For all n>=0: sum(i:0..n) 2^i = 2^(n+1) – 1

1) Prove the base case, i.e. the case of the smallest n. In this case: n = 0.
   Sum(i:0..0)2^i = 2^0 = 1
   2^(1)-1 = 1
   Yes, the base case holds.

2) State the inductive hypothesis. Suppose the expression hold for some arbitrary k. In other words: sum(i:0..k) 2^i = 2^(k+1)-1

3) Prove the inductive step. In other words, show that based on the inductive hypothesis, the expression holds for k+1.

   Write the expression for k+1:
   Sum(i:0..(k+1)) 2^i = 2^((k+1)+1) – 1

   Tease apart the summation:
   Sum(i:0..(k+1) 2^i = sum(i:0..k) 2^i + 2^(k+1)

   Use inductive hypothesis:
   sum(i:0..k) 2^i + 2^(k+1) = 2^(k+1)-1 + 2^(k+1) = 2*(2^(k+1)) – 1 = 2^(k+2) – 1.

For all N ≥ 3: 2N + 1 < N^2.

1) Base case: N = 3: 2*3+1 < 3^2. Yes.
2) Inductive hypothesis: Assume for arbitrary k: 2k+1 < k^2
3) Inductive step: Show that if 2k+1 < k^2 then 2(k+1)+1 < (k+1)^2
   2(k+1)+1 = 2k + 2 + 1 < k^2 + 2 < k^2 + 2k + 1, for k>=3

   Note: (k+1)^2 = k^2 + 2k + 1