CSSE 230 Day 16

Tree Properties (size *vs* height)
Balanced Binary Trees

CSSE 230 Day 16 Announcements

- Due now:
  - WA 7 (This is the end of the grace period).
  - Note that a grace period takes the place of a late day.
  - Now is the last time you can turn this in for credit.
- Due at the beginning of Day 17:
  - OO Queens exercise from Day 15 class
- Due at the beginning of Day 18:
  - WA8
  - Sliding Blocks Milestone 1
- I eliminated the NameThatSort proramming problem, to leave you more time to work on SlidingBlocks and Scrabble.

Agenda

- Touch Base with your SlidingBlocks partner.
- Height *vs* size of binary trees
- The need for balanced trees
- Keeping trees completely balanced
- Height-Balanced (H-B) trees

What are your questions? About…

- WA8
- Exhaustive search & Backtracking
- Sliding Blocks Problem
- Anything else

Solution to PreIn Exam Problem

```java
public static BinaryNode buildFromPreOrderInorder(String pre,
                                                  String in)  {
   if (pre.length() == in.length())
      return preIn(pre, in);
   throw new IllegalArgumentException();
}

private static BinaryNode preIn(String pre, String in) {
   if (pre.equals(""))
      return null;
   char rootChar = pre.charAt(0);
   int rootPos = in.indexOf(rootChar);
   return new BinaryNode(rootChar,
     preIn(pre.substring(1, rootPos+1), in.substring(0, rootPos)),
     preIn(pre.substring(rootPos+1), in.substring(rootPos+1)));
}
```

Properties of Binary Trees

Size vs Height

Binary Tree: Recursive definition

- A Binary Tree is either
  - empty,   or
  - consists of:
    - a distinguished node called the root, which contains an element and two disjoint subtrees
    - A left subtree T_L, which is a binary tree
    - A right subtree T_R, which is a binary tree

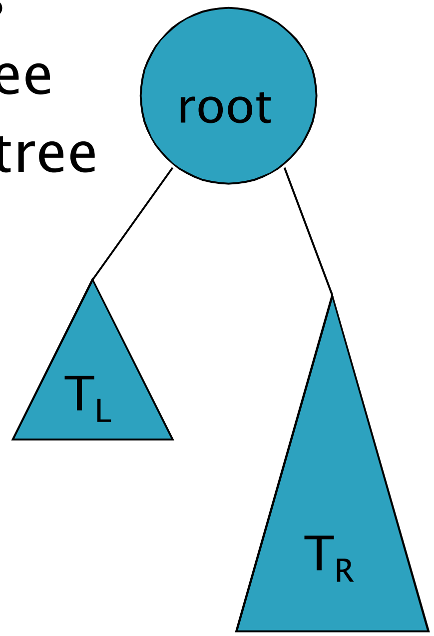
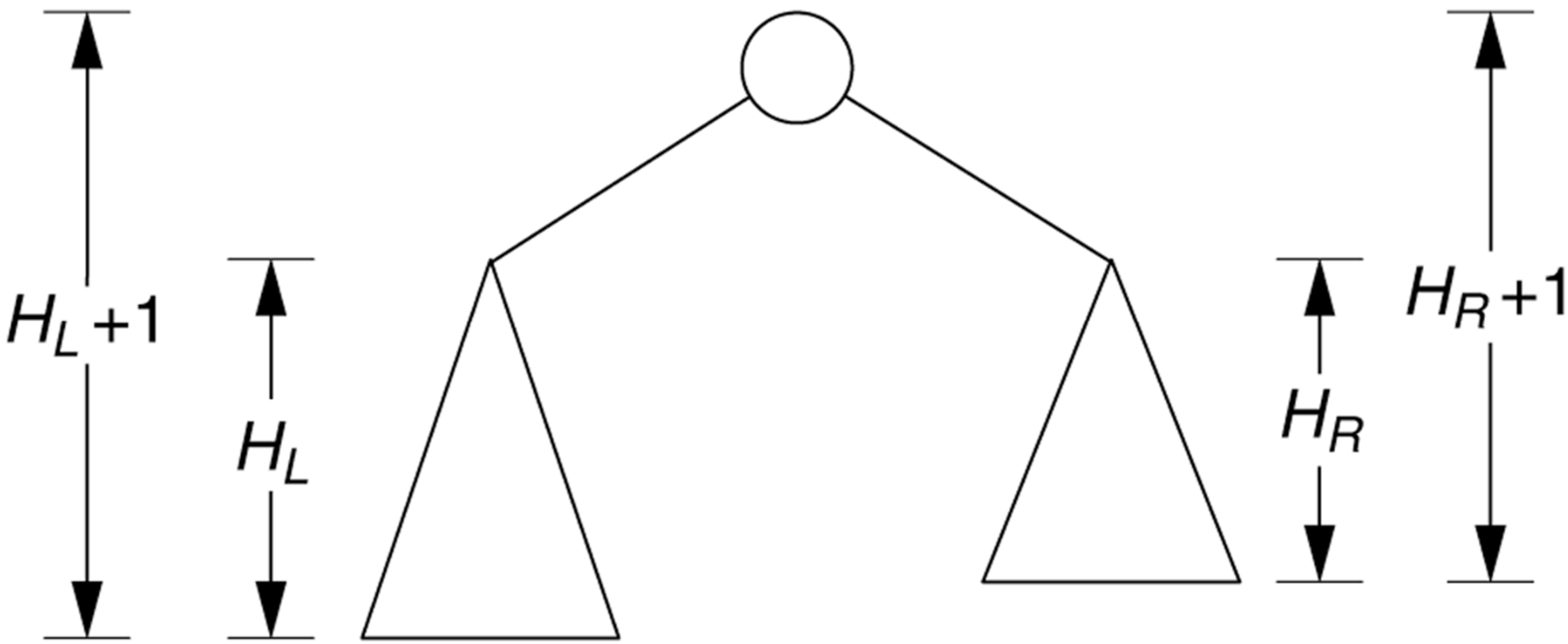


Figure 18.20

Recursive view of the node height calculation:

$H_T = \text{Max}\ (H_L + 1, H_R + 1)$



Size and Height of Binary Trees

- If T is a tree, we'll often write h(T) for the height of the tree, and N(T) for the number of nodes in the tree
- For a particular h(T), what are the upper and lower bounds on N(T)?
  - Lower: N(T) ≥ ________ (prove it by induction)
  - Upper N(T) ≤ ________ (prove it by induction)
  - Thus ________ ≤ N(T) ≤ __________
- Write bounds for h(T) in terms of N(T)
  - Thus ________ ≤ h(T) ≤ __________

Extreme Trees

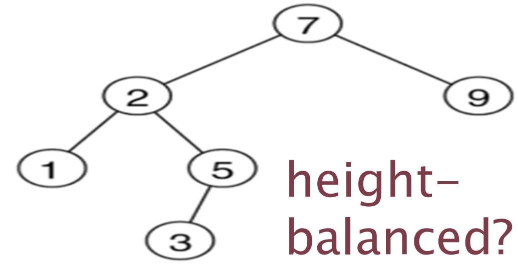
- A tree with the maximum number of nodes for its height is a full tree.
  - Its height is O(log N)
- A tree with the minimum number of nodes for its height is essentially a ______ ______
  - Its height is O(N)
- Height matters!
  - We saw that the algorithms for search, insertion, and deletion in a Binary Search Tree are O(h(T))

Introduction to Balanced Trees

BST algorithms and their efficiency

- Review:
  - Efficiency of insertion, deletion, find for
    - Array list
    - Linked list
- BST insertion algorithm – O(height of tree)
- BST deletion algorithm – O(height of tree)
- BST search algorithm – – O(height of tree)
- Efficiency (worst case)?
  - Can we get a better bound?
  - What about balancing the tree each time?
  - What do we mean by completely balanced?
  - Insert E C G B D F A into a tree in that order.
  - What is the problem?
  - How might we do better? (less is more!)

What have we discovered about BSTs so far?



height-balanced?

- We'd like the worst-case time for find, insert, and delete to be O(log N).
- The running time for find, insert, and delete are all proportional to the height of the tree.
- Height of the tree can vary from log N to N.
- Keeping the tree completely balanced is too expensive. Can require O(N) time to rebalance after insertion or deletion.
- Height-balanced trees may provide a solution.
  - A BST T is *height balanced* if T is empty, or if
    - $|\ \text{height}(T_L) - \text{height}(T_R)\ | \le 1$, and
    - T_L and T_R are both height-balanced.
- What can we say about the maximum height of an height-balanced tree with N nodes?

Details: next slide

Why do we use absolute value in the formula?

Height-Balanced trees

- We want to calculate the maximum height of a height-balanced tree with N nodes.
- It's not the shortest possible tree, but how close is it?
- We first look at the dual concept: find the minimum number of nodes in a HB tree of height h.
- Make a table of heights and # of nodes.
- What can we say in general about height as a function of number of nodes?

What is an AVL tree?

- Named for authors of original paper, Adelson-Velskii and Landis (1962).
- It is a height-balanced Binary Search Tree.
- Recall: A BST T is *height balanced* if
  - T is empty, or if
    - $|\ \text{height}(T_L) - \text{height}(T_R)\ | \leq 1$, and
    - T_L and T_R are both height-balanced.
- Maximum height of an AVL tree with N nodes is O(log N).

Recap: Why we study AVL trees

- For a Binary Search Tree (BST), worst-case for *insert*, *delete*, and *find* operations are all O(height of tree).
- Height of tree can vary from O(log N) to O(N).
- We showed that the height of a height-balanced tree is always O(log N).
- Thus all three operations will be O(log N) if we can rebalance after insert or delete in time O(log N)

More on AVL trees

- An AVL tree is
  1. height-balanced
  2. a Binary search tree
- We saw that the maximum height of an AVL tree with N nodes is O(log n).
- We want to show that after an insertion or deletion (also O(log n) since the height is O(log n)), we can rebalance the tree in O(log n) time.
  - If that is true, then find, insert, and remove, will all be O(Log N).
- An extra field is needed in each node in order to achieve this speed. Values: / = \
  We call this field the *balance code.*
- The balance code could be represented by only two bits.

Balancing an AVL tree after insertion

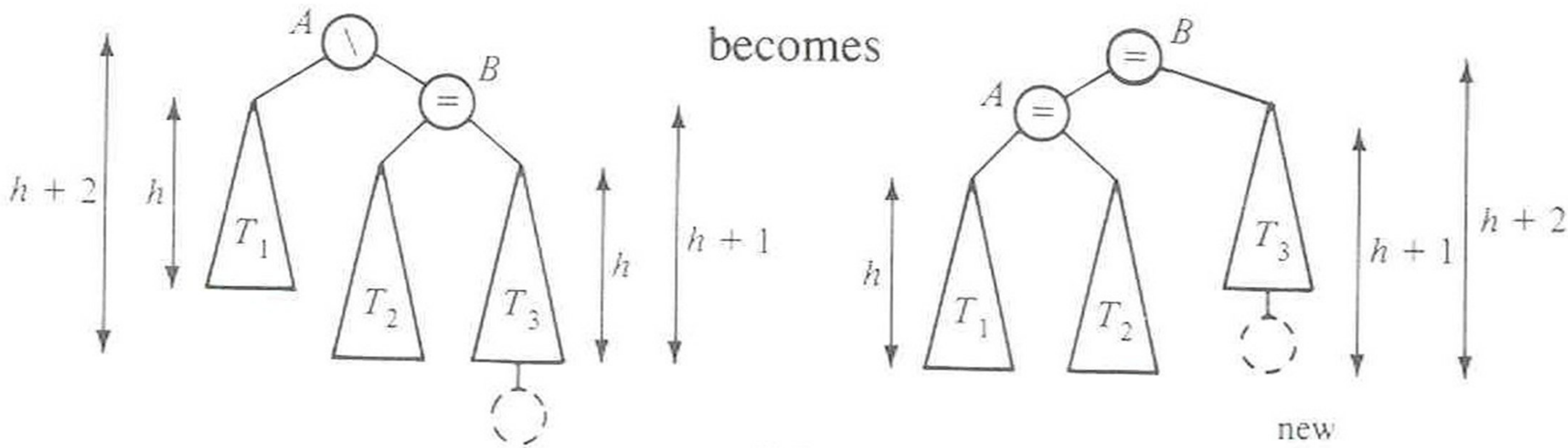
- Assume that the tree is height-balanced before the insertion.
- Start at the inserted node (always a leaf).
- Move back up the tree to the first (lowest) node (if any) where the heights of its subtrees now differ by more than one.
  - We'll call that node A in our diagrams.
- Do the appropriate single or double rotation to balance the subtree whose root is at this node.
- If a rotation is needed, we will see that the combination of the insertion and rotation leaves this subtree with the same height that it had before insertion.
- So why is the algorithm O(log N)?

Which kind of rotation to do?

Depends on the first two links in the path from the node with the imbalance (A) down to the newly-inserted node.

First link (down from A)	**Second link** (down from A's child)	**Rotation type** (rotate "around A's position")
Left	Left	Single right
Left	Right	Double right
Right	Right	Single left
Right	Left	Double left

Single left rotation (right is the mirror image of this picture)



Diagrams are from *Data Structures* by E.M. Reingold and W.J. Hansen.