473 Levitin problems and hints HW 09

Problem 1: 4.4.8 [5.5.2]

2. Consider *ternary search*—the following algorithm for searching in a sorted array $A[0..n-1]$: if $n = 1$, simply compare the search key K with the single element of the array; otherwise, search recursively by comparing K with $A[\lfloor n/3 \rfloor]$, and if K is larger, compare it with $A[\lfloor 2n/3 \rfloor]$ to determine in which third of the array to continue the search.

   a. What design technique is this algorithm based on?

   b. Set up a recurrence relation for the number of key comparisons in the worst case. (You may assume that $n = 3^k$.)

   c. Solve the recurrence for $n = 3^k$.

   d. Compare this algorithm's efficiency with that of binary search.

Author's Hints:

2. The algorithm is quite similar to binary search, of course. In the worst case, how many key comparisons does it make on each iteration and what fraction of the array remains to be processed?

Problem 2: 4.4.10[5.5.3]

3. a. Write a pseudocode for the divide-into-three algorithm for the fake-coin problem. (Make sure that your algorithm handles properly all values of n, not only those that are multiples of 3.)

   b. Set up a recurrence relation for the number of weighings in the divide-into-three algorithm for the fake-coin problem and solve it for $n = 3^k$.

   c. For large values of n, about how many times faster is this algorithm than the one based on dividing coins into two piles? (Your answer should not depend on n.)

Author's Hints:

3. While it is obvious how one needs to proceed if $n \bmod 3 = 0$ or $n \bmod 3 = 1$, it is somewhat less so if $n \bmod 3 = 2$.

Problems 3-5: Not from textbook

3. (5) Which permutation immediately follows 37246510 in lexicographic order?
Show how you use the algorithm from Day 18 class to get your answer.

4. (5) If the permutations of the numbers 0-7 are numbered from 0 to 8!-1,
what is the (lexicographic ordering) sequence number of the permutation 37246510?

5. (5) Which permutation of 01234567 is number 25000 in lexicographic order?

5.6.10: This problem may be harder than it looks at first. Perhaps it is no accident that the problem is in section 5.6, and also very close to Chapter 6.
"Transform and conquer" is a good way to find a complete solution. However, If you can't solve the general case, to get some partial credit solve some cases that you can solve, and talk about what you tried for other cases.

Problem 6: 4.5.11a[5.6.10a]

10. ▷a. *Moldy chocolate* Two payers take turns by breaking an m-by-n chocolate bar, which has one spoiled 1-by-1 square. Each break must be a single straight line cutting all the way across the bar along the boundaries between the squares. After each break, the player who broke the bar last eats the piece that does not contain the spoiled corner. The player left with the spoiled square loses the game. Is it better to go first or second in this game?

Author's Hints:

10. Play several rounds of the game on the graphed paper to become comfortable with the problem. Considering special cases of the spoiled square's location should help you to solve it.