CSSE 230

Name:

Problems

1. Design an algorithm to find all the common elements in two sorted lists of numbers. For example, for the lists 2, 5, 5, 5 and 2, 2, 3, 5, 5, 7, the output should be 2, 5, 5. What is the maximum number of comparisons your algorithm makes if the lengths of the two given lists are m and n respectively?

2. Use **either the formal definition or the limit definitions** of O, Θ, Ω to determine whether the following assertions are true or false.

   (a) $n(n+1)/2 \in O(n^3)$

   (b) $n(n+1)/2 \in O(n^2)$

   (c) $n(n+1)/2 \in \Theta(n^3)$

   (d) $n(n+1)/2 \in \Omega(n)$

3. Prove the following assertions by using the "c, n_0" (non-limit) definitions of the notations involved, or disprove them by giving a specific counterexample.

   (a) If $t(n) \in O(g(n))$, then $g(n) \in \Omega(t(n))$.

   (b) $\Theta(\alpha g(n)) = \Theta(g(n))$, where $\alpha > 0$.

   (c) $\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$.

4. Consider the following algorithm.

```
ALGORITHM Secret($A[0..n-1]$)
    //Input: An array $A[0..n-1]$ of $n$ real numbers
    $minval \leftarrow A[0]$; $maxval \leftarrow A[0]$
    for $i \leftarrow 1$ to $n-1$ do
        if $A[i] < minval$
            $minval \leftarrow A[i]$
        if $A[i] > maxval$
            $maxval \leftarrow A[i]$
    return $maxval - minval$
```

   (a) What does the algorithm compute?

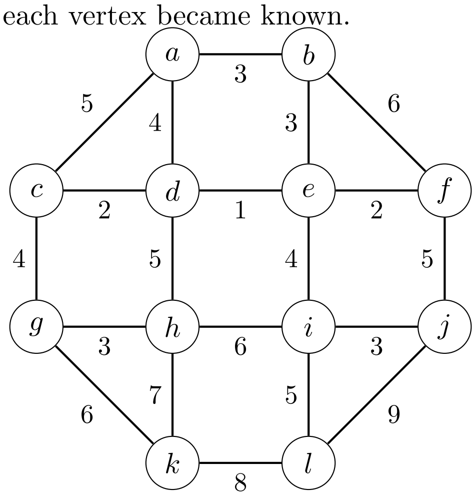
(b) What is its basic operation?

(c) How many times is the basic operation executed?

(d) What is the efficiency class of this algorithm?

(e) Suggest a better algorithm (one that runs fewer basic operations, even in the worst case!) Note: this will take a bit of ingenuity. There is an easy improvement to the algorithm, but it unfortunately won't improve the *worst-case* number of basic operations—can you extend this idea?

5. For the following graph, run Dijkstra's algorithm to find the single-source shortest paths from vertex a. To show your work and solution, write the final Known/Cost/Path table, and indicate the order that each vertex became known.


a
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4
3
6
c
d
e
f
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1
2
4
5
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5
g
h
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j
3
6
3
7
5
6
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k
l
8