473 HW 02 Problems from Levitin

2.1

4. a. *Glove selection* There are 22 gloves in a drawer: 5 pairs of red gloves, 4 pairs of yellow, and 2 pairs of green. You select the gloves in the dark and can check them only after a selection has been made. What is the smallest number of gloves you need to select to have at least one matching pair in the best case? in the worst case? (after [Mos01], #18)

   b. *Missing socks* Imagine that after washing 5 distinct pairs of socks, you discover that two socks are missing. Of course, you would like to have the largest number of complete pairs remaining. Thus, you are left with 4 complete pairs in the best-case scenario and with 3 complete pairs in the worst case. Assuming that the probability of disappearance for each of the 10 socks is the same, find the probability of the best-case scenario; the probability of the worst-case scenario; the number of pairs you should expect in the average case. (after [Mos01], #48)

5. a. Prove formula (2.1) for the number of bits in the binary representation of a positive decimal integer.

   b. What would be the analogous formula for the number of decimal digits?

   c. Explain why, within the accepted analysis framework, it does not matter whether we use binary or decimal digits in measuring n's size.

2.2

3. For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.) Prove your assertions.

   a. $(n^2+1)^{10}$ b. $\sqrt{10n^2+7n+3}$

   c. $2n\lg(n+2)^2+(n+2)^2\lg\frac{n}{2}$ d. $2^{n+1}+3^{n-1}$

   e. $\lfloor \log_2 n \rfloor$

only parts a and d are required

7. Prove (by using the definitions of the notations involved) or disprove (by giving a specific counterexample) the following assertions.

   a. If $t(n) \in O(g(n))$, then $g(n) \in \Omega(t(n))$.

   b. $\Theta(\alpha g(n)) = \Theta(g(n))$ where $\alpha > 0$.

   c. $\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$.

   d. For any two nonnegative functions $t(n)$ and $g(n)$ defined on the set of nonnegative integers, either $t(n) \in O(g(n))$, or $t(n) \in \Omega(g(n))$, or both.

2.2

10. *Door in a wall* You are facing a wall that stretches infinitely in both directions. There is a door in the wall, but you know neither how far away nor in which direction. You can see the door only when you are right next to it. Design an algorithm that enables you to reach the door by walking at most $O(n)$ steps where n is the (unknown to you) number of steps between your initial position and the door. [Par95], #652 An efficient algorithm.

2.3

2. Find the order of growth of the following sums.

   a. $\sum_{i=0}^{n-1}(i^2+1)^2$ b. $\sum_{i=2}^{n-1}\lg i^2$

   c. $\sum_{i=1}^{n}(i+1)2^{i-1}$ d. $\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j)$

   Use the $\Theta(g(n))$ notation with the simplest function $g(n)$ possible.

2.3

10. Consider the following version of an important algorithm that we will study later in the book.

ALGORITHM $GE(A[0..n-1, 0..n])$
    //Input: An n-by-$n+1$ matrix $A[0..n-1, 0..n]$ of real numbers
    for $i \leftarrow 0$ **to** $n-2$ **do**
        for $j \leftarrow i+1$ **to** $n-1$ **do**
            for $k \leftarrow i$ **to** n **do**
                $A[j,k] \leftarrow A[j,k] - A[i,k] * A[j,i] / A[i,i]$

a. Find the time efficiency class of this algorithm.

b. What glaring inefficiency does this pseudocode contain and how can it be eliminated to speed the algorithm up?