Show the following by induction.

$\forall N \geq 0$: $\sum_{i=0}^{N} 2^{i} = 2^{N+1}-1$

*Step 1: Prove the base case. The base case is the lowest N for which the expression holds. In our case, that is 0.*

Base case: N=0. $\sum_{i=0}^{0} 2^{i} = 1$. This is the same as $2^{0+1}-1$. The base case holds.

*Step 2: State the inductive hypothesis. Notice that we state that we assume that the expression holds for an arbitrarily chosen k. We can assume anything. Sometimes we assume we are superheroes, we can do that.*

Inductive Hypothesis. Assume the expression holds for arbitrary k:

$$\sum_{i=0}^{k} 2^{i} = 2^{k+1}-1$$

*Step 3: Prove the inductive step. Here is where we do all the heavy hitting. We will show that if our assumption, i.e. step 2 is true then the expression also holds for k+1. In other words, we are proving a conditional. Here is a fun conditional: If the moon is made from green cheese, I am a millionaire. Try to prove that; actually, never mind. However, notice that we are NOT proving that the moon is made from green cheese, just that if it were, then something else would be true. Back to work now.*

Inductive Step. Show that if the expression holds for k, it also holds for k+1. Here is the expression rewritten for k+1.

$$\sum_{i=0}^{k+1} 2^{i} = 2^{(k+1)+1}-1$$

Let's start with the left side and unroll it by one iteration:

$$\sum_{i=0}^{k+1} 2^{i} = \sum_{i=0}^{k} 2^{i} + 2^{(k+1)}$$

Now let's use the inductive hypothesis. Btw. if you do not use the inductive hypothesis, it is not an inductive proof.

$$\sum_{i=0}^{k} 2^{i} + 2^{(k+1)} = 2^{(k+1)} - 1 + 2^{k+1}$$

Let's rearrange a few things to get:

$$2^{(k+1)} - 1 + 2^{k+1} = 2 * 2^{(k+1)} - 1 = 2^{k+2} - 1$$

This completes the proof.

*Taking steps 1, 2 and 3 together, we have shown that the expression holds for all N >=0.*