



* Mesh analysis:

Mesh 1: $-20 + 200 i_1 - 0.1 \frac{di_2}{dt} + 0.5 \frac{di_1}{dt} = 0$

Mesh 2: $0.2 \frac{di_2}{dt} - 0.1 \frac{di_1}{dt} + 100 i_2 + 1000 i_2 = 0$

Initial conditions: $i_1(0) = 0$ & $i_2(0) = 0$

* Solve the system for $i_1(t)$ & $i_2(t)$, & find $v(t) = 1000 i_2(t)$:

$$v(t) = 3.90 e^{-397t} - 3.90 e^{-6159t} \text{ V}$$

(See next page for Maple worksheet)

Worksheet for Example 3

Equations from mesh current analysis, including the initial conditions:

```
> restart:
  Digits := 4:
  eqs := {
  -20 + 200*i1(t) - 0.1*diff(i2(t),t) + 0.5*diff(i1(t),t),
  0.2*diff(i2(t),t) - 0.1*diff(i1(t),t) + 1100*i2(t),
  i1(0) = 0,
  i2(0) = 0
  };
```

$$eqs := \{-20 + 200\,i1(t) - .1\left(\frac{\partial}{\partial t} i2(t)\right) + .5\left(\frac{\partial}{\partial t} i1(t)\right),\ .2\left(\frac{\partial}{\partial t} i2(t)\right) - .1\left(\frac{\partial}{\partial t} i1(t)\right) + 1100\,i2(t),$$
$$i1(0) = 0,\ i2(0) = 0\}$$

Mesh current names:

```
> currents := {i1(t),i2(t)};
```

$$currents := \{i1(t),\ i2(t)\}$$

Solve the system of differential equations:

```
> soln := dsolve(eqs,currents,method=laplace);
```

$$soln := \{i1(t) = .1000 - .1000\,e^{(-3278.\,t)}\cosh(2881.\,t) - .09838\,e^{(-3278.\,t)}\sinh(2881.\,t),$$
$$i2(t) = .007717\,e^{(-3278.\,t)}\sinh(2881.\,t)\}$$

```
> assign(soln);
```

Display the voltage of interest:

```
> v(t) := 1000*i2(t);
```

$$v(t) := 7.717\,e^{(-3278.\,t)}\sinh(2881.\,t)$$

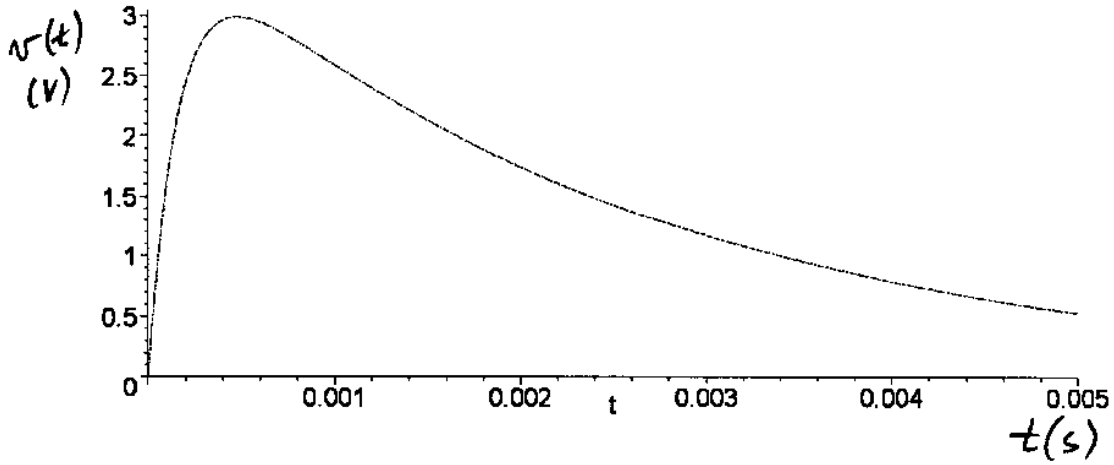
Show in a simpler form (eliminate the hyperbolic terms):

```
> v(t) := simplify(expand(convert(v(t),exp)));
```

$$v(t) := 3.859\,e^{(-397.\,t)} - 3.859\,e^{(-6159.\,t)}$$

Plot the voltage vs. time:

```
> plot(v(t),t=0..5E-3);
```




```
>
```