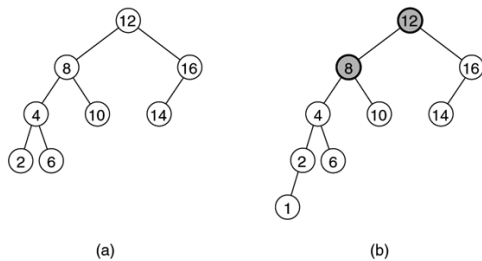


# AVL Trees

- An AVL tree is a BST with the additional requirement that:

For any node in the tree, the height of its left and right subtrees differs by at most 1.

- The height of an empty subtree is -1.



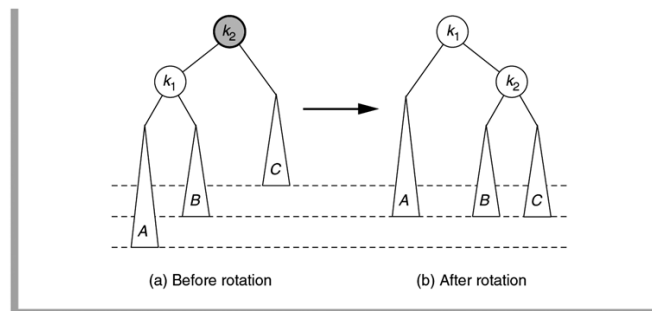
**figure 19.21**

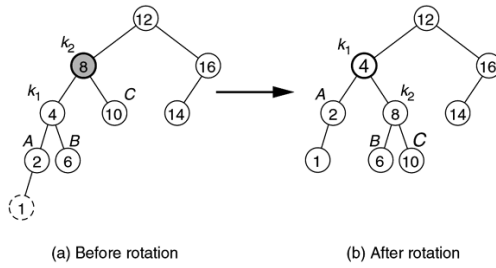
Two binary search trees: (a) an AVL tree; (b) not an AVL tree (unbalanced nodes are darkened)

## Four Cases to Consider

- A height imbalance occurs because:
  - An insertion in the left subtree of the left child
  - An insertion in the right subtree of the left child
  - An insertion in the left subtree of the right child
  - An insertion in the right subtree of the right child

**figure 19.23**  
Single rotation to fix  
case 1

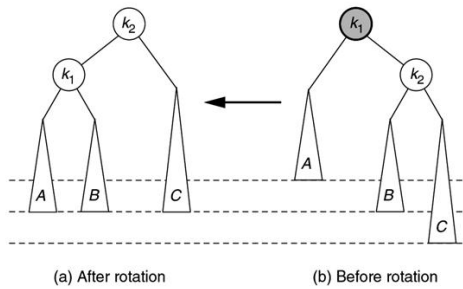




**figure 19.25**  
Single rotation fixes an AVL tree after insertion of 1.

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19-5

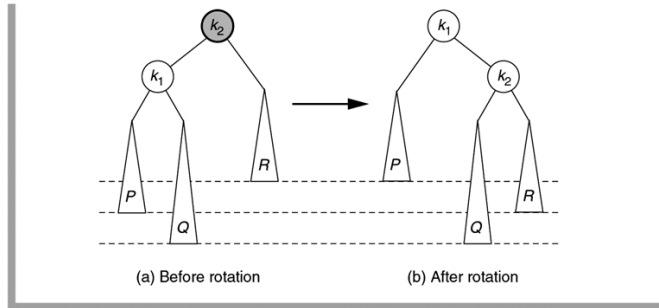


**figure 19.26**  
Symmetric single rotation to fix case 4

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19-6

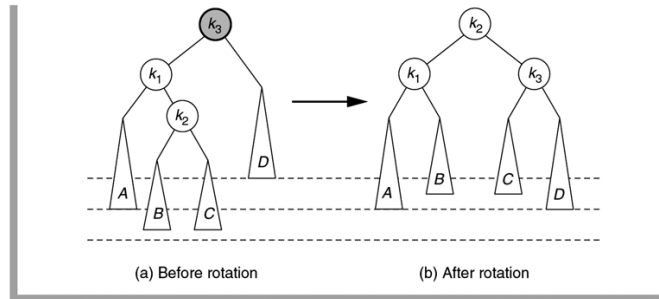
**figure 19.28**  
Single rotation does not fix case 2.



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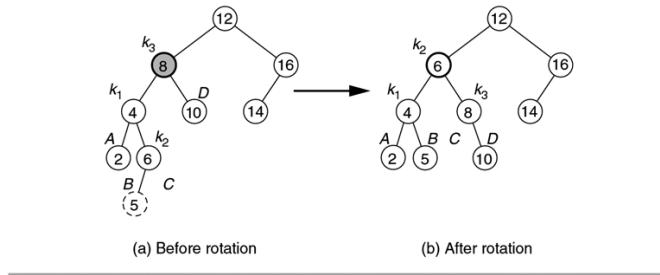
19-7

**figure 19.29**  
Left-right double rotation to fix case 2



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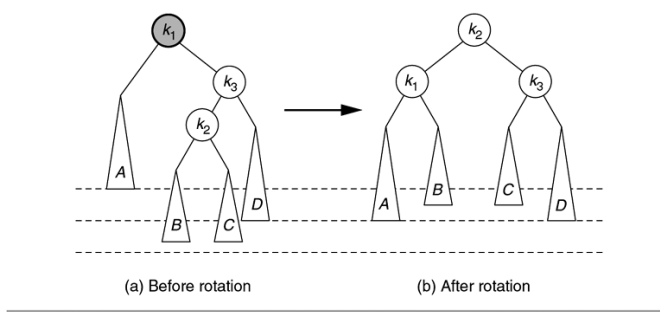
19-8



**figure 19.30**  
Double rotation fixes AVL tree after the insertion of 5.

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**figure 19.31**  
Right-Left double rotation to fix case 3.

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