Data Compression

Variable-length character codes
Huffman Trees

Data Compression

YOU SAY GOODBYE. I SAY HELLO. HELLO, HELLO. I DON'T KNOW WHY YOU SAY GOODBYE, I SAY HELLO.

Letter frequencies

SPACE	17		A	4		U	2
O	12		S	4		W	2
Y	9		I	3		N	2
L	8		D	3		K	1
E	6		COMMA	2		T	1
H	5		B	2		APOSTROPHE	1
PERIOD	4		G	2			

- There are 90 characters altogether.
- How many total bits in the ASCII representation of this string?
- We can get by with fewer bits per character (custom code)
  - How many bits per character? How many for entire message?
  - Do we need to include anything else in the message?
  - How to represent the table?
    1. count
    2. ASCII code for each character How to do better?

Data Compression

- Main Principle:
  - *Declaration of Independence* doesn't apply to characters
  - Not all characters are created equal!

Variable-length codes for characters

- **Principles for determining a scheme for creating character codes:**
  1. Less-frequent characters have longer codes so that more-frequent characters can have shorter codes
  2. No code can be a prefix of another code
     - Why is this restriction necessary?
- We assume that we have some routines for packing sequences of bits into bytes and writing them to a file, and for unpacking bytes into bits when reading the file
  - Weiss has a very clever approach:
    - **BitOutputStream** and **BitInputStream**
    - methods `writeBit` and `readBit` allow us to logically read or write a bit at a time

Compression algorithm: Huffman encoding

- Named for David Huffman
  - http://en.wikipedia.org/wiki/David_A._Huffman
  - Invented while he was a graduate student at MIT.
  - Huffman never tried to patent an invention from his work. Instead, he concentrated his efforts on education. In Huffman's own words, "My products are my students."
- Principles of variable-length character codes:
  - Less-frequent characters have longer codes
  - No code can be a prefix of another code
- We build a tree (based on character frequencies) that can be used to encode and decode messages

A Huffman code: HelloGoodbye message

```
C:\Personal\Courses\CS-230\java-source>type HelloGoodbyeOneLine
YOU SAY GOODBYE. I SAY HELLO. HELLO, HELLO. I DON'T KNOW WHY YOU SAY GOODBYE, I SAY HELLO.

C:\Personal\Courses\CS-230\java-source>java HuffmanDS <HelloGoodbyeOneLine
Encoding of   is 00 (frequency was 17, length of code is 2)
Encoding of . is 0100 (frequency was 4, length of code is 4)
Encoding of H is 0101 (frequency was 5, length of code is 4)
Encoding of Y is 011 (frequency was 9, length of code is 3)
Encoding of K is 100000 (frequency was 1, length of code is 6)
Encoding of T is 1000010 (frequency was 1, length of code is 7)
Encoding of ' is 1000011 (frequency was 1, length of code is 7)
Encoding of D is 10001 (frequency was 3, length of code is 5)
Encoding of E is 1001 (frequency was 6, length of code is 4)
Encoding of O is 101 (frequency was 12, length of code is 3)
Encoding of I is 11000 (frequency was 3, length of code is 5)
Encoding of B is 110010 (frequency was 2, length of code is 6)
Encoding of , is 110011 (frequency was 2, length of code is 6)
Encoding of S is 11010 (frequency was 4, length of code is 5)
Encoding of A is 11011 (frequency was 4, length of code is 5)
Encoding of U is 111000 (frequency was 2, length of code is 6)
Encoding of G is 111001 (frequency was 2, length of code is 6)
Encoding of N is 111010 (frequency was 2, length of code is 6)
Encoding of W is 111011 (frequency was 2, length of code is 6)
Encoding of L is 1111 (frequency was 8, length of code is 4)
Total bits required for message: 351
```

Draw part of the Tree

Decode a "message"

Build the tree for a smaller message

I	1
R	1
N	2
O	3
A	3
T	5
E	8

- Start with a separate tree for each character (in a priority queue).
- Repeatedly merge the two lowest (total) frequency trees
- Use the tree to encode NATION.
- How would we decode this message?

Huffman trees are provably optimal among single-character codes.

What About the Code Table?

- When we send a message, the code table can basically be just the list of characters and frequencies
  - Why?

Huffman Java Code Overview

- This code provides human-readable output to help us understand the Huffman algorithm.
- We will deal with it at the abstract level; "real" code to do file compression is found in DS chapter 12.
- I am confident that you can figure out the other details if you need them.
- This code is based on code written by Duane Bailey, in his book *JavaStructures.*
- A great thing about this example is the use of various data structures (Binary Tree, Hash Table, Priority Queue).

New Classes used by **Huffman**

- **Leaf.** Represents a leaf node in a Huffman tree.
  - Contains the character and a count of how many times it occurs in the text.
- **HuffmanTree**: Each node contains the total weight of all characters in the tree, and either a leaf node or a binary node with two subtrees that are Huffman trees.
  - The contents field of a non-leaf node is never used; we only need the total weight.
  - `compareTo` returns its result based on comparing the total weights of the trees.

Classes used by Huffman, part 2

- **Huffman**: Contains **main** The algorithm:
  - Count character frequencies and build a list of Leaf nodes containing the characters and their frequencies.
  - Use these nodes to build a sorted list (treated like a priority queue) of single-character Huffman trees.
  - **do**
    - Take two smallest (in terms of total weight) trees from the sorted list
    - Combine these nodes into a new tree whose total weight is the sum of the weights of the new tree
    - Put this new tree into the sorted list.

    while there is more than one tree left.

  The one remaining tree will be an optimal tree for the entire message.

Look at the code …

- … and answer quiz questions 8-12
- You can do this with one or two other people
- Each of you should write the answers on your quiz paper
- Code is in your repository
  - Project name: Huffman-Bailey-JFC

Code Details – several slides.

- These are mainly here so that
  - You can see an overview of the most important parts of the code before looking at the code on-line.
  - After you have looked at the on-line code, if you ask any questions about it in class, we can easily refer to the code together.

Leaf node class for Huffman Tree

```
class Leaf  { // Leaf node of a  Huffman tree.

    char ch; // the character represented
             // by this node.
    int frequency; // frequency of this
                   // character in message.

    public Leaf(char c, int freq) {
        ch = c;
        frequency = freq;
    }
}
```

Highlights of the HuffmanTree class

```
class HuffmanTree implements Comparable<HuffmanTree> {
  BinaryNode root; // root of tree
  int totalWeight; // weight of tree
  static int totalBitsNeeded;
       // bits needed to represent entire message
       // (not including code table).

  public HuffmanTree(Leaf e) {
    root = new BinaryNode(e, null, null);
    totalWeight = e.frequency;
  }

  public HuffmanTree(HuffmanTree left, HuffmanTree right) {
    // pre: left and right non-null
    // post: merge two trees together and add their weights
    this.totalWeight = left.totalWeight + right.totalWeight;
    root = new BinaryNode(null, left.root, right.root);
  }
          public int compareTo(HuffmanTree other) {
              return (this.totalWeight - other.totalWeight);
          }
```

Printing a HuffmanTree

```
public void print() {
  // print out strings associated with characters in tree
  totalBits = 0;
  print(this.root, "");
  System.out.println("Total bits for entire message: "+ totalBits);
 }

protected static void print(BinaryNode r,
                        String representation) {
  //  print out strings associated with chars in tree r,
  //  prefixed by representation
  if (r.getLeft() != null) {  // interior node
    print(r.getLeft(),  representation + "0"); // append a 0
    print(r.getRight(), representation + "1"); // append a 1
  } else { // leaf; print its code
    Leaf e = (Leaf) r.getElement();
    System.out.println("Encoding of " + e.ch + " is " +
       representation + " (frequency was " +  e.frequency +
       ", length of code is " +  representation.length() + ")");
    totalBits += (e.frequency * representation.length());
  }
}
```

Highlights of Huffman class 1

```
public static void main(String args[]) throws Exception {

  BufferedReader r = new BufferedReader(
                   new InputStreamReader(System.in));
  HashMap<Character, Integer> freq =
     new HashMap<Character,Integer>();
  String oneLine;  // current input line.

  // First read the data and count characters
  // Go through the input line, one character at a time.
  while ((oneLine = r.readLine()) != null) {
     for (int i = 0; i<oneLine.length(); i++) {
     char c = oneLine.charAt(i);
     if (freq.containsKey(c))
        freq.put(c, freq.get(c)+1);
     else // first time we've seen c
        freq.put(c, 1);
      }
  }
```

Highlights of Huffman class 2

```
// Now the table of frequencies is complete.
// put each character into its own Huffman tree

PriorityQueue<HuffmanTree> treeQueue =
             new  PriorityQueue<HuffmanTree>();
for (char c : freq.keySet())
   treeQueue.add(new HuffmanTree(new Leaf(c, freq.get(c))));

HuffmanTree smallest, secondSmallest;
// merge trees in pairs until only one tree remains
while (true) {
  smallest = treeQueue.poll();
  secondSmallest = treeQueue.poll();
  if (secondSmallest == null) break;
    // add bigger tree containing both to the sorted list.
    treeQueue.add(new HuffmanTree(smallest, secondSmallest));
}
 // print the only tree left in the list of Huffman trees.
 smallest.print();
}
```

Representingthe code table

- Three or four bytes per character
  - The character itself.
  - The frequency count.
- End of table signaled by 0 for char and count.
- Tree can be reconstructed from this table.
- The rest of the file is the compressed message.

Summary

- The Huffman code is provably optimal among all single-character codes for a given message.
- Going farther:
  - Look for frequently occurring sequences of characters and make codes for them as well.

Graphs

Definitions
Representations
Algorithms

Example Graph

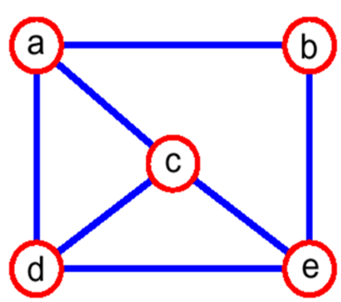
A **graph G** = (**V**,**E**) is composed of:

V: set of *vertices*

E: set of *edges* connecting the *vertices* in **V**

An **edge e** = (u,v) is a pair of vertices

Example:

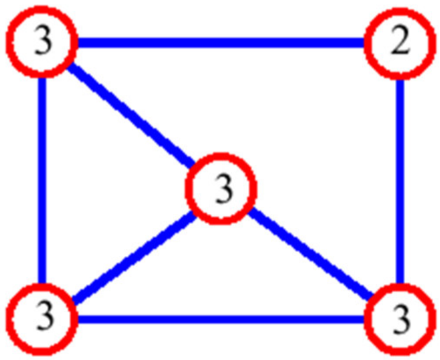


V= {a,b,c,d,e}

E= {(a,b),(a,c),(a,d), (b,e),(c,d),(c,e), (d,e)}

Graph Terminology

- adjacent vertices: connected by an edge
- degree (of a vertex): # of adjacent vertices

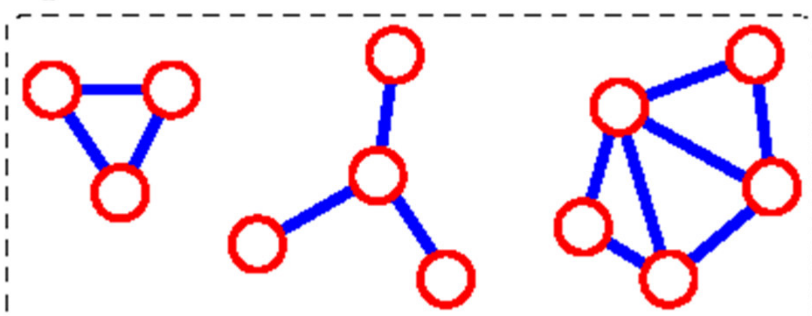


$$\sum_{v \in V} \deg(v) = 2(\#\ \text{edges})$$

- Since adjacent vertices each count the adjoining edge, it will be counted twice

Continuing Graph Terminology

connected component: maximal connected subgraph. E.g., the graph below has 3 connected components.



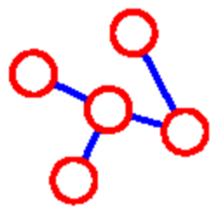
More Connectivity

n = #vertices

m = #edges

For a tree m = n - 1

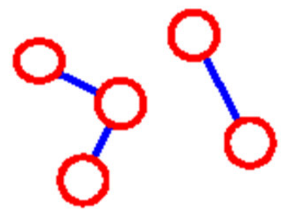
A necessary but not sufficient condition for a graph to be a tree.



n = 5

m = 4

If m < n - 1, G is not connected



n = 5

m = 3

Graph representations

- Adjacency matrix.
- Adjacency list.
- Look at both for the same graph.
- What does the square of an adjacency matrix give us?
- Boolean adjacency matrix?
- Edge Listing.