MA/CSSE 473 – Design and Analysis of Algorithms

Homework 9 (51 points total) Updated for Summer, 2014

Problems for enlightenment/practice/review (not to turn in, but you should think about them):

How many of them you need to do serious work on depends on you and your background. I do not want to make everyone do one of them for the sake of the (possibly) few who need it. You can hopefully figure out which ones you need to do.

5.5.4 (multiplication à la Russe)
5.5.7 (Josephus problem for N=40)
5.5.9 (Prove properties of Josephus solutions)

Problems to write up and turn in:

1. (8) (Nim strategy) In class (Day 23 in in the Summer, 2014 schedule) we stated that

   in n-pile Nim, a player is guaranteed to be able to win if and only if the nim sum (as defined in class) is nonzero at the beginning of that player's turn.

   We proved three lemmas (Slide 8) that can be used to prove this statement (see the ICQ solution from that day for details). Use one or more of these lemmas to construct a proof by induction (on the total number of chips in all of the piles?) that the statement is correct for any nonnegative number of piles and any non-negative number of chips.

   These are the lemmas:

   – Let $x_1, \ldots ,x_n$ be the sizes of the piles before a move, and $y_1, \ldots ,y_n$ be the sizes of the piles after that move.
   – Let $s = x_1 \oplus \ldots \oplus x_n$, and $t = y_1 \oplus \ldots \oplus y_n$.
   - **Lemma 1:** $t = s \oplus x_k \oplus y_k$.
   - **Lemma 2:** If $s = 0$, then $t \neq 0$.
   - **Lemma 3:** If $s \neq 0$, it is possible to make a move such that $t=0$.

2. (5) Using the algorithm from class (and referenced in the previous problem) consider the following situation:

Pile #	Chips
1	77
2	46
3	27
4	74

   Which pile should the player take chips from and how many chips should be taken in order to guarantee a win? Show your work.

3. (6) 4.4.8 [5.5.2] (Ternary Search)

4. (12) 4.4.10 [5.5.3] (fake coin divide-into-three) Levitin made me do it!

5. (20) 4.5.11a [5.6.10a] (moldy chocolate) This problem may be harder than first appears to be. "Transform and conquer" is a good way to find a complete solution, so you may want to look ahead to Chapter 6. However, if you can't solve the general case, get some partial credit; solve some cases that you can solve, and write about what you tried for other cases. **In the past, several students said that this problem took them longer than any previous problem in the course**.