Proof of upper bound of AVL tree height

We can show that an AVL tree with n nodes has O(log n) height. Consider figure 19.22 from our book:

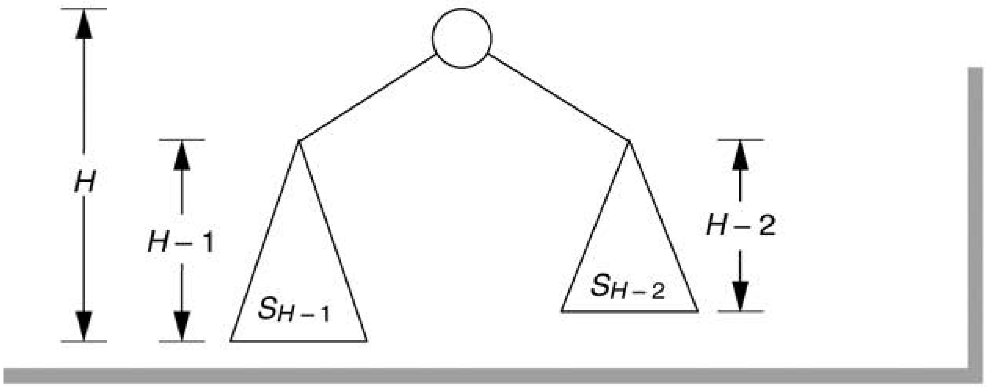

figure 19.22

Minimum tree of height *H*

In order to construct an AVL tree of height h with a minimum number of nodes, one child out to have a height h – 1 and the other child one ought to have a height h − 2.

Let N_h represent the minimum number of nodes that can form an AVL tree of height h. If we know N_{h-1} and N_{h-2}, we can determine N_h. By creating a tree with a root whose left sub-tree has N_{h-1} nodes and whose right sub-tree has N_{h-2} nodes, we have constructed the AVL tree of height h with the least nodes possible.

This AVL tree has a total of $N_{h-1} + N_{h-2} + 1$ nodes. Using this formula, we can then reduce as such:

$N_h = N_{h-1} + N_{h-2} + 1$
$N_{h-1} = N_{h-2} + N_{h-3} + 1$
$N_h = (N_{h-2} + N_{h-3} + 1) + N_{h-2} + 1$

$N_h > 2N_{h-2}$
$N_{h-2} > 2N_{h-4}$
$N_h > 2*2N_{h-4}$
$N_h > 2*2*2N_{h-6}$
…
$N_h > 2^{h/2}$

$\log(N_h) > \log(2^{h/2})$
$\log(N_h) > h/2$

$2\log(N_h) > h$