ME 410 Day 20

Topics

- Review of Otto Cycle
- Review of Constant Pressure and Limited Pressure
- Examples

Constant Volume Combustion. (Otto Cycle)

- Unthrottled
- Ideal Gas, Constant Specific Heat Ratio γ
- Compression Ratio, $r_c = V_1 / V_2$

1-2:

Start with T_1 and P_1.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad \text{or} \quad \frac{T_2}{T_1} = (r_c)^{\gamma-1} \quad T_1 = T_2\, r_c^{1-\gamma}$$

ideal gas law: $P_1 v_1 = RT_1$ and $P_2 v_2 = RT_2$

$$r_c = \frac{v_1}{v_2}$$

2-3:

Let m be the mass of the mixture, and m_f the mass of the fuel.

$q^* = \frac{m_f}{m} Q_{LHV}$ is the heat released during isothermal combustion, per mass of working fluid.

$$c_v\left(T_3 - T_2\right) = q^*$$

$$v_3 = v_2 \text{ and } \frac{P_3}{P_2} = \frac{T_3}{T_2}$$

3-4

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \text{ or } \frac{T_3}{T_4} = (r_c)^{\gamma-1} \quad T_4 = T_3\, r_c^{1-\gamma}$$

ideal gas law: $P_3 v_3 = RT_3$ and $P_2 v_2 = RT_2$

$$r_c = \frac{v_1}{v_2}$$

4-1:

$$v_4 = v_1$$

Heat transfer to exhaust: $q_{out} = c_v\left(T_4 - T_1\right)$

We assume that there is a constant volume heat rejection, and we should be back to starting point.

Calculations based on these processes

Specific work per cycle: $q^* - q_{out} - w = \Delta u = 0$

$$w = q^* - q_{out} = c_v(T_3 - T_2) - c_v(T_4 - T_1)$$

Now let us consider the fuel conversion efficiency

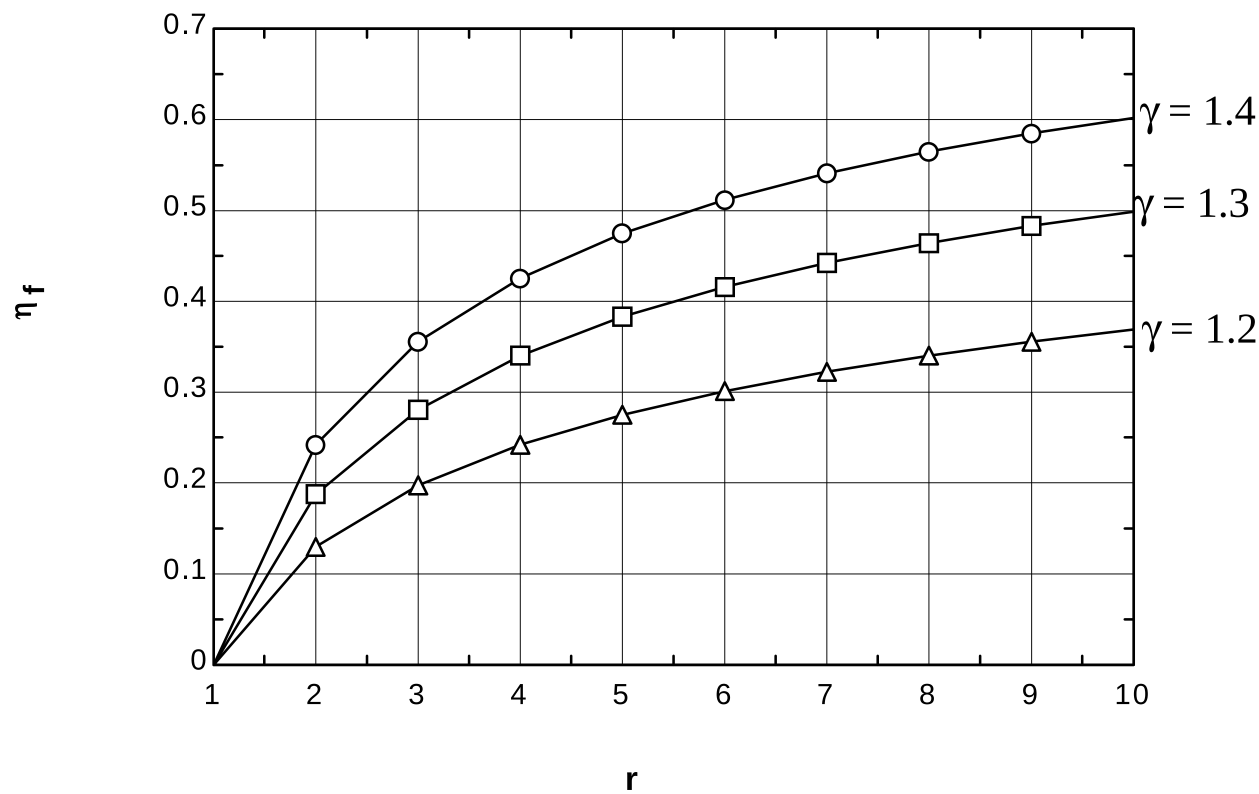
$$\eta_f = \frac{w}{q^*} = \frac{c_v(T_3 - T_2) - c_v(T_4 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

Next, we bring back the temperature relationships.

$$\eta_f = 1 - \frac{r_c^{1-\gamma}\, T_3 - r_c^{1-\gamma}\, T_2}{T_3 - T_2} = 1 - r_c^{1-\gamma}$$

This simple result gives the fuel conversion efficiency in terms of two things:

- specific heat ratio of the working fluid
- the compression ratio of the engine



It is also possible to calculate the imep based on this information.

$$imep = \frac{W_c}{V_d} = \frac{\eta_f m_f Q_{LHV}}{V_1 - V_2} = \frac{\eta_f m q^*}{V_1\left(1 - \frac{1}{r_c}\right)}$$

$$imep = \frac{\eta_f q^*}{\frac{V_1}{m}\left(1 - \frac{1}{r_c}\right)} = \frac{\eta_f q^*}{\frac{RT_1}{P_1}\left(\frac{r_c - 1}{r_c}\right)}$$

Now, $R = c_p - c_v = (\gamma - 1)c_v$

$$imep = \frac{\eta_f q^*}{\frac{c_v T_1}{P_1}(\gamma - 1)\left(\frac{r_c - 1}{r_c}\right)} = \left(\frac{q^*}{c_v T_1}\right)\left(\frac{1}{\gamma - 1}\right)\left(\frac{r_c}{r_c - 1}\right)\eta_f P_1$$

It's common to express this as a ratio, see eq. 5.32

$$\frac{imep}{P_1} = \left(\frac{q^*}{c_v T_1}\right)\left(\frac{1}{\gamma - 1}\right)\left(\frac{r_c}{r_c - 1}\right)\eta_f$$

It can also be shown that,

$$\frac{imep}{P_3} = \left(\frac{1}{(\gamma - 1)r_c}\right)\left(\frac{r_c}{r_c - 1}\right)\frac{1 - 1/r_c^{\gamma - 1}}{\left(\frac{c_v T_1}{q^*}\right) + 1/r_c^{\gamma - 1}}$$

A high value of this quantity is an indication of good design, by the way. Why? imep is good, the higher the better. P at 3 is bad, (it's the peak pressure), and the engine has to be designed mechanically to withstand it. Remember stress and metal fatigue!

Further thermodynamic calculations can be carried out. Here's what we find. (See pages 171-172)

First of all, the calculation of residual mass fraction.

$$x_b = x_r = \frac{1}{r_c}\frac{(p_e/p_i)^{1/\gamma}}{\left[1+\frac{q^*}{c_v T_1 r_c^{\gamma-1}}\right]^{\frac{1}{\gamma}}}$$

Secondly, it is possible to derive a relationship between the temperature T_1 and the temperature T_i. These will be different, because of the residual gas

$$\frac{T_1}{T_i} = \frac{1-x_r}{1-\left(\frac{1}{\gamma r_c}\right)\left[p_e/p_i + (\gamma-1)\right]}.$$