

MA/CSSE 473 – Design and Analysis of Algorithms

Homework 3 43 points total [45 points for Fall, 2010]

Summer: in drop box. These are to be turned in as hard copy. You can write solutions out by hand, or write them on your computer and print them. If there are multiple pages, please staple them together.

When a problem is given by number, it is from the textbook. 1.1.2 means “problem 2 from section 1.1” .

Problems for enlightenment/practice/review (not to turn in, but you should think about them):

How many of them you need to do serious work on depends on you and your background. I do not want to make everyone do one of them for the sake of the (possibly) few who need it. You can hopefully figure out which ones you need to do.

3.1.5 (selection sort practice)

3.1.10 (Is bubble sort stable?)

Problems to write up and turn in:

1. (5) Prove by mathematical induction that the following formula is true for every positive integer n .

$$\sum_{i=1}^n (-1)^{i+1} i^2 = \frac{(-1)^{n+1} n(n+1)}{2}$$

2. (8) Prove (not necessarily directly by mathematical induction) that $\sum_{i=1}^n i \cdot r^i < \frac{r}{(1-r)^2}$ for all $n \geq 1$ and $0 < r < 1$.

3. (4) Let F_n be the n^{th} Fibonacci number (recall that $F_0 = 0$ and $F_1 = 1$ in our formulation). Show by mathematical induction that that for all $n > 0$,

$$\sum_{i=1}^n F_i^2 = F_n F_{n+1}$$

4. (6) Prove by mathematical induction that F_n (as defined above) is even if and only if n is divisible by 3.

5. (2) 3.1.2 (algorithms for computing a^n) [this will count 4 points for Fall, 2010]

6. (7) 3.1.4 (polynomial evaluation)

7. (1) 3.1.6 (stability of selection sort)

8. (2) 3.1.7 (selection sort linked list)

9. (8) 3.1.11 (alternating disks) Come up with the best solution that you can, and come up with a formula for the number of moves as a function of N , the total number of disks. You may assume that N is even. If it makes the analysis easier, assume that N is a power of 2.