ROSE-HULMAN INSTITUTE OF TECHNOLOGY
Department of Electrical and Computer Engineering

EC 300 Signals and Systems Fall 2000
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Measurement of Fourier Coefficients

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Objectives

- To become acquainted with the Tektronix 7L5 Spectrum Analyzer.
- To measure the Fourier coefficients of several waveforms and compare the measured values with theoretical values.

Equipment

Tektronix 7L5 Spectrum Analyzer
HP8116A Function Generator
Tektronix 2215A Oscilloscope
BNC T-Connector

Background

Long ago we learned to calculate the spectrum of a periodic signal by using the Fourier series. We have in our lab spectrum analyzers that can display the spectrum of a signal in real time. The Tektronix 7L5 Spectrum Analyzer can be used to view the power spectrum of any signal having a bandwidth of less than 5 MHz. The 7L5 displays a one-sided spectrum in decibels versus frequency. In lab we will observe the spectra of sinusoids, square and triangle waves, and pulse trains, but first we must learn how to convert the Fourier series coefficients that we calculate to the dB values displayed by the spectrum analyzer.

Recall that any periodic signal $x(t)$ can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_0 t}, \text{ where } f_0 = 1/T_0 .$$

Writing out a few terms gives

$$\begin{aligned} x(t) &= \cdots + c_{-2} e^{-j2\pi 2 f_0 t} + c_{-1} e^{-j2\pi f_0 t} + c_0 + c_1 e^{j2\pi f_0 t} + c_2 e^{j2\pi 2 f_0 t} + \cdots \\ &= \cdots + |c_2| e^{-j\measuredangle c_2} e^{-j2\pi 2 f_0 t} + |c_1| e^{-j\measuredangle c_1} e^{-j2\pi f_0 t} + c_0 + |c_1| e^{j\measuredangle c_1} e^{j2\pi f_0 t} + |c_2| e^{j\measuredangle c_2} e^{j2\pi 2 f_0 t} + \cdots \end{aligned}$$

where we have used the fact that $c_{-k} = c_k^*$ whenever $x(t)$ is real-valued. Notice that, aside from c_0, the terms come in pairs. We can combine each positive-frequency term with its matching negative frequency term to obtain

$$x(t) = c_0 + 2|c_1| \cos(2\pi f_0 t + \measuredangle c_1) + 2|c_2| \cos(2\pi 2 f_0 t + \measuredangle c_2) + \cdots.$$

A power spectrum for $x(t)$ based on the Fourier series is shown in Fig. 1. This is a *two-sided* spectrum, in which the power associated with the complex exponential at frequency kf_0 is seen to be $|c_k|^2$. The corresponding *one-sided* power spectrum is shown in Fig. 2. To make the one-sided spectrum, the powers associated with complex exponentials at frequencies kf_0 and $-kf_0$ are added. The result, representing the average power in the sinusoid $2|c_k|\cos(2\pi f_k t + \measuredangle c_k)$, is shown at frequency kf_0.

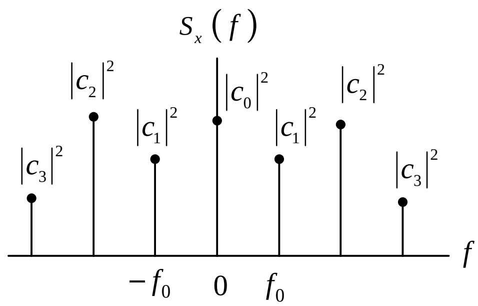

Figure 1: Two-Sided Power Spectrum

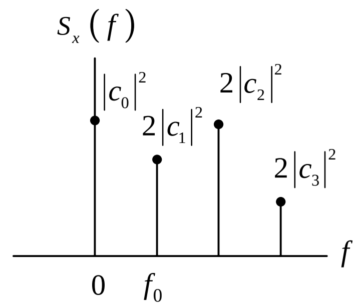

Figure 2: One-Sided Power Spectrum

The Tektronix 7L5 Spectrum Analyzer displays a one-sided spectrum as shown in Fig. 2, but instead of showing the value of $2|c_k|^2$ at each frequency, the spectrum analyzer shows average power in decibels with respect to a one volt reference. For the sinusoid at frequency kf_0, the average power in decibels is given by

$$P_k\big|_{dB} = 10\log_{10}\frac{P_k}{P_{ref}},$$

where the power P_k represents the power spectrum coefficient $2|c_k|^2$, and the power P_{ref} is the average power delivered to a one-ohm resistor by a one-volt RMS sinusoid. We have

$$P_k\big|_{dB} = 10\log\frac{2|c_k|^2}{1} \text{ dBV}.$$

The units "dBV" indicate that the reference for the decibels is a one-volt sinusoid.

Note: The Tektronix spectrum analyzer will not display the DC term $|c_0|^2$ even when one is present. Also, because it is showing a power spectrum, the spectrum analyzer does not measure or display the phase angles $\measuredangle c_k$.

Pre-Lab

Make the following two calculations for each of the three signals given below:

1. Calculate the Fouier series coefficients c_k, $k = 0, \pm 1, \pm 2, \ldots, \pm 9$.
2. Calculate the decibel values that you expect will be displayed by the spectrum analyzer.

The signals are:

a) $x_1(t) = 0.1\cos(2\pi 10\times 10^3 t)$ V

b) $x_2(t)$ is a square wave of period 100 μs and peak-to-peak amplitude 0.2 V.

c) $x_3(t)$ is a triangle wave of period 100 μs and peak-to-peak amplitude 0.2 V.

For each signal create a table in your lab notebook containing a column of values of c_k and a column of values of predicted decibels. Leave two additional blank columns, one for measured decibels and one for dB difference.

Read the first two sections of *The Spectrum Analyzer: Operating Principles and Instructions*. Pay particular attention to the "Cautions" on the first page.

Procedure

Getting Ready

Before connecting any input to the spectrum analyzer, be sure that the *TerminZ* switch is set to provide an input impedance of 1 MΩ. The spectrum analyzer is a little more robust when its input impedance is high, and it will not be subject to damage from *small* DC voltages.

Follow the calibration steps in Section 2 of *The Spectrum Analyzer: Operating Principles and Instructions*. Be sure to calibrate the spectrum analyzer every time you turn it on.

Measuring the Spectrum

1. Use the HP8116A Function Generator to generate a sinusoid of frequency 10 kHz and (open circuit) amplitude 0.1 V. Use the oscilloscope to verify the amplitude. Now observe the sinusoid on the spectrum analyzer. Measure the amplitude and frequency. Compare with your pre-lab calculation.

   Informally, vary the frequency and the amplitude of the sinusoid and observe how the spectrum analyzer display changes.

2. Use the function generator to generate a square wave of period 100 μs and peak-to-peak amplitude 0.2 V. Using the spectrum analyzer, measure the level of the first nine harmonics. Compare with the values you calculated in the pre-lab.

3. Use the function generator to generate a triangle wave of period 100 μs and peak-to-peak amplitude 0.2 V. Use the spectrum analyzer to measure the level of the first nine harmonics. Compare with the values you calculated in the pre-lab.

4. Now use the function generator to generate a pulse train having a period of 100 μs, a peak amplitude of 0.5 V and a duty cycle of 10%. (The "duty cycle" is the pulse width divided by the period.) Measure the spectrum using the spectrum analyzer. Compare the spectrum with the spectrum of the square wave. Take data that will allow you to *quantitatively* answer the following two questions:

   (a) What happens to the spectrum when the pulse width is decreased with the pulse period held constant?

   (b) What happens when the pulse width is held constant and the pulse period is increased?

   Pay particular attention to the frequency spacing between spectral lines, and to the lowest frequency at which the envelope of the spectral lines is zero.

Report

Present your theoretical results, measured results, comparisons, comments, and answers to the questions posed above. Be sure that all members of your lab group sign the lab notebook, and hand the notebook in at the end of lab.