It is also possible to calculate the imep based on this information.

$$imep = \frac{W_c}{V_d} = \frac{\eta_f m_f Q_{LHV}}{V_1 - V_2} = \frac{\eta_f m q^*}{V_1\left(1 - \frac{1}{r_c}\right)}$$

$$imep = \frac{\eta_f q^*}{\frac{V_1}{m}\left(1 - \frac{1}{r_c}\right)} = \frac{\eta_f q^*}{\frac{RT_1}{P_1}\left(\frac{r_c - 1}{r_c}\right)}$$

Now, $R = c_p - c_v = (\gamma - 1)c_v$

$$imep = \frac{\eta_f q^*}{\frac{c_v T_1}{P_1}(\gamma - 1)\left(\frac{r_c - 1}{r_c}\right)} = \left(\frac{q^*}{c_v T_1}\right)\left(\frac{1}{\gamma - 1}\right)\left(\frac{r_c}{r_c - 1}\right)\eta_f P_1$$

It's common to express this as a ratio, see eq. 5.32

$$\frac{imep}{P_1} = \left(\frac{q^*}{c_v T_1}\right)\left(\frac{1}{\gamma - 1}\right)\left(\frac{r_c}{r_c - 1}\right)\eta_f$$

It can also be shown that,

$$\frac{imep}{P_3} = \left(\frac{1}{(\gamma - 1)r_c}\right)\left(\frac{r_c}{r_c - 1}\right)\frac{1 - 1/r_c^{\gamma - 1}}{\left(\frac{c_v T_1}{q^*}\right) + 1/r_c^{\gamma - 1}}$$

A high value of this quantity is an indication of good design, by the way. Why? imep is good, the higher the better. P at 3 is bad, (it's the peak pressure), and the engine has to be designed mechanically to withstand it. Remember stress and metal fatigue!

Further thermodynamic calculations can be carried out. Here's what we find. (See pages 171-172)

First of all, the calculation of residual mass fraction.

Start with some thoughts about what happens at blowdown.

- Most of the gas leaves the cylinder through the exhaust valve.
- This exhaust gas exits to a pressure P_e.
- Each element of this gas exits with constant enthalpy. (We assume no heat transfer during the exit.)
- Each element of this gas attains a different temperature and entropy at the end of the exit. Exit process is irreversible.
- Gas left behind expands isentropically to fill the vacated space left in the cylinder.
- Next, the piston moves from BC to TC forcing a displacement of this gas. Additional gas leaves, but what is left behind has the same thermodynamic state that it had at the end of the blowdown process.

The residual mass m_r occupies the volume V_6 (at TC)
But the thermodynamic property v_6 is the same as v_5.

$$v_6 = \frac{V_6}{m_r} = v_5 = \frac{(1/r_c)}{m_r} V_4 \qquad \text{and } v_4 = \frac{V_4}{m}$$

$$v_5 = \frac{(1/r_c)}{m_r} m v_4 \qquad \text{solving} \qquad x_r = \frac{m_r}{m} = \frac{v_4}{r_c v_5}$$

Since there is an isentropic expansion of the residual gas between 4 and 5

$$\frac{v_4}{v_5} = \left(\frac{p_5}{p_4}\right)^{\frac{1}{\gamma}} = \left(\frac{p_e}{p_4}\right)^{\frac{1}{\gamma}}$$

So, since $P_1 = P_i$.

$$x_r = \frac{1}{r_c}\left(\frac{p_e}{p_4}\right)^{\frac{1}{\gamma}} = \frac{1}{r_c}\left(\frac{p_e}{p_i}\right)^{\frac{1}{\gamma}}\left(\frac{p_1}{p_4}\right)^{\frac{1}{\gamma}}$$

Next

$$\frac{p_1}{p_4} = \frac{p_1}{p_2}\frac{p_2}{p_3}\frac{p_3}{p_4}$$

Use $\frac{p_1}{p_2} = \frac{1}{r_c^{\gamma}}$ and $\frac{p_3}{p_4} = r_c^{\gamma}$ and also $\frac{p_2}{p_3} = \frac{T_2}{T_3}$

OK, now recall that

$$c_v(T_3 - T_2) = q^* \qquad T_3 - T_2 = \frac{q^*}{c_v}$$

$$\frac{T_3}{T_2} - 1 = \frac{q^*}{c_v T_2} \qquad T_2 = T_1 r_c^{\gamma-1}$$

$$\frac{T_3}{T_2} = 1 + \frac{q^*}{c_v T_1 r_c^{\gamma-1}}$$

Putting these pieces together gives the final result.

$$x_b = x_r = \frac{1}{r_c} \frac{(p_e/p_i)^{1/\gamma}}{\left[1 + \frac{q^*}{c_v T_1 r_c^{\gamma-1}}\right]^{\frac{1}{\gamma}}}$$

Secondly, it is possible to derive a relationship between the temperature T_1 and the temperature T_i.

The manipulations are quite tedious and will be passed over for now.

These two temperatures will be different, because of the residual gas

$$\frac{T_1}{T_i} = \frac{1 - x_r}{1 - \left(\frac{1}{\gamma r_c}\right)[p_e/p_i + (\gamma - 1)]}.$$

At this point we have completed the derivations related to the Otto cycle. See the EES document Otto1.

IMPORTANT:

Note that

$$m = m_a + m_f + m_r = m_a + m_f + x_r m$$

$$(1 - x_r)m = \left(\frac{A}{F} + 1\right)m_f$$

$$\frac{m_f}{m} = \frac{1 - x_r}{1 + \frac{A}{F}}$$

Pumping Work

If the engine is running unthrottled, there is no pumping work.

If there is throttling there will be pumping work.

$$W_p = (p_i - p_e)(V_1 - V_2)$$

If we do not include this work in our calculations, the work per cycle is called gross indicated work. If we do add the pumping work, what we will have is the net indicated work.

Exercise: (Could be a future test problem.)

I'm going to supply you with some pieces. It's your job to put these pieces together to fill in the blanks.

Given:

- Constant volume specific heat, c_v = 0.946 kJ/(kg K)
- $\gamma = 1.3$
- Fuel QLHV = 44000 kJ/kg
- Assume that the mass flow through each cylinder is 0.00045 kg/cycle.
- AF ratio = 16
- Temperature T_1 = 360 K.
- Intake pressure P_i = 50 kPa
- Temperature T_2 = 645.4 K.
- Temperature T_3 = 3451 K
- Exhaust Pressure P_e = 100 kPa

Find:

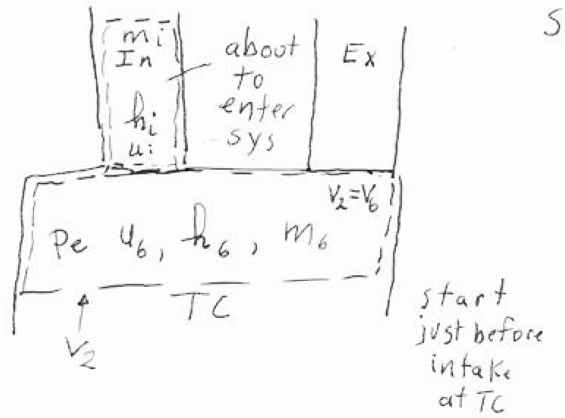
- P_1
- V_1
- P_2
- r_c
- V_2
- x_r
- P_3
- T_4
- P_4
- P_5
- T_i
- Gross indicated work per cycle per cylinder
- Net indicated indicated work per cycle per cylinder
- Pump work per cycle per cylinder
- Fuel Conversion Efficiency
- Imep, Imep/P_1, and Imep/P_3

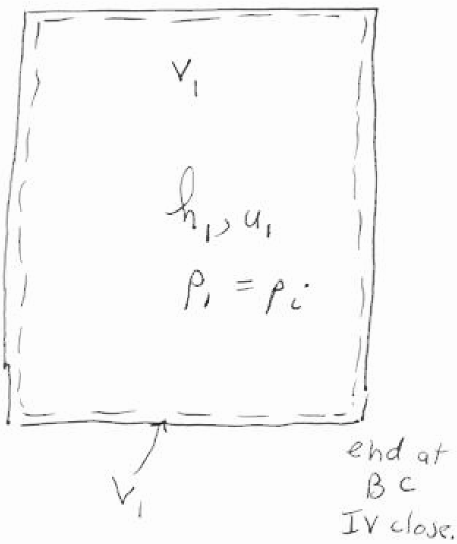
- What's the gross indicated power for the whole engine @ 2400 rpm if there are 4 cylinders?
- Also, find the pumping power.
- What's the net indicated power?
- What is the indicated specific fuel consumption? Base it on gross indicated power. Make your units micrograms/J.
- The density of the fuel is 0.75 kg/liter. The vehicle is travelling at 100 km/hr. Calculate the "mileage" in km/liters. Convert to miles per gallon if you desire.

The values you get are not 100% realistic, but they are in the ball park.

An EES file to solve this problem will appear at the website.

What follows is an appendix containing the derivations of some of the formulas we have used today.





System:

residual mass, m_6 trapped in cyl. ⊕

m_i - mass of mixture to be inducted $m_i = m_1 - m_6$

Assume: Adiabatic

$$\Delta U = \cancel{Q} - W + m_i h_i$$

$$m_1 u_1 - m_6 u_6 = (m_1 - m_6) h_i - p_i (V_1 - V_6)$$

$$m_1 u_1 + p_1 V_1 = m_6 u_6 + p_e V_6 \overset{\text{add \& subtract}}{-} p_e V_2 + (m_1 - m_6) h_i + p_i V_2$$

$$\underline{m_1 h_1 = m_6 h_6 + (m_1 - m_6) h_i + V_2 (p_i - p_e)}$$

W is p_i working through V_6 to V_1.

Derivation of 5.36

$$\frac{p_6 V_6}{p_1 V_1} = \frac{m_6 R T_6}{m_1 R T_1}$$

$$\frac{p_6}{p_1 r_c} = x_r \frac{T_6}{T_1} \qquad \text{but } p_6 = p_e \quad p_1 = p_i$$

$$\frac{T_6}{T_1} = \frac{1}{r_c x_r} \frac{p_e}{p_i} \qquad \text{recall } x_r = \frac{1}{r_c} \frac{(p_e/p_i)^{\frac{1}{\gamma}}}{[1 + Q^*/(c_v T_1 r_c^{\gamma-1})]^{1/\gamma}}$$

$$\frac{T_6}{T_1} = \frac{p_e/p_i}{r_c \cdot \frac{1}{r_c} \frac{(p_e/p_i)^{\frac{1}{\gamma}}}{[1 + q^*/(c_v T_1 r_c^{\gamma-1})]^{\frac{1}{\gamma}}}}$$

so

$$\frac{T_6}{T_1} = \left(\frac{p_e}{p_i}\right)^{1-\frac{1}{\gamma}} \left(1 + \frac{q^*}{c_v T_1 r_c^{\gamma-1}}\right)^{\frac{1}{\gamma}}$$

Derivation of 5.38

It's a single fluid!!! Start with h

$$m_1 h_1 = m_6 h_6 + (m_1 - m_6) h_i + V_2 (p_i - p_e) \qquad V_2 = V_1/r_c$$

$$m_1 c_p T_1 = x_r m_1 c_p T_6 + m_1 (1 - x_r) c_p T_i + p_i V_2 \left(1 - \frac{p_e}{p_i}\right)$$

$$m_1 c_p T_1 = x_r m_1 c_p T_6 + m_1 (1 - x_r) c_p T_i - \frac{m_1 R T_1}{r_c}\left(\frac{p_e}{p_i} - 1\right)$$

$$c_p T_1 = x_r c_p T_6 + (1 - x_r) c_p T_i - \frac{c_p (1 - \frac{1}{\gamma}) T_1}{r_c}\left(\frac{p_e}{p_i} - 1\right)$$

divide by T_1

$$1 = x_r \frac{T_6}{T_1} + (1 - x_r)\frac{T_i}{T_1} - \frac{\gamma - 1}{\gamma r_c}\left(\frac{p_e}{p_i} - 1\right)$$

recall that $\frac{T_6}{T_1} = \frac{1}{r_c x_r}\frac{p_e}{p_i}$

$$1 = x_r \left(\frac{1}{r_c x_r}\right)\frac{p_e}{p_i} + (1 - x_r)\frac{T_i}{T_1} - \frac{1}{r_c}\frac{p_e}{p_i} + \frac{1}{r_c} + \frac{1}{\gamma r_c}\frac{p_e}{p_i} - \frac{1}{\gamma r_c}$$

$$1 - \frac{1}{\gamma r_c}\frac{p_e}{p_i} - \frac{\gamma}{\gamma r_c} + \frac{1}{\gamma r_c} = (1 - x_r)\frac{T_i}{T_1}$$

$$\frac{T_i}{T_1} = \frac{1 - \frac{1}{\gamma r_c}\left(\frac{p_e}{p_i} + \gamma - 1\right)}{1 - x_r}$$

or

$$\frac{T_1}{T_i} = \frac{1 - x_r}{1 - \frac{1}{\gamma r_c}\left(\frac{p_e}{p_i} + \gamma - 1\right)}$$