Fourier Transform Pairs

Time Function	**Fourier Transform**
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}\left(fT\right)$
$\mathrm{sinc}\left(2Bt\right)$	$\frac{1}{2B}\mathrm{rect}\left(\frac{f}{2B}\right)$
$\exp\left(-at\right)\mathrm{u}\left(t\right),\ a>0$	$\frac{1}{a+j2\pi f}$
$\exp\left(-a\lvert t\rvert\right),\ a>0$	$\frac{2a}{a^2+\left(2\pi f\right)^2}$
$\exp\left(-\pi t^2\right)$	$\exp\left(-\pi f^2\right)$
$\Lambda\left(\frac{t}{T}\right)=\begin{cases}1-\frac{\lvert t\rvert}{T}, & \lvert t\rvert<T\\ 0, & \lvert \mathrm{t}\rvert\geq T\end{cases}$	$T\,\mathrm{sinc}^2\left(fT\right)$
$B\,\mathrm{sinc}^2\left(fB\right)$	$\Lambda\left(\frac{f}{B}\right)=\begin{cases}1-\frac{\lvert f\rvert}{B}, & \lvert f\rvert<B\\ 0, & \lvert f\rvert\geq B\end{cases}$
$\delta\left(t\right)$	1
1	$\delta\left(f\right)$
$\delta\left(t-t_0\right)$	$\exp\left(-j2\pi ft_0\right)$
$\exp\left(j2\pi f_c t\right)$	$\delta\left(f-f_c\right)$
$\cos\left(2\pi f_c t\right)$	$\frac{1}{2}\left[\delta\left(f-f_c\right)+\delta\left(f+f_c\right)\right]$
$\sin\left(2\pi f_c t\right)$	$\frac{1}{2j}\left[\delta\left(f-f_c\right)-\delta\left(f+f_c\right)\right]$
$\mathrm{sgn}\left(t\right)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}\left(f\right)$
$\mathrm{u}\left(t\right)$	$\frac{1}{2}\delta\left(f\right)+\frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty}\delta\left(t-iT_0\right)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_0}\right)$

Fourier Transform Properties

1. **Linearity**
   If $x(t) \leftrightarrow X(f)$ and $y(t) \leftrightarrow Y(f)$,
   then $ax(t) + by(t) \leftrightarrow aX(f) + bY(f)$.

2. **Hermitian Symmetry**
   If $g(t)$ is *real-valued*,
   then $G(-f) = G^*(f)$.

   This means that $|G(f)|$ is an *even* function of *f*, and $\measuredangle G(f)$ is an *odd* function of *f*.

   Also, if $g(t)$ is *real and even*, then $G(f)$ is also *real and even*.

3. **Time Shifting**
   If $g(t) \leftrightarrow G(f)$,
   then $g(t - t_0) \leftrightarrow G(f)e^{-j2\pi f t_0}$.

4. **Frequency Shifting**
   If $g(t) \leftrightarrow G(f)$,
   then $g(t)e^{j2\pi f_0 t} \leftrightarrow G(f - f_0)$.

5. **Differentiation**
   If $g(t) \leftrightarrow G(f)$ and $x(t) = \dfrac{dg}{dt}$,
   then $X(f) = (j2\pi f)G(f)$.

6. **Integration**
   If $g(t) \leftrightarrow G(f)$ and
   $y(t) = \int_{-\infty}^{t} g(\alpha)d\alpha$,
   then $Y(f) = \dfrac{G(f)}{j2\pi f} + \tfrac{1}{2}G(0)\delta(f)$.

7. **Scaling**
   If $g(t) \leftrightarrow G(f)$,
   then $g(at) \leftrightarrow \dfrac{1}{|a|}G\left(\dfrac{f}{a}\right)$.

8. **Duality**
   If $h(f) = \mathcal{F}\left[g(t)\right]$,
   then $g(-f) = \mathcal{F}\left[h(t)\right]$.

9. **Convolution**
   If $x(t) \leftrightarrow X(f)$ and $h(t) \leftrightarrow H(f)$,
   and if $Y(f) = X(f)H(f)$,
   then $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$.

10. **Convolution**
    If $x(t) \leftrightarrow X(f)$ and $z(t) \leftrightarrow Z(f)$,
    and if $y(t) = x(t)z(t)$,
    then $Y(f) = \int_{-\infty}^{\infty} X(\nu)Z(f - \nu)d\nu$.