Growable array analysis

Continued

Growable array exercise

- Doubling each time: Assume that N = 5 (2^k) + 1.
- Total # of array elements copied:

k	N	#copies
0	6	5
1	11	
2	21	
3	41	
4	81	
k	= 5 (2^k) + 1	

Growable array exercise

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- Total # of array elements copied:

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0	6	5
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Growable array exercise

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2	21	5+10 + 20
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Growable array exercise

- Doubling each time: Assume that $N = 5 (2^k) + 1$.
- Total # of array elements copied:

k	N	#copies
0	6	5
1	11	5 + 10
2	21	5+10 + 20
3	41	5+10 + 20 + 40
4	81	5+10 + 20 + 40 + 80
k	$5 (2^k) + 1$	???

- Simplify the sum as a closed-form expression in terms of K.
- Then express the result in terms of N.

Growable array exercise

- Adding one each time:
- Total # of array elements copied:

N	#copies
6	5
7	5 + 6
8	5+6 + 7
9	5+6 + 7 + 8
10	5+6 + 7+ 8+ 9
N	

- Simplify the sum as a closed-form expression.

Average overhead cost of adding an additional string to the array:

- In the doubling case
- In the add-one case
- Conclusions?

Mathematical Induction

What it is?
Why is it a legitimate proof method?
How to use it?

Important Sets of Integers

- Z all integers (whole numbers)
- Z^+ the positive integers
- Z^- the negative integers
- N Natural numbers: non-negative integers
  - Note that some people define Natural numbers as Z^+

Getting to be good at Mathematical Induction

- For some of you, that will take a lot of time, many explanations, and many examples.
- We will do a few simple induction problems in class
- Then some for homework, WA3.
  - After that, I will have one or two on each of several written assignments.
    - More and more they will be to prove something about the things we are studying.
  - And probably at least one induction problem on each exam.

Mathematical Induction What's it all about? (Outline of the next slides)

- Most of the time our intuition is good enough so we don't have to resort to formal proofs of things. But sometimes ...
- What kind of thing do we try to prove *via* induction?
- What is the approach? (How does it work?)
- Why does this really prove the infinite set of statements that we want to prove?
  - Start with the Well-ordering Principle
  - proof of Mathematical Induction Principle by contradiction.

What kind of things do we try to prove *via* induction?

- In this course, it will be a property of positive integers (or non-negative integers, or integers larger than some specific number).
- A ***property*** p(n) is a boolean statement about the integer *n*. [p: int→ boolean]
  - Example: p(n) could be "n is an even number".
  - Then p(4) is true, but p(3) is false.
- If we believe that some property p is true for **all** positive integers, induction gives us a way of proving it.

What is the approach? (How does it work?)

- To prove that p(n) is true for all n $\geq n_0$:
  - Show that p(n_0) is true.
  - Show that for all k $\geq n_0$,

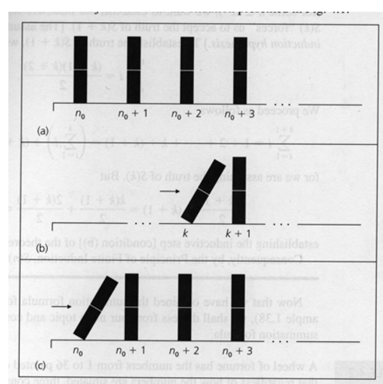
    p(k) implies p(k+1).

    I.e, show that whenever p(k) is true,

    then p(k+1) is true also.

Why does induction work? (Informal look)

- To prove that p(n) is true for all n $\geq n_0$:
  - Show that p(n_0) is true.
  - Show that for all k $\geq n_0$,
    p(k) implies p(k+1).
    I.e, if p(k) is true, then
    p(k+1) is true also



From Ralph Grimaldi's discrete math book.

Why does induction work?

- Next we focus on a formal proof of this, because:
  - Some people may not be convinced by the informal one
  - The proof itself illustrates an important proof technique

The Well-ordering principle

- It's an axiom, not something that we can prove.
- WOP: Every non-empty set of non-negative integers has a smallest element.
- Note the importance of "non-empty", "non-negative", and "integers".
  - The empty set does not have a smallest element.
  - A set with no lower bound (such as the set of all integers) does not have a smallest element.
    - In the statement of WOP, we can replace "positive" with "has a lower bound"
  - Unlike integers, a set of rational numbers can have a lower bound but no smallest member.
- Assuming the well-ordering principle, we can prove that the principle of mathematical induction is true.