MA/CSSE 473 – Design and Analysis of Algorithms

Homework 4

Summer: Drop box. These are to be turned in as hard copy. You can write solutions out by hand, or write them on your computer and print them. If there are multiple pages, please staple them together.

All of these problems are variations on the "Tiling with Trominoes" example from Day 7 class. For another brief discussion of this example (and Odd Pie Fights), see
http://mathdemos.gcsu.edu/mathdemos/tromino/tromino.html
Also http://www.rose-hulman.edu/class/csse/csse473/201040/Homework/JohnsonbaughTrominoes.pdf

Here is pseudocode for the tiling algorithm from class. Many details are omitted, but it conveys the idea.

```
def tileWithTrominoes(n, m):
    """n is the dimension of the deficient board to be tiled, and assumed to be a power of 2.
       m gives the location of the missing square that makes the board deficient."""
    if n == 2:
        # place the tromino so it covers the three squares
        return
    # Consider the four squares in the center of the board.
    # Three of them are in quadrants of the board that do
    # not contain m.  Place a tromino on them.  Now let
    # m1, m2, m3, and m4 be m and the three squares just covered.

    tileWithTrominoes (n/2, m1)  # assume that we have a mechanism
    tileWithTrominoes (n/2, m2)  # for translating these tilings
    tileWithTrominoes (n/2, m3)  # to the appropriate quadrants
    tileWithTrominoes (n/2, m4)  # of the original board.
```

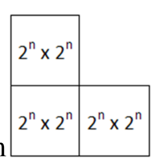
1. (5) Write a recurrence relation for the running time of this algorithm, and use the Master Theorem to show that its solution is $\Theta(n^2)$. Thanks to Mike Jones for the following reference: http://www.math.dartmouth.edu/archive/m19w03/public_html/Section5-2.pdf

2. (2) Show that 9×9 deficient boards cannot be tiled with trominoes.

3. (8) Show that a 5×5 board with the upper-left corner square missing can be tiled with trominoes.

4. (10) Show a deficient 5×5 board that cannot be tiled with trominoes, and prove that this board cannot be tiled.

5. (5) Show that any 2i×3j board (not deficient) can be tiled with trominoes, for all positive integers i and j.

6. (10) A $2^n \times 2^n$ L-shape is a figure of the form 

with no missing squares, or any rotation of this figure by a multiple of 90°. Write a recursive algorithm that tiles any $2^n \times 2^n$ L-shape with trominoes. The recursive calls should tile smaller L-shapes. You may express it in English, pseudo code, or code from any programming language that is likely to be known by most people in this course. Feel free to use diagrams as part of your algorithm description.

7. (10) Use the preceding exercise as the basis for a different algorithm for tiling (with trominoes) any $2^n \times 2^n$ deficient board with trominoes.

Extra credit problem. (20) Show that any deficient 7×7 board can be tiled with trominoes. You will probably want to use the results of some of the previous problems. There will be several different cases, depending on the location of the missing square.