Mathematical Induction

Recap of the principle
Example proofs

Recap: The Principle of Mathematical Induction

- To prove that p(n) is true for all $n \geq n_0$:
  a) Show that $p(n_0)$ is true.
  b) Show that for all $k \geq n_0$,
  p(k) implies p(k+1).
  I.e, show that whenever p(k) is true, then p(k+1) is true also.
- Let's do it!

Induction example

- To prove: If $n \geq 1$, then $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- What is the base case?
- Induction hypothesis?
- What do we need to show in the induction step?
- Procedural matters:
  - Start with one side of the equation/inequality and work toward the other side.
  - DO NOT use the " write what we want to prove and do the same thing to both sides of the equation to work backwards to what we are assuming" approach.
  - Clearly indicate the step in which you use the induction hypothesis.

Another example:

Show by induction that $2n + 1 < n^2$ for all integers $n \geq 3$.

There are other ways that we could show this (using calculus, for example).

But for now the goal is to have a simple example that can illustrate how to do proofs by induction.

Your Turn! Work with another student

- Consider the sum below. First, do a few examples to figure out a formula for the sum. Then use induction to show that your formula is correct for all n≥1.

$$\sum_{i=1}^{n}(2i-1)$$

Induction is like Recursion

- When we write a recursive program, we make it work for the base case(s).
  - Then, assuming that it works for smaller values, we use the solution for a smaller value to construct a solution for a larger value.
- To prove that a property p(n) is true for all n>0 using (strong) mathematical induction,
  - we show that
    - it is true for the base case (typically n=0 or n=1), and that
    - the truth of p(k) for larger values of k can be derived from the truth of p(j) for all j with $n_0 \le j < k$.