

# MA/CSSE 473 – Design and Analysis of Algorithms

## Homework 7 67 points total **updated for summer, 2012**

When a problem is given by number, it is from the textbook. 1.1.2 means “problem 2 from section 1.1” .

### Problems for enlightenment/practice/review (not to turn in, but you should think about them):

How many of them you need to do serious work on depends on you and your background. I do not want to make everyone do one of them for the sake of the (possibly) few who need it. You can hopefully figure out which ones you need to do. These problems should all be review from CSSE 230.

- 5.1.1 [4.1.1] (divide-and-conquer array max for unsorted array)
- 5.1.2 [4.1.2] (divide-and-conquer array max/min for unsorted array)
- 5.1.7 [4.1.7] (Mergesort stability)
- 5.1.9 [4.1.9] ( $O(n \log n)$  algorithm to count inversions in an array)
- 5.2.1 [4.2.1] (quicksort example)
- 5.2.4 [4.2.4] (quicksort sentinel)
- 5.2.6 [4.2.6] (increasing arrays in quicksort)

### Problems to write up and turn in:

Problems 1-3 are based on the Dasgupta excerpt that is available on ANGEL (in the Reading Materials folder). Most of the material is also covered in Weiss, Sections 7.4 and 9.6, which are also on ANGEL

1. (15) (Miller-Rabin test) Let  $N = 1729$ .
  - (a) How many values of  $a$  in the range  $1..1728$  pass the Fermat test [i.e.  $a^{1728} \equiv 1 \pmod{1729}$ ]?
  - (b) For how many of those values does the Miller-Rabin test provide a witness that  $N$  is composite?
  - (c) For  $N=1729$ , if we pick  $a$  at random, what is the probability that running the test on  $a$  will show that  $N$  is composite? Note: Rabin showed that for any  $N$ , the probability is at least 75%, what is it for this case?  
[Hint: writing a few lines of code may help you here].
2. (15) (RSA decoding). If small primes are used, it is computationally easy to "crack" RSA codes. Suppose my public key is  $N=703$ ,  $e= 53$ . You intercept an encrypted message intended for me, and the encrypted message is 361. What was the original message?  
How did you get your answer?
3. ( 6) (RSA attacks) Read about various ways of attacking the RSA cryptosystem. Write about two attacks that interest you. Explain how they work.
4. ( 3) 5.1.4 [4.1.4] (logarithm base in the Master Theorem)
5. ( 6) 5.1.5 [4.1.5] (Simple application of the Master Theorem)
6. ( 6) 5.2.2 [4.2.2] (Quicksort partition scan properties)
7. (10) Solve the average-case recurrence for quicksort. The recurrence is given on page 180 [133] of Levitin. Feel free to look up a solution, understand it, and write it in your own words (and symbols). The Weiss Data Structures book (Section 8.6.2) is one possible source. You should write a reasonable amount of detail.
8. ( 6) 5.2.8 [4.2.8] (Negatives before positives)