

CSE 230 /

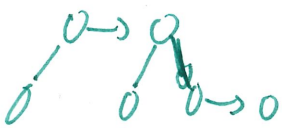
Proof of log-height of AA trees.

Consider the following AA trees for a given level L .
They have the largest height H w/ the least # of nodes N .

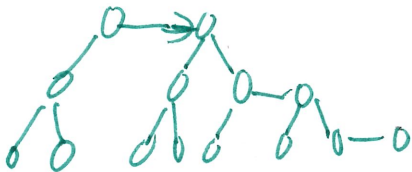
AA Tree

$0 \rightarrow 0$

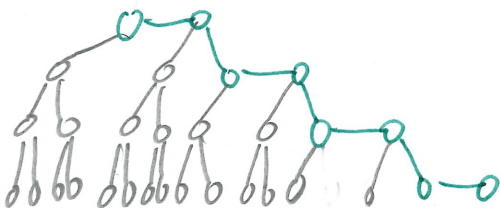
$$\begin{array}{cccc} 2^{L+1} - 2 & N & L & H \\ 2 & 2 & 1 & 1 \end{array}$$



$$\begin{array}{cccc} 2^{L+1} - 2 & 6 & 2 & 3 \end{array}$$



$$\begin{array}{cccc} 2^{L+1} - 2 & 14 & 3 & 5 \end{array}$$



$$\begin{array}{ccc} 2^{L+1} - 2 & 30 & 4 \quad 7 \end{array}$$

$$L = \frac{H+1}{2}$$

$\uparrow$
Expressed in terms of L

$$N = 2^{L+1} - 2 = 2^{\frac{H+1}{2} + 1} - 2 = 2^{\frac{H+3}{2}} - 2$$

+ 26
/ apply log₂

$$\log(N+2) = \frac{H+3}{2}$$

$$2 \log(N+2) - 3 = H \rightarrow H < 2 \log_2(N+2)$$